Improved Solution Techniques for Multi-Period Area-Based Forest Harvest Scheduling Problems

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1 Introduction

Mathematical modeling has been frequently used for harvest schedule planning. This has allowed several regulations and requirement to be incorporated in the planning process. These regulations are generally incorporated as restrictions to a Linear Integer Programming model and often make the problem more difficult to solve.

Regulations limiting spatial disturbances have lead to restrictions typically know as maximum area restrictions, which limit the size of clear cut areas (Thompson et al.1973, Murray 1999). Several models using these restrictions have been proposed over the years, but the model known as the Area Restriction Model (ARM) has been shown to deliver the most profitable harvest schedules (Murray and Weintraub 2002). Unfortunately the ARM has proven to be very difficult to solve computationally. Although several heuristics to solve this model have been proposed (Hokans 1983, Lockwood and Moore 1993, Barrett et al. 1998, Clark et al. 1999, Richards and Gunn 2000, Boston and Bettinger 2001), exact methods have only recently been able to solve small and medium sized instances of this problem. One such method is that developed in Goycoolea et. al. (2003), who focused on a strengthened formulation know as the Cluster Packing Problem. They were able to solve modest sized problems using a commercial integer programming solver for a single period instance.

The model's solvability properties are generally preserved for multi-period instances, but by only adding volume production restrictions the problem becomes very difficult to solve.

In this work we present an alternative way of structuring volume restrictions in order to restore most of the favorable properties of the single period Cluster Packing Problem. Application results are presented which demonstrate that near optimal solutions can be obtained quickly using the developed modeling approach.

2 Cluster Packing Harvest Scheduling Model

2.1 Single and Multi-Period Model without volume constraints

Forest information is generally obtained from GIS generated maps. These maps partition a forest region into small units for which area, volume and harvest profit information is available.

The area of each unit is generally smaller that the maximum clear cut size specified for the Maximum Area Restrictions, so some groups of adjacent units may be harvested together.

We will define the set of Feasible Clusters (Λ) as all groups of adjacent units whose combined area doesn't exceed the maximum clear cut size. For this definition we will generally consider two units to be adjacent if they share an edge in the forest map.

We will say that two clusters are incompatible is they share a unit or if they are adjacent. Forbidding the simultaneous harvesting of incompatible clusters will assure compliance with the maximum clear cut restrictions. For this definition we will generally consider two clusters to be adjacent if they share an edge or a vertex in the forest map.

With these definitions, to comply with the Maximum Area Restrictions we only need to forbid two incompatible clusters to be harvested at the same time. This restriction is strengthened further in Goycoolea et. al. (2003) by replacing it by restrictions based on maximal cliques in the forest map. These restrictions give the formulation integrality properties that make it very easy to solve. Almost all instances of the single period problem are solved to optimality in the root *Branch & Bound (B&B)* node by CPLEX 8.1.

The multi-period version of the model allows harvesting over several periods, but only allows each unit to be harvested once in the planning horizon.

The following formulation shows the multi-period model with the strengthened incompatible cluster restrictions.

Indices

- s = Feasible Clusters: $1,\ldots,|\Lambda|$.
- t = Time Periods: $1,\ldots,T$.

Index Sets

- $\Lambda(K)$ = Clusters that intersect clique K.
- $\Lambda(u)$ = Clusters that intersect unit u.

Parameters

- c_{st} = net present value of profit obtained if cluster s is harvested in period t.

Decision variables

- x_{st} = 1 if cluster S is harvested in period t.

Model *CPPT*

1) Minimise $\sum_{t=1}^{T} \sum_{s=1}^{|\Lambda|} c_{st}\, x_{st}$..
2) $\sum_{s \text{ in } \Lambda(K)} x_{st} \leq 1$ for $t = 1,\ldots,T$.
3) $\sum_{t=1}^{T} \sum_{s \text{ in } \Lambda(u)} x_{st} \leq 1$.
4) x_{st} in $\{0,1\}$.

Explanation

1) Objective is to maximize net present value of profit
2) Cannot harvest two incompatible clusters in the same period. Modelled with clique constraints .
3) Each unit u may only be harvested once.
4) Variables x_{st} are binary.

This formulation preserves most of the good properties of the single period formulation and is easily solvable, as the computational results will show.

2.2 Volume Restrictions

The multi-period model can be complemented with different kinds of restrictions over the volume harvested in each period.

One typical restriction over the harvested volume is to ask that the volume harvested in a period is within ±Δ% of the volume harvested in the previous period. This can be achieved by adding the following restrictions to the multi-period model for each time period t>1:

$$(1-(\Delta/100)) \sum^{|\Lambda|}_{s=1} v_{s(t-1)}\, x_{s(t-1)} \leq \sum^{|\Lambda|}_{s=1} v_{st}\, x_{st} \leq (1+(\Delta/100)) \sum^{|\Lambda|}_{s=1} v_{s(t-1)}\, x_{s(t-1)}$$

Where v_{st} the volume harvested if cluster s is selected to be harvested in period t.

If we add these restrictions to model *CPPT* we obtain the strict volume constraint model *CPPT-V.*

It is usual that one of the inequalities is active and hence acts as a fractional generating cut on the LP polytope. As the computational results will show this makes the problem very difficult to solve.

3 Elastic Volume Constraint Method

One technique that can be used to eliminate most of the fractional generating effects of the volume constraints is to use an elastic version of the constraints. The elastic constraint allows a violation of the restriction, but penalizes this violation in the objective function. In this manner the volume constraints will no longer act as cuts and hence will not generate new fractional extreme points to the LP polytope. This will restore practically all the integrality properties of the multi-period model without volume constraints. Elastic constraints have been successfully used in similar problem like in Ehrgott and Ryan (2003).

It is very difficult to find penalties that will lead to *integer* solutions that don't violate the volume restrictions. For this reason it is a good idea to start penalizing before the restrictions are really violated. So, for example, if we wanted to solve the problem with ±15% non-decreasing volume constraints (i.e. with Δ=15% in the original model), we could add a ±14% non-decreasing volume constraint (i.e. with Δ_E=14% in the elastic models), allow violations to these ±14% constraints and penalize the violation in the objective function. In this way, if we just keep the violations controlled (bellow 1%) instead of non-existing we will be complying with our target 15% volume constraint.

3.1 Elastic Volume Constraint Model

In the following section we will describe the elastic type volume constraints.

Indices

s = Feasible Clusters: 1,…,|Λ|.

t = Time Periods: 1,…,T.

Index Sets

$\Lambda(K)$ = Clusters that intersect clique K.

$\Lambda(u)$ = Clusters that intersect unit u.

Parameters

c_{st} = net present value of profit obtained if cluster s is harvested in period t

v_{st} = the volume harvested if cluster s is selected to be harvested in period t.

l_t, u_t = penalties for violating the volume restrictions.

Decision variables

x_{st} = 1 if cluster S is harvested in period t.

l_t = lower volume constraint violations.

u_t = upper volume constraint violations.

Model *CPPT-EV*

1) Minimise $\sum^{T}_{t=1} \sum^{|\Lambda|}_{s=1} c_{st}\, x_{st} - \sum^{T}_{t=2} p_t l_t - \sum^{T}_{t=2} q_t u_t.$
2) $\sum_{s \text{ in } \Lambda(K)} x_{st} \leq 1$ for $t = 1,\ldots,T$.
3) $\sum^{T}_{t=1} \sum_{s \text{ in } \Lambda(u)} x_{st} \leq 1$.
4) $(1-(\Delta_E/100)) \sum^{|\Lambda|}_{s=1} v_{s(t-1)}\, x_{s(t-1)} - \sum^{|\Lambda|}_{s=1} v_{st}\, x_{st} \leq l_t$ for $t = 2,\ldots,T$.
5) $\sum^{|\Lambda|}_{s=1} v_{st}\, x_{st} - (1+(\Delta_E/100)) \sum^{|\Lambda|}_{s=1} v_{s(t-1)}\, x_{s(t-1)} \leq u_t$ for $t = 2,\ldots,T$.
6) x_{st} in $\{0,1\}$.
7) $l_t, u_t \geq 0$.

Explanation

1) Objective is to maximize net present value of profit minus the penalties for violating volume constraints
2) Cannot harvest two incompatible clusters in the same period. Modelled with clique constraints .
3) Each unit u may only be harvested once.
4) Variables l_t will measure the lower volume constraint violations
5) Variables u_t will measure the upper volume constraint violations
6) Variables x_{st} are binary.
7) Variables l_t, u_t are positive.

3.2 Integer Allocation

Although penalties can be easily adjusted to control volume constraint violations for the root B&B node it might be very difficult to do this for integer solutions. General purpose LP based heuristics tend to have problems generating solutions with small volume constraint violations. For this reason a custom integer allocation heuristic was developed.

The heuristic simply fixes variables and re-solves the linear relaxation of the model while trying to correct any violations that are too big.

The elastic volume constraints are crucial for the performance of the heuristic. The fractional generating effect of the volume constraints makes it very difficult to develop an LP based heuristic for the strict volume constraint model. Fixing some fractional variables to integrality in this model generally ends in the appearance of an alternate set of fractional variables, making the integer allocation process very slow. This doesn't happen with the elastic constraint model as the fractional generating effect of the strict volume constraints is not present in this model. On the other hand, if the penalties are big enough the violations will probably be reasonably controlled. Some corrections of the violations are still necessary, but thanks to the penalties they are very few.

4 Computational Results

Computational tests were run over two instances. A real forest in Northern California called El dorado and a randomly generated square grid with 12 units in each side. Table 1 shows a summary of the instances characteristics.

Instance	# of Cells	# of Feasible Clusters	# of Strangthened Adjacency Restrictions	Total # of restrictions for 15 period model withouth volume constraints	Total # of variables for 15 period model withouth volume constraints
Eldorado	1363	21412	2105	32938	321180
rand12by12t15	144	2056	121	1959	30840

Table 1. Instance Characteristics

Multi-period models for 12 and 15 periods where tested for both instances and the runs were made in a Pentium 4 2.0Ghz PC with 2.0 Gb of RAM with Linux. CPLEX 8.1 was used as a MIP solver and problem generation and heuristics were programmed in C++.

4.1 Multi-period model without volume constraints

Map	Time Periods	IP Time [s]	B&B Nodes	GAP [%]	1st sol under 1% Time [s]	1st Feasible Time [s]	1st Feasible GAP [%]
Eldorado15	12	720	30	Optimal	448	173	6.58
Eldorado15	15	147	0	Optimal	101	77	3.02
ran12by12	12	501	789	Optimal	24	14	33.61
ran12by12	15	524	732	Optimal	32	19	38.32

Table 2. Multi-period model without volume constraints results

The integrality properties of this model help CPLEX 8.1 find solutions early and also to declare optimality very quickly.

4.2 Non-decreasing volume constraint model

Table 3 shows the results for the non-decreasing volume constraint model as solved directly by CPLEX 8.1. All tests for this table were run for 8 hours.

Map	Time Periods	Δ	LP Solution	IP Time	B&B Nodes	Best Solution Time [s]	GAP [%]	1st sol under 1% GAP [s]	1st Feasible Time [s]	1st Feasible GAP
Eldorado15	12	10%	6292354	28800	2133	18606	1.47	-	18606	1.47
Eldorado15	15	10%	6812231	28800	1575	18315	0.83	10839	10839	1.00
ran12by12	12	10%	17959328	28800	388	-	-	-	-	-
ran12by12	15	10%	23071854	28800	394	-	-	-	-	-
Eldorado15	12	15%	6413097	28800	2087	11211	0.50	10719	2323	1.51
Eldorado15	15	15%	6931024	28800	2067	20733	0.59	20274	20274	0.77
ran12by12	12	15%	18970483	28800	634	-	-	-	-	-
ran12by12	15	15%	25012357	28800	342	-	-	-	-	-

Table 3 Volume Constraint Model Results

It can bee seen that CPLEX has a lot of trouble finding integer solutions. Although eventually it does find good solutions for Eldorado it does take plenty of time, furthermore, no integer solutions are found for the grid.

4.3 Non-decreasing elastic volume constraint method

Table 4 shows the results for the elastic constraint based method. This method is essentially *B&B* over the multiple penalties elastic constraint model with constraint branching plus the integer allocation heuristic ran on each *B&B* node. Penalties for each constraint are set independently so that the root LP has less than 1% violation (again we use Δ_E=14% and allow only 1% of violation to solve the exact volume constraint with Δ=15%), but they are then kept fixed along the *B&B* tree. The GAP's are calculated with respect to the LP solution of the corresponding exact Δ% volume constraint model, so they can be compared to the GAP's in table 3. Solutions for Δ=10% using Δ_E=9% are also presented.

Map	Time Periods	Δ	IP Time	B&B Nodes	Best Solution Time [s]	GAP [%]	1st sol under 1% GAP [s]	1st Feasible Time [s]	1st Feasible GAP
Eldorado15	12	0.10	14400	23	5555	0.41	1706	1706	0.43
Eldorado15	15	0.10	14400	13	12541	0.44	4307	4307	0.45
ran12by12	12	0.10	14400	75	5059	3.43	-	663	8.70
ran12by12	15	0.10	14400	25	13856	4.52	-	614	14.42
Eldorado15	12	0.15	14400	18	12216	0.30	1160	1160	0.33
Eldorado15	15	0.15	14400	13	9916	0.29	2387	2387	0.34
ran12by12	12	0.15	14400	199	9684	2.29	-	312	5.07
ran12by12	15	0.15	14400	20	9124	4.97	-	504	7.99

Table 4 Elastic Volume Constraint Method Results

With these methods we can get good solutions fast for Eldorado and we can also get integer solutions for the grid quickly, although their quality is not that good. It should be noted though, that the grids where purposefully generated so that it was very difficult to get integer solutions that comply with the volume constraints tightly, so the are very strong suspicions that there is a big GAP between the IP and LP solutions to the grids.

If we compare these results with Table 3 we can see that the elastic constraint method is much faster than CPLEX over the strict volume constraint model. Integer feasible solutions with similar objective values are found up to 150 times faster with this method (CPLEX was run for 24 hours over the strict volume constraint model for the ran12by12 instance with 15 periods. Only one solution with 9% GAP was found after 22 hours).

5 Conclusions

By eliminating the fractional generating effect of the strict volume constraints it is much easier to obtain integer feasible solutions from solutions to the lineal relaxation of the model. For this reason the elastic constraint method allows good solutions to be obtained much earlier than when solving the strict volume constraint model directly.

It should be noted tough that restrictions over the harvested volume are generally guides instead of strict requirements, so small violations would be acceptable. It is clear that allowing these small violations (for example by allowing violations slightly over 1% of the 14% volume constraint in the computational results) will give even better results. This leads to a double reason for not using strict volume constraints, namely that they make the problem very difficult and that it is not necessary for them to be strict.

During the computational results it was found that the integer allocation heuristic worked better when the initial LP solved had little or no violations of the target volume constraints. Because of this, it might be useful to adjust penalties each time a volume restriction is violated along the *B&B* tree. This would also guaranty that integer solutions found in leafs of the *B&B* tree would comply with the target volume constraints. We are currently implementing this dynamic adjustment of penalties to be added to the *B&B* based integer allocation method.

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