Integer Programming Approaches for Imposing Connectivity in Forest Management

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SSAFR, 2011 – Maitencillo, Chile

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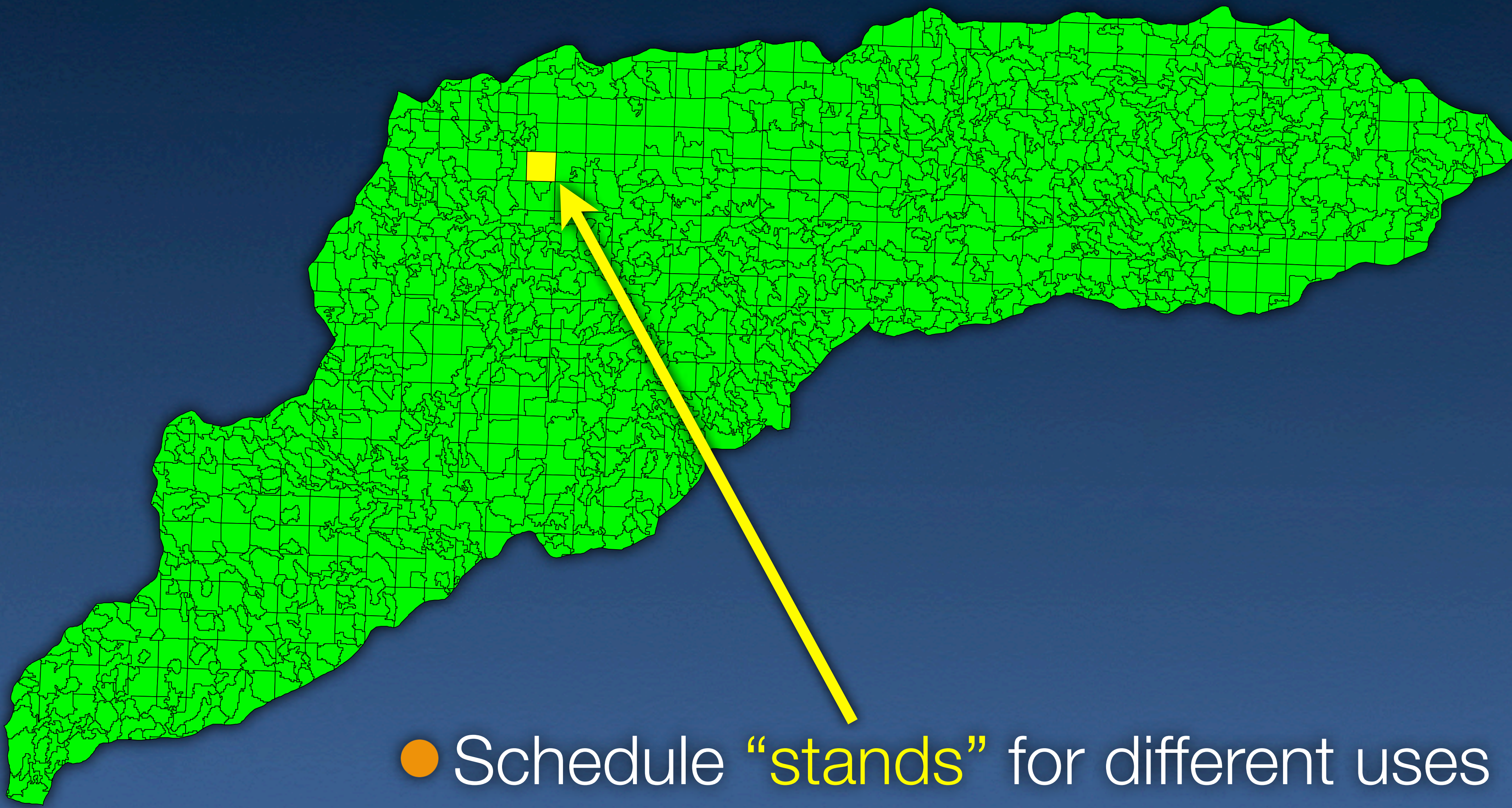
Andres Weintraub
University of Chile

Outline

- Introduction
- IP Models
- Connectivity in Forestry
  - Models
  - Computation
- Final Thoughts

Forest Management

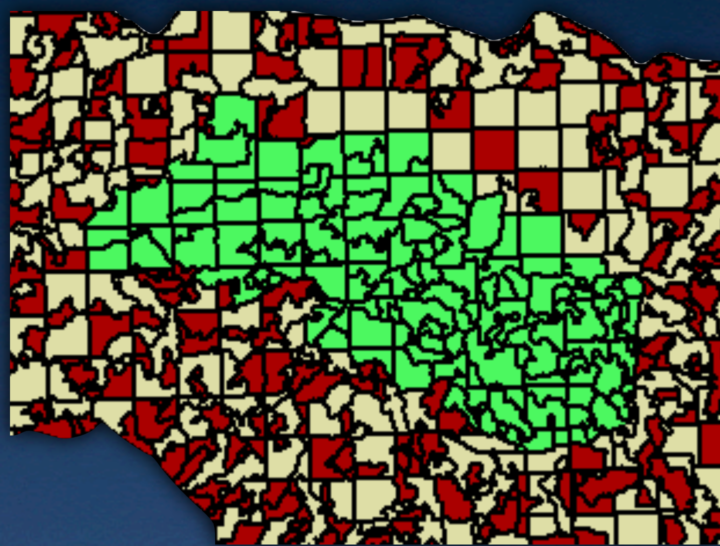
Forest Management



- Schedule "stands" for different uses

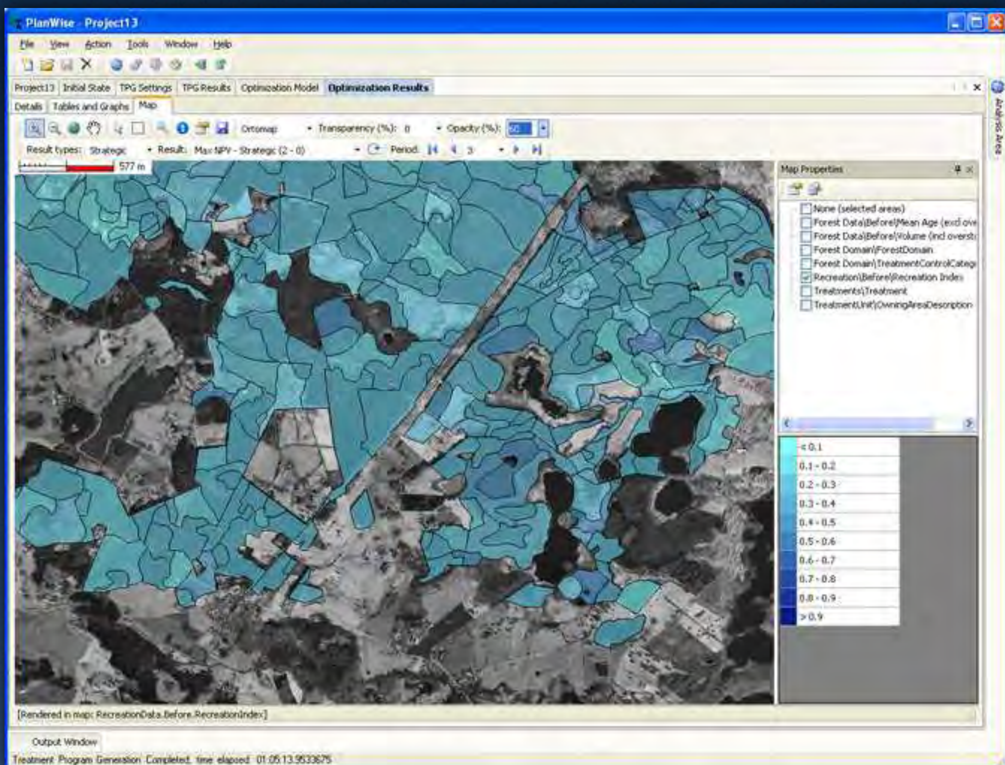
Connectivity in Forestry

- Clearcut size constraints
- Old growth patches
- Reserve selection
- Wildlife corridors



Where are we?

☑ Clearcut constraint:



Peder Wikström

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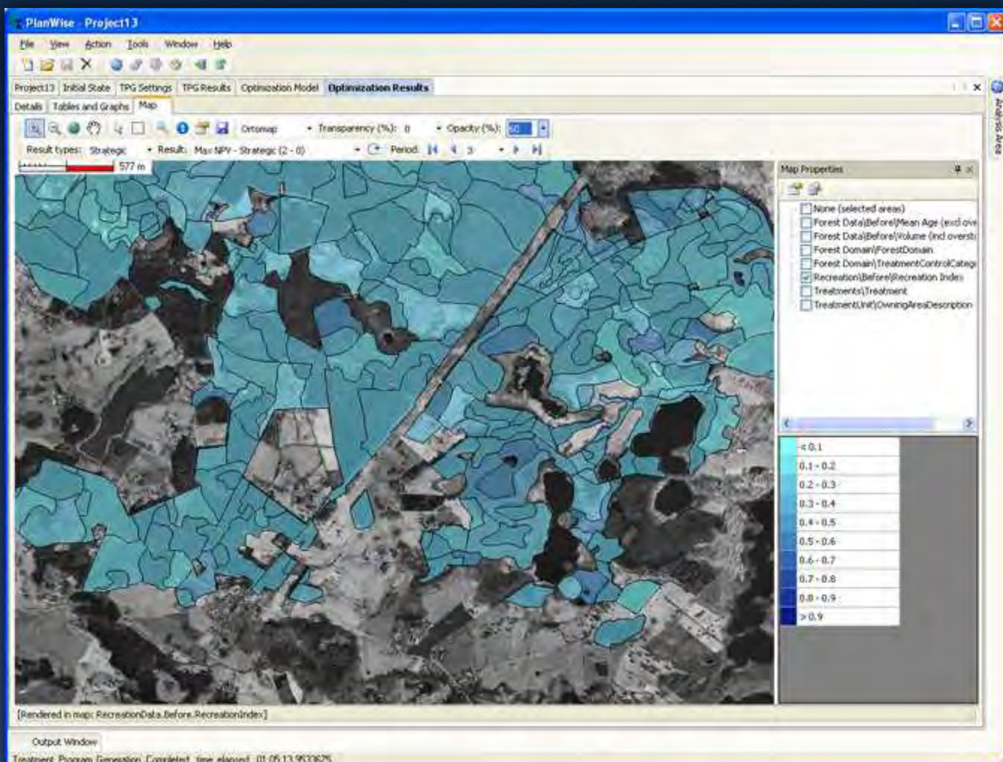
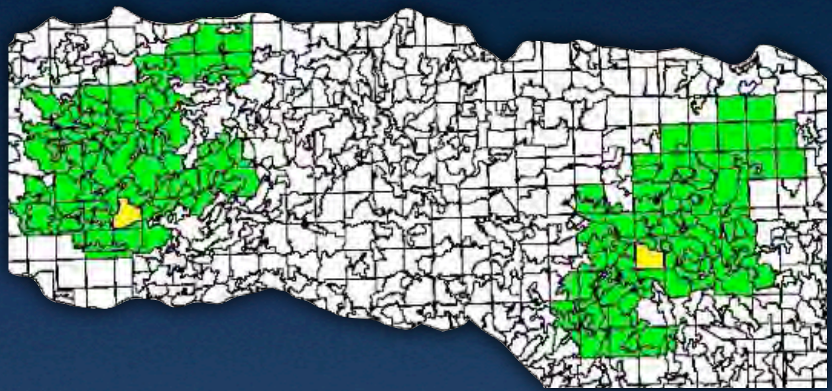


heureka!

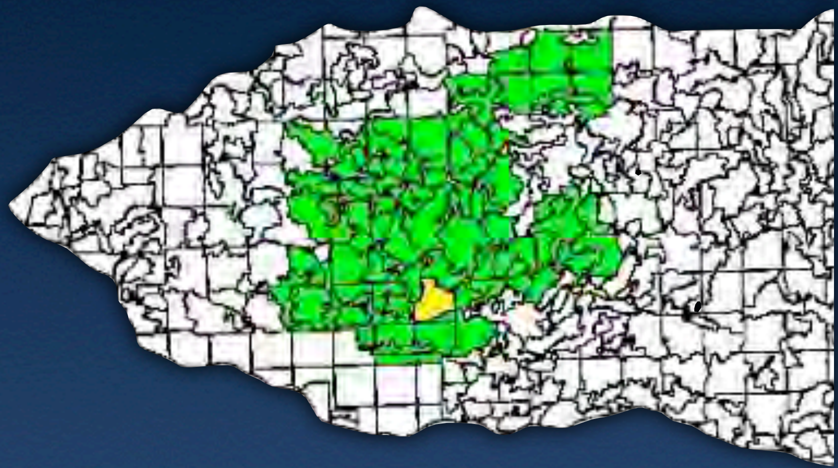
Peder Wikström

☐ Other connectivity?

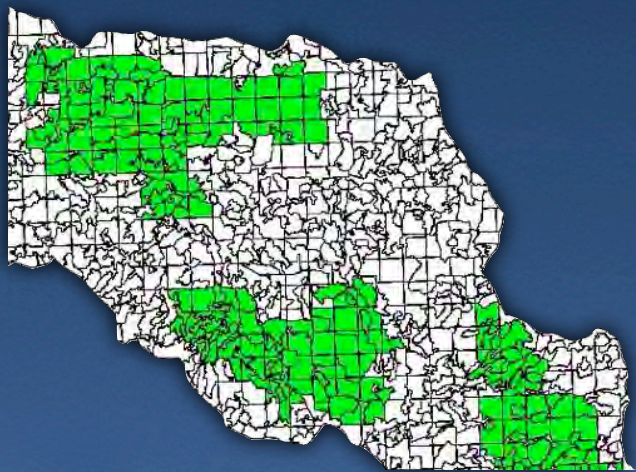
Types of Connectivity 1



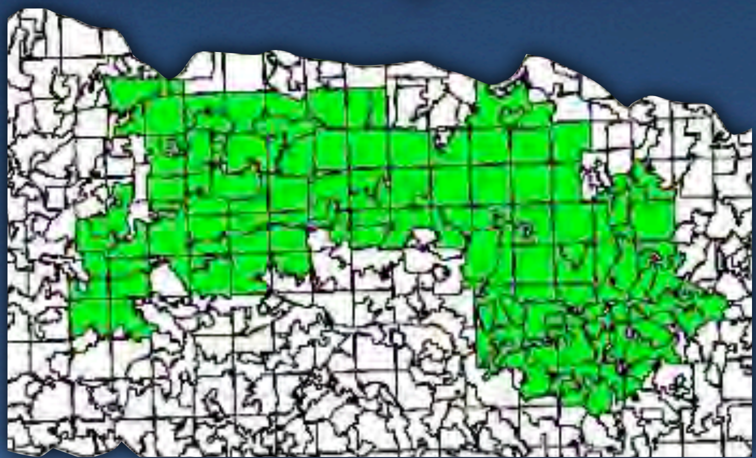
- Rooted Multi-Patch



- Rooted Single Patch



- Unrooted Multi-Patch



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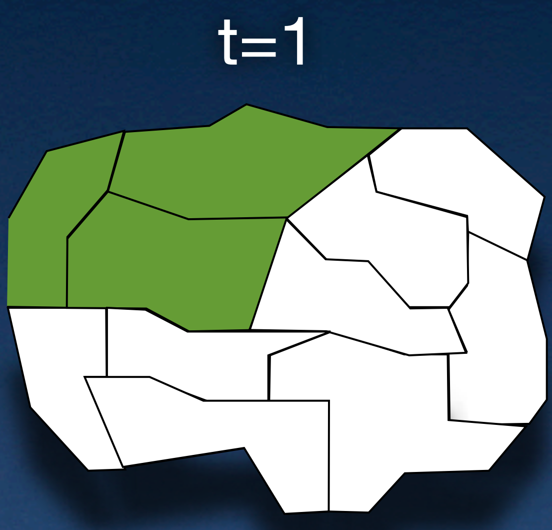
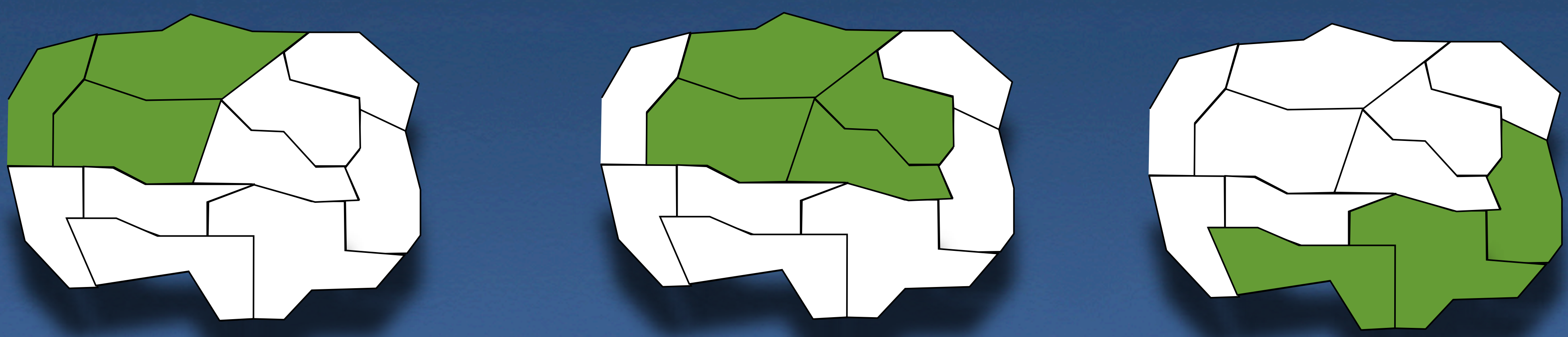
Types of Connectivity 2

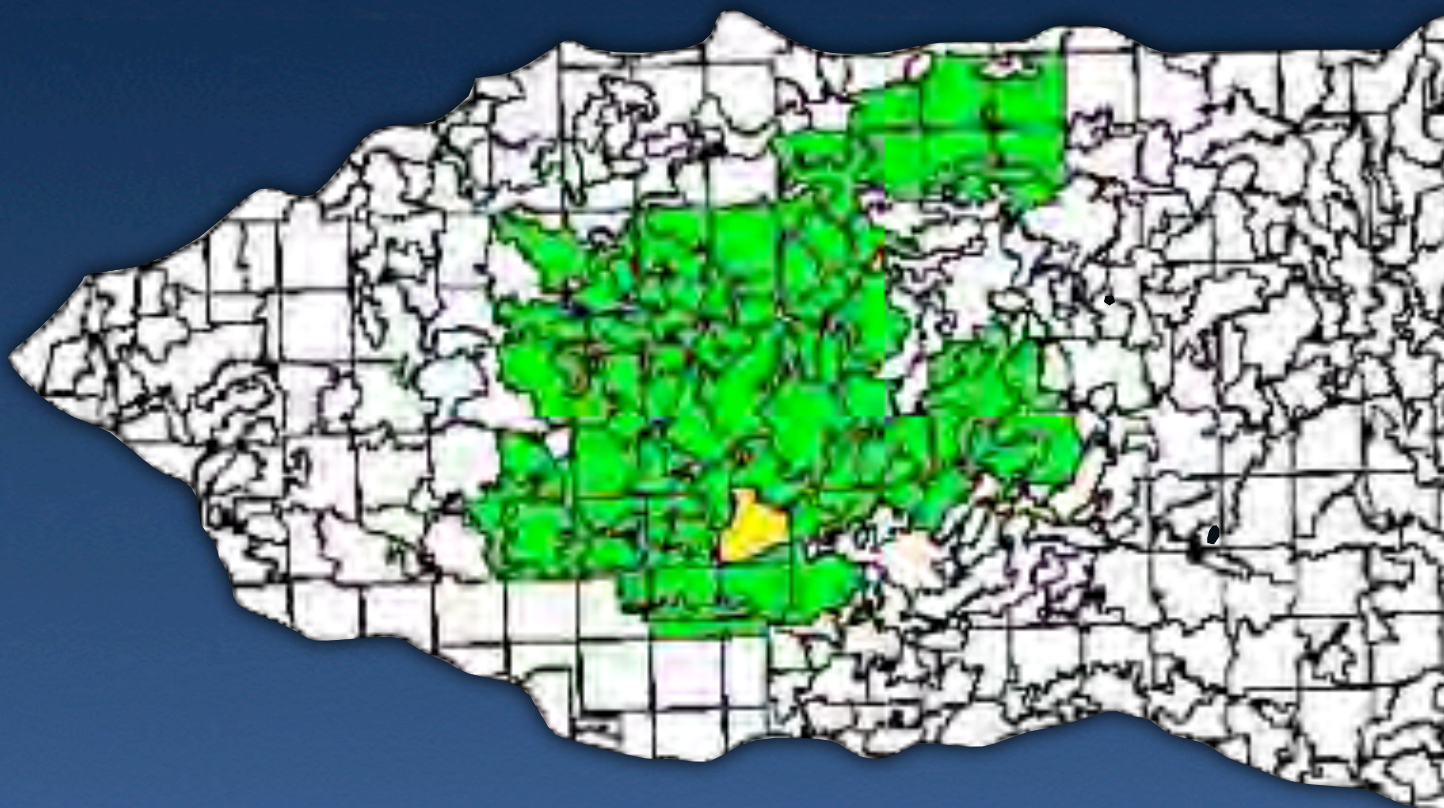


- Static Patch

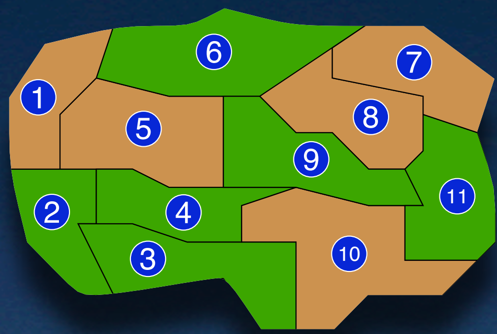


- Dynamic Patch

Today: Rooted Single Patch



IP Models for Forest Management



$$y_{v,t} = \begin{cases} 1 & \text{if stand } v \text{ is harvested} \\ & \text{in period } t. \\ 0 & \text{otherwise} \end{cases}$$

$$z_{v,t} = \begin{cases} 1 & \text{if stand } v \text{ is old-growth} \\ & \text{or reserve in period } t \\ 0 & \text{otherwise} \end{cases}$$

$$z_{v,t} + y_{v,t} \leq 1 \quad \forall t, v$$

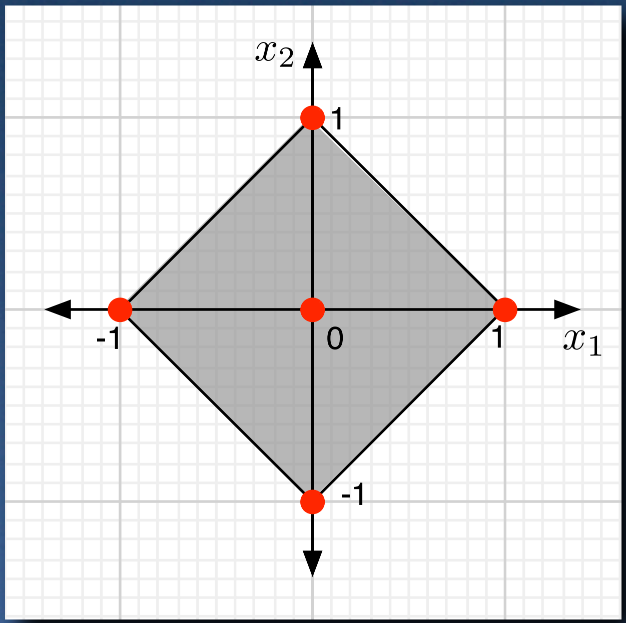
$\{v : z_{v,t} = 1\}$ is connected $\forall t$

- Linear Constraints/Objective:
  - Profits, timber flow, ending age of forest, etc.

IP Models?

Two Types of (Strongest) IP Models

$$S := \left\{ x \in \mathbb{Z}^n \ : \ \sum_{i=1}^{n} |x_i| \leq 1 \right\}$$



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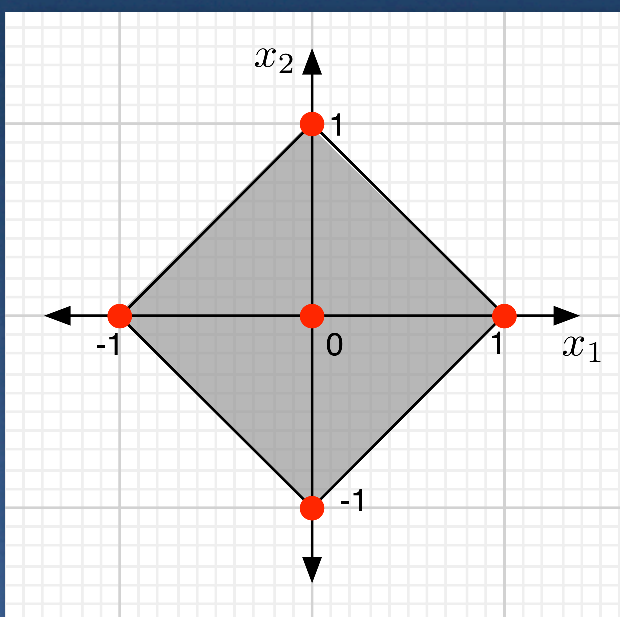

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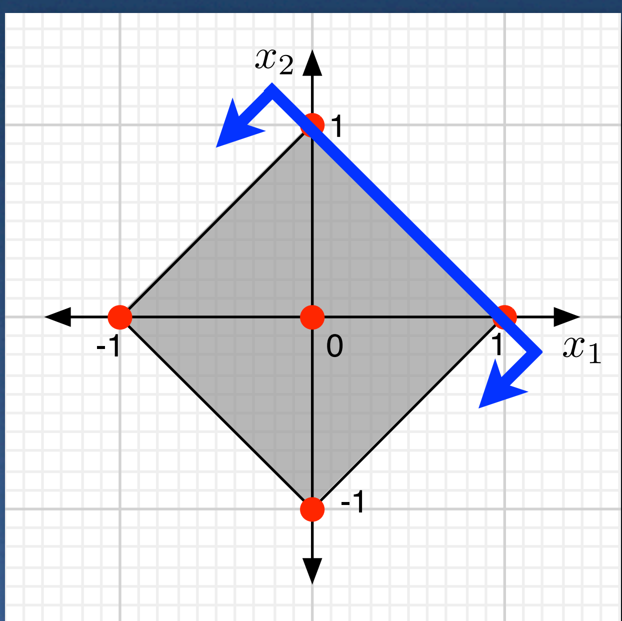

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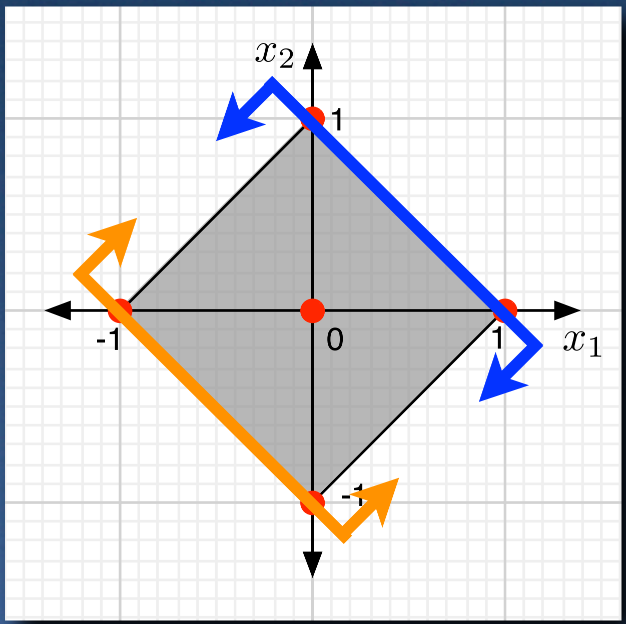

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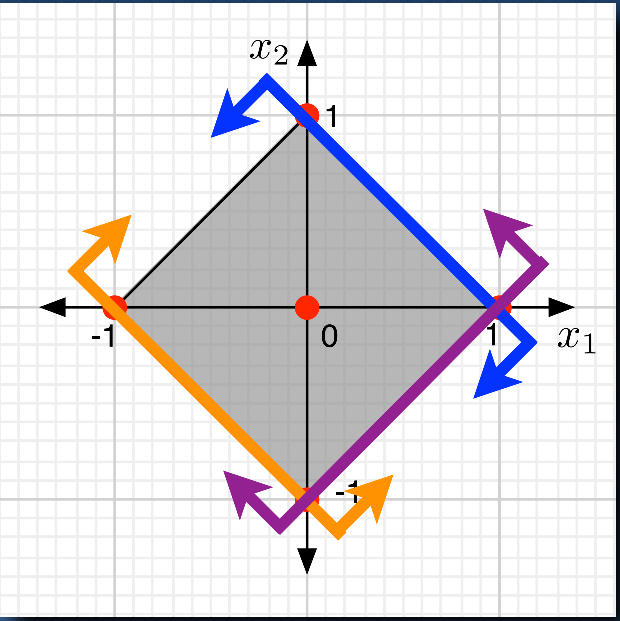

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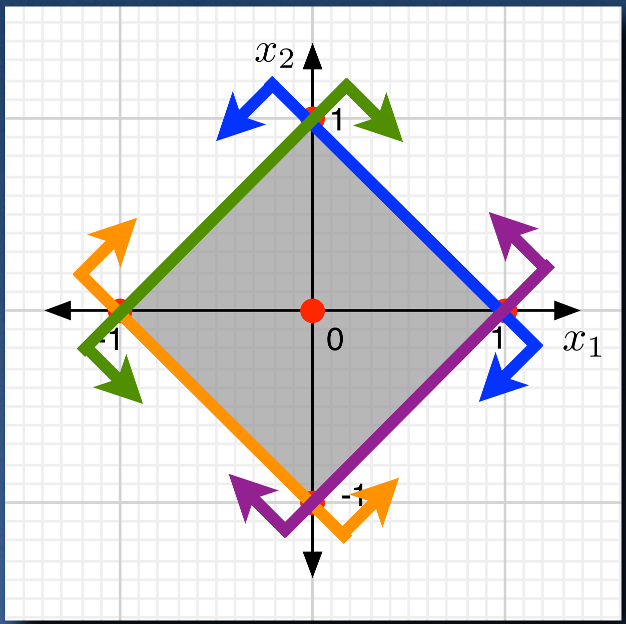

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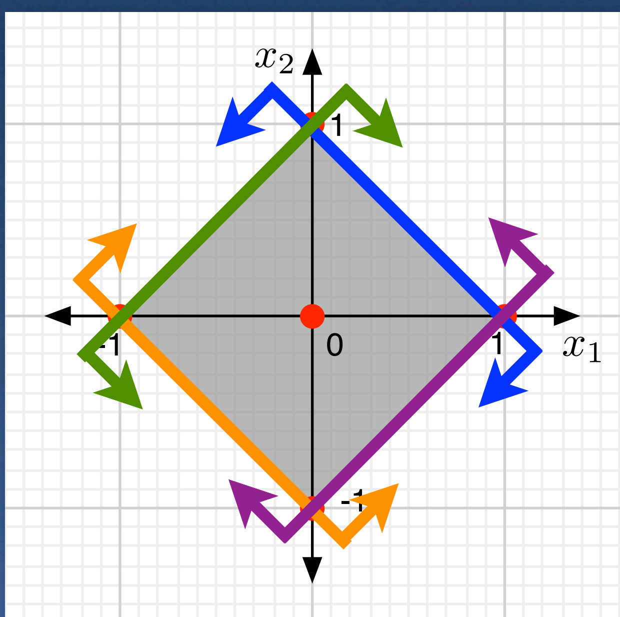
Original Space: Size= $O\left(2^n\right)$



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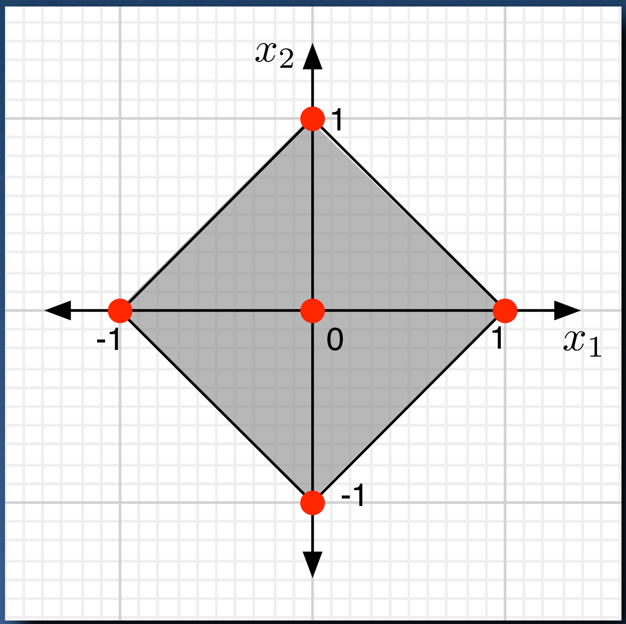

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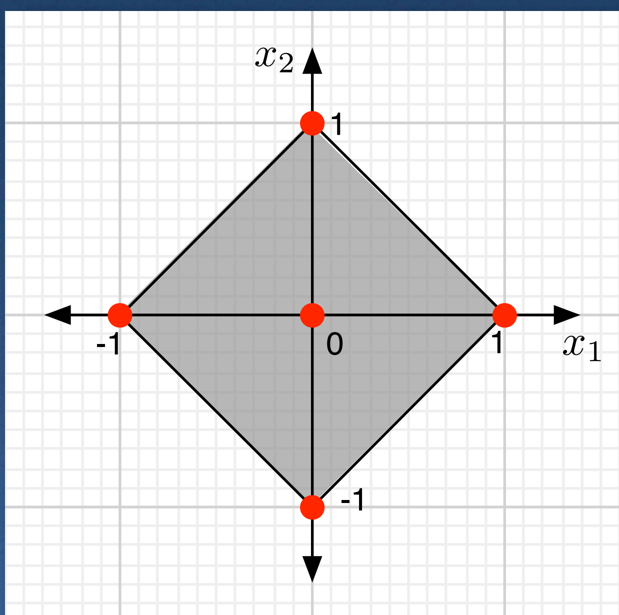

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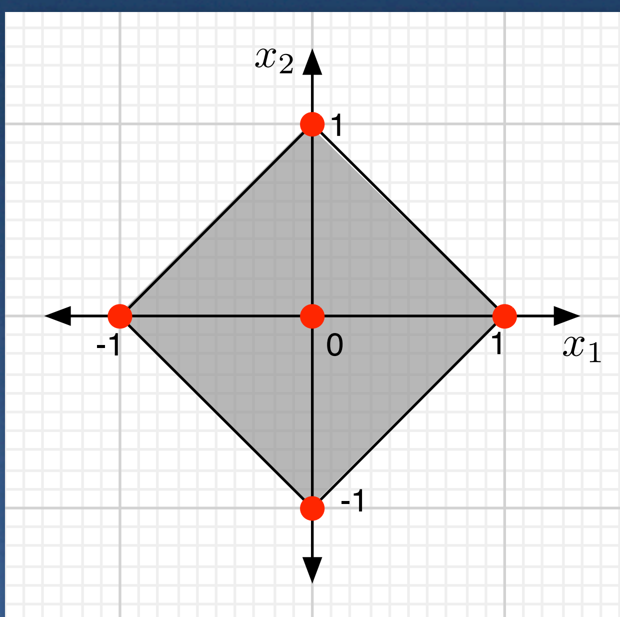

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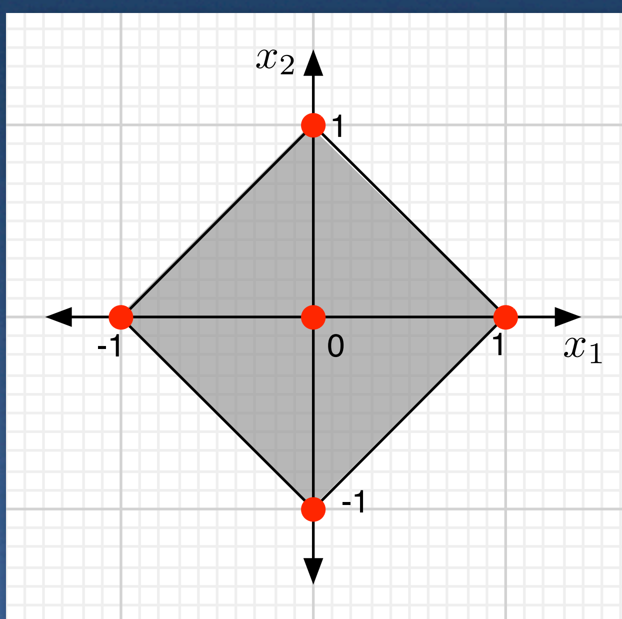

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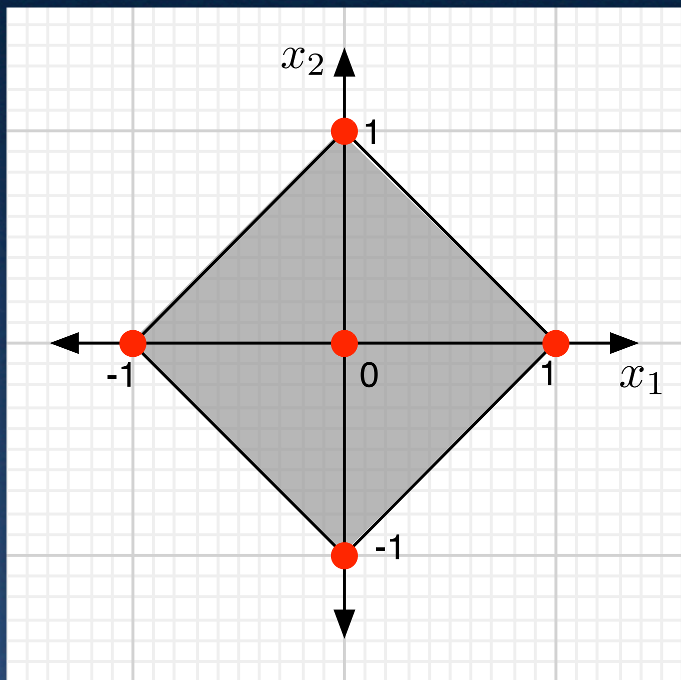
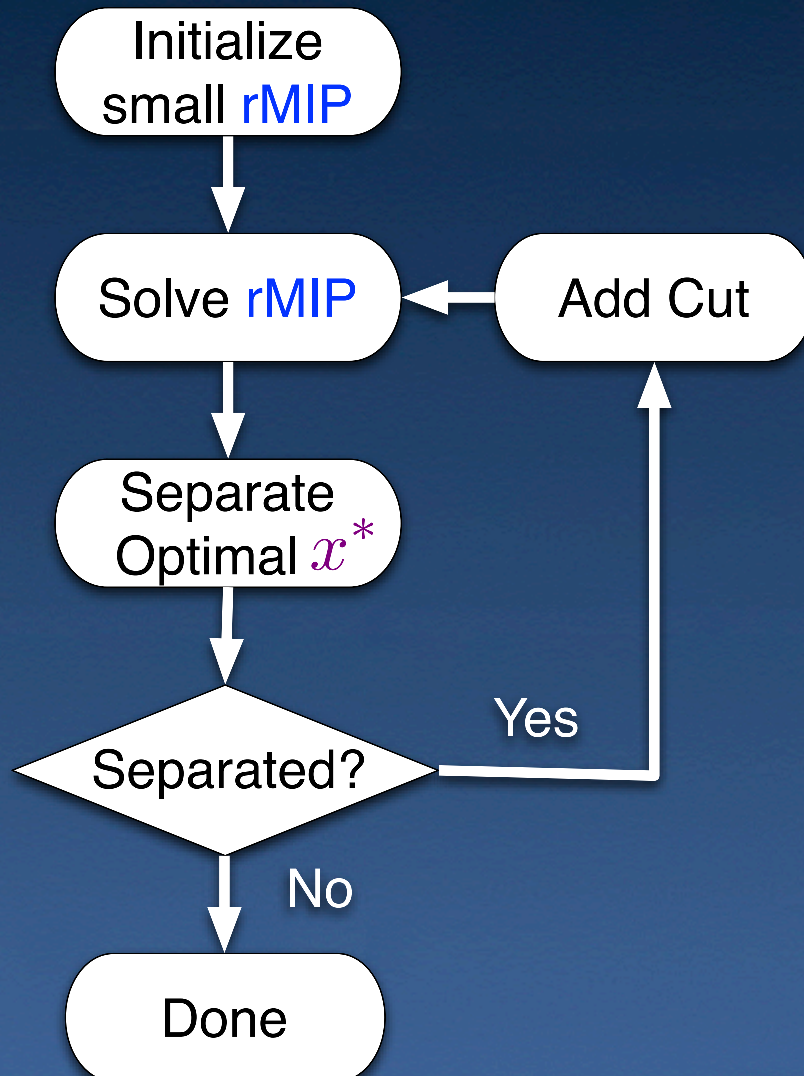
Using Large Formulations: Separate





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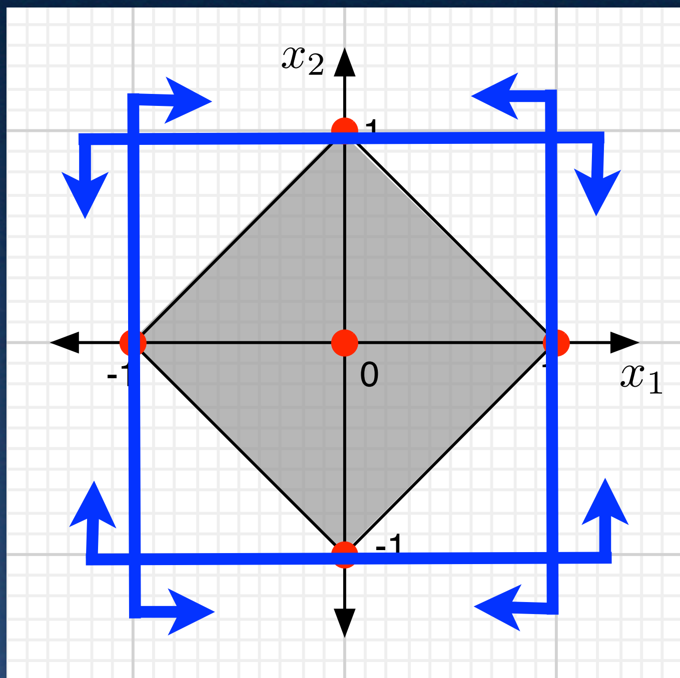

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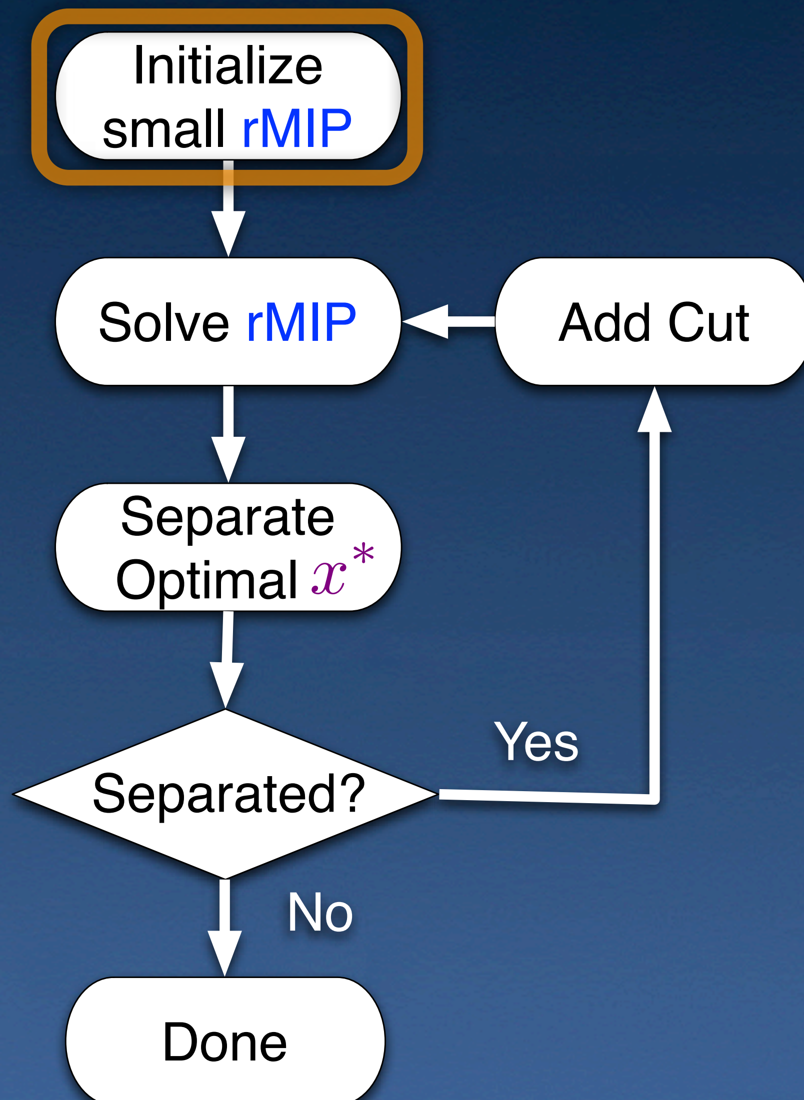

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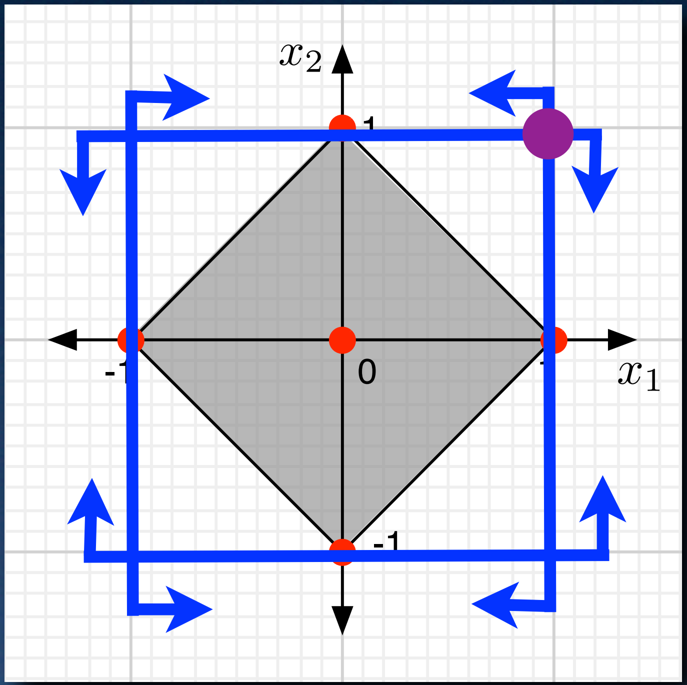

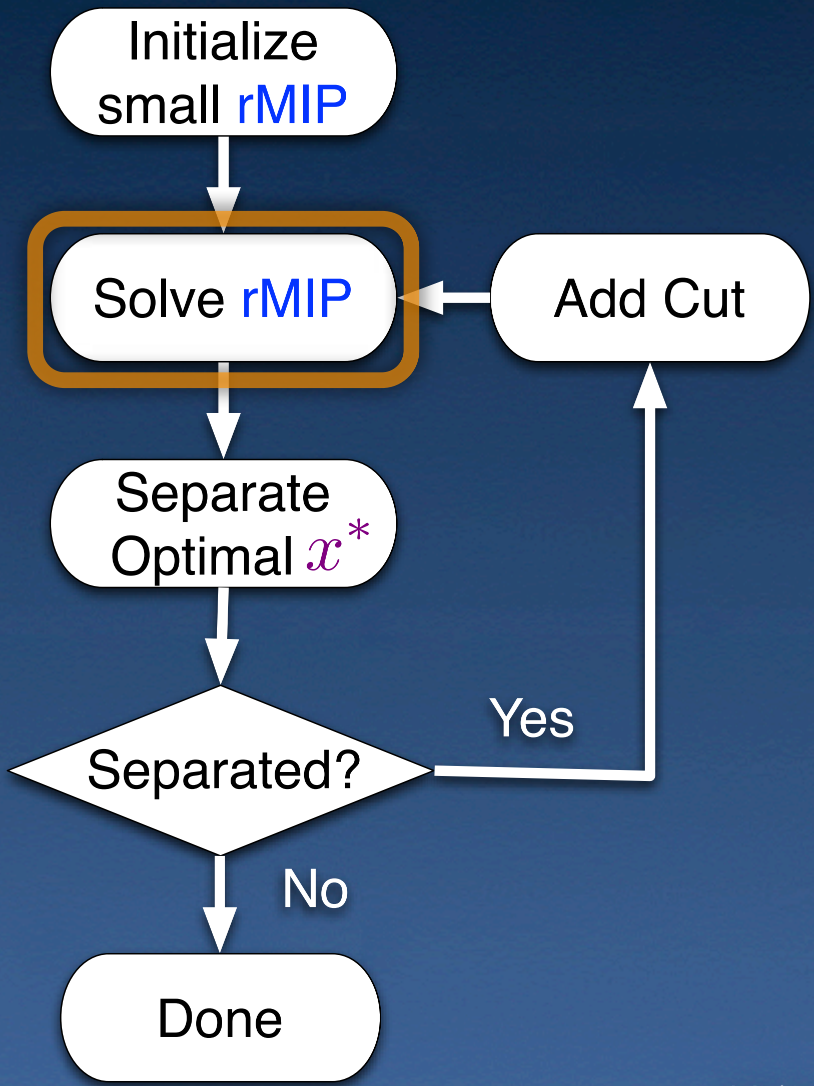

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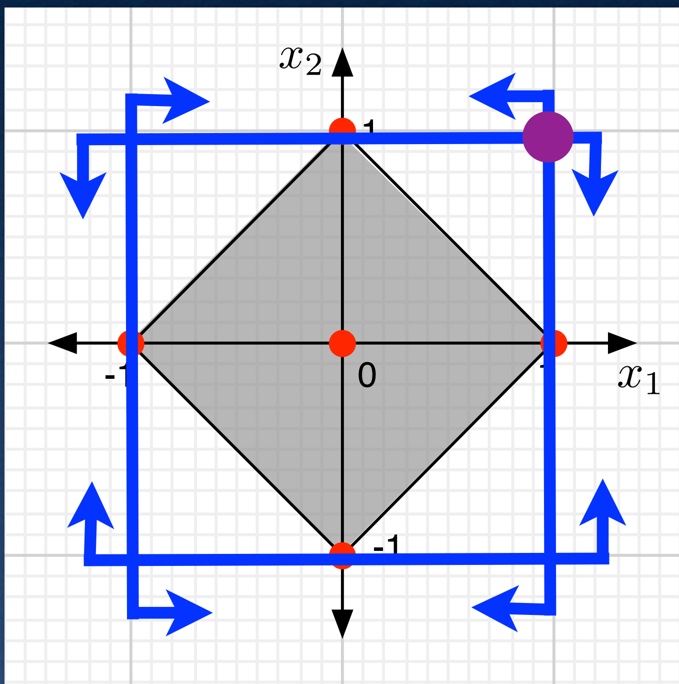

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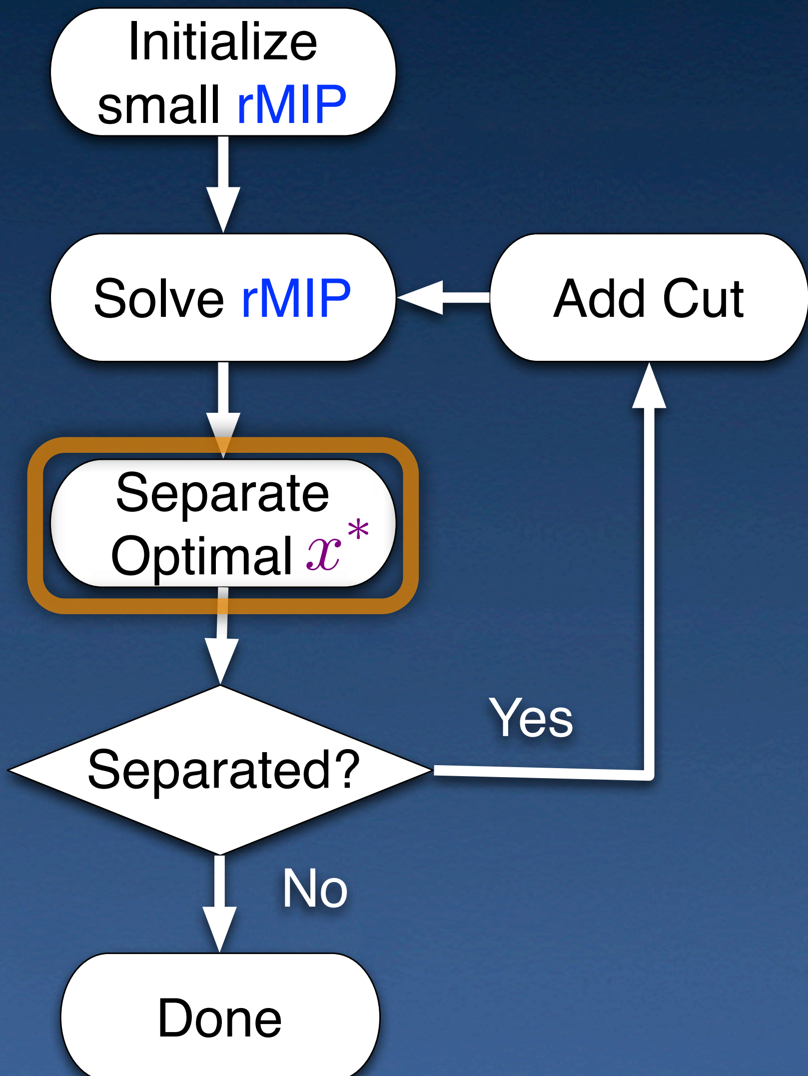

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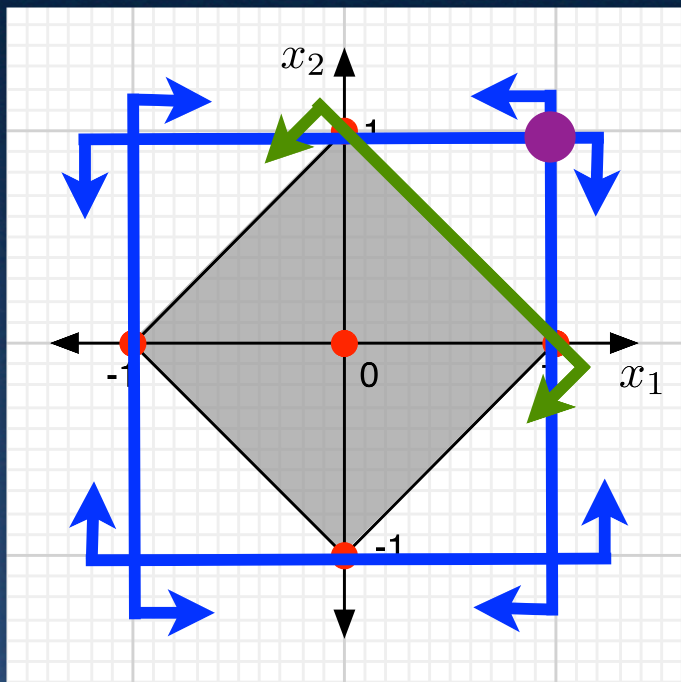

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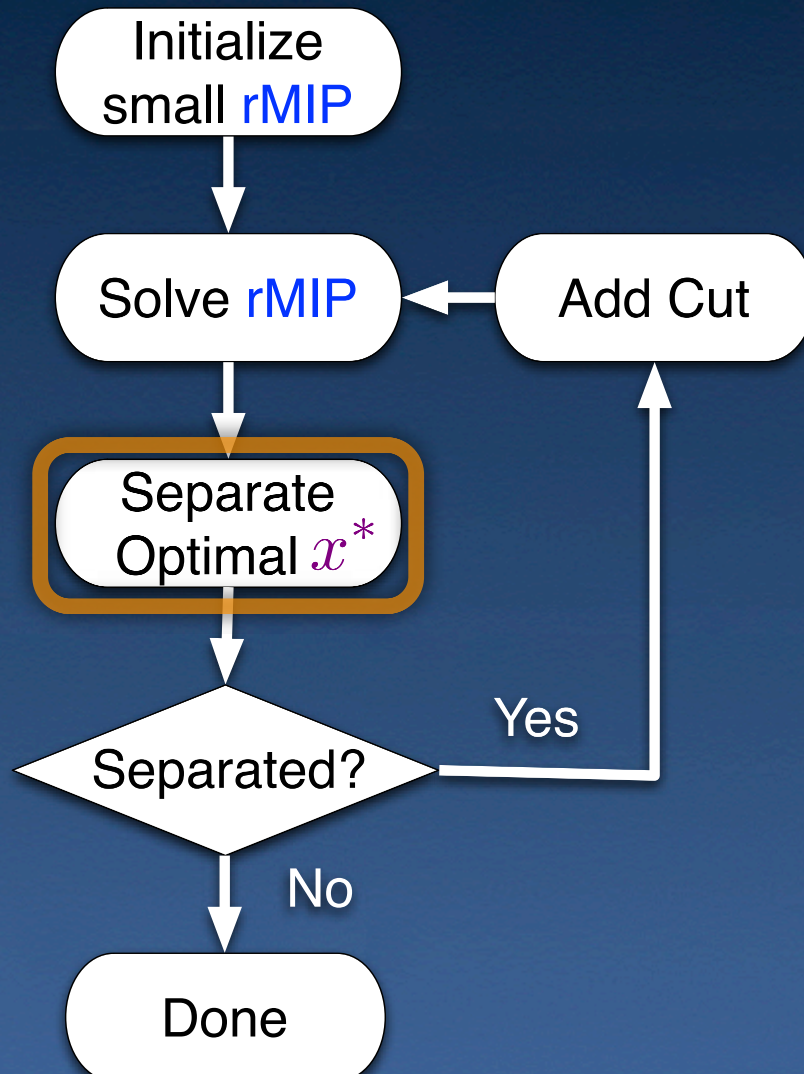

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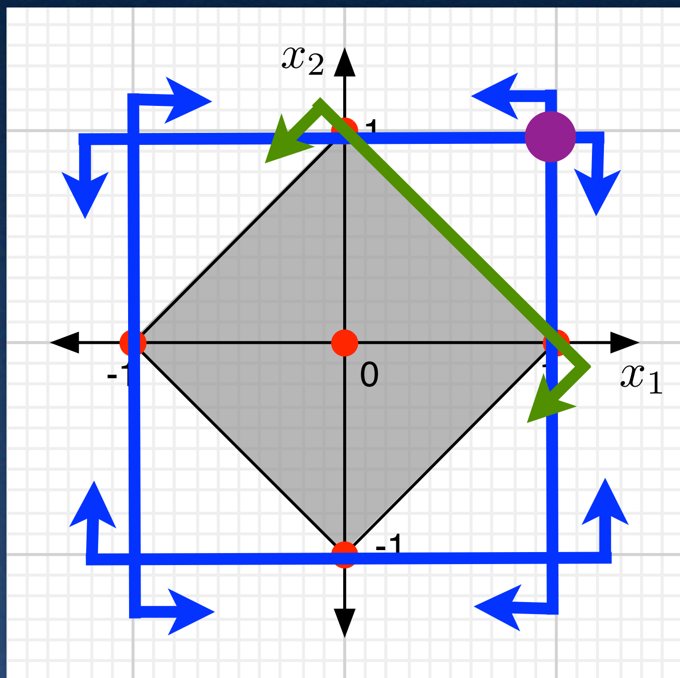

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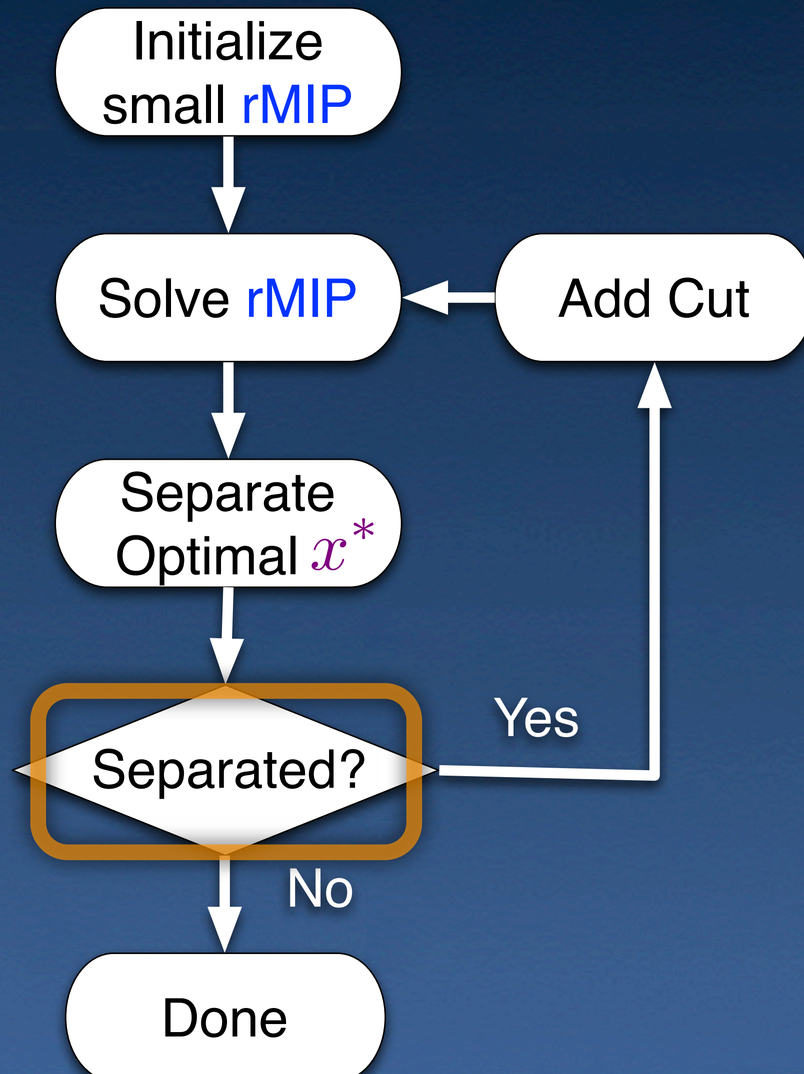

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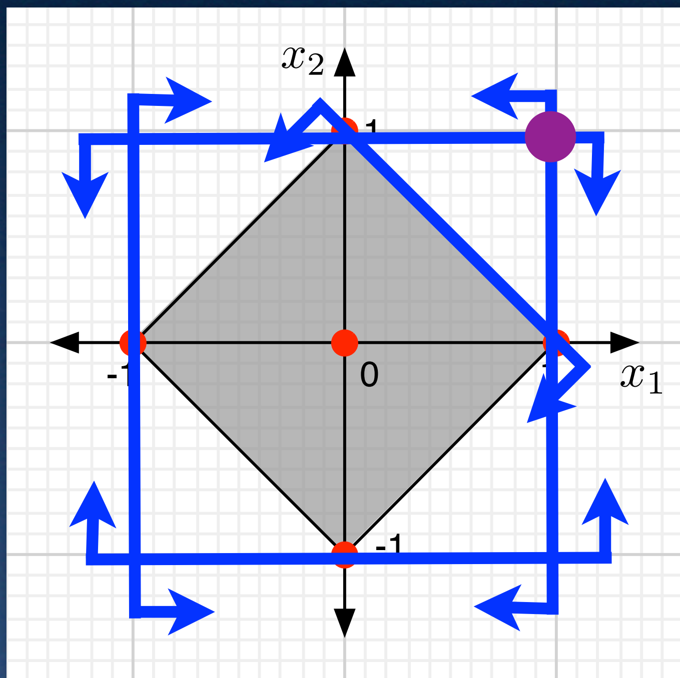

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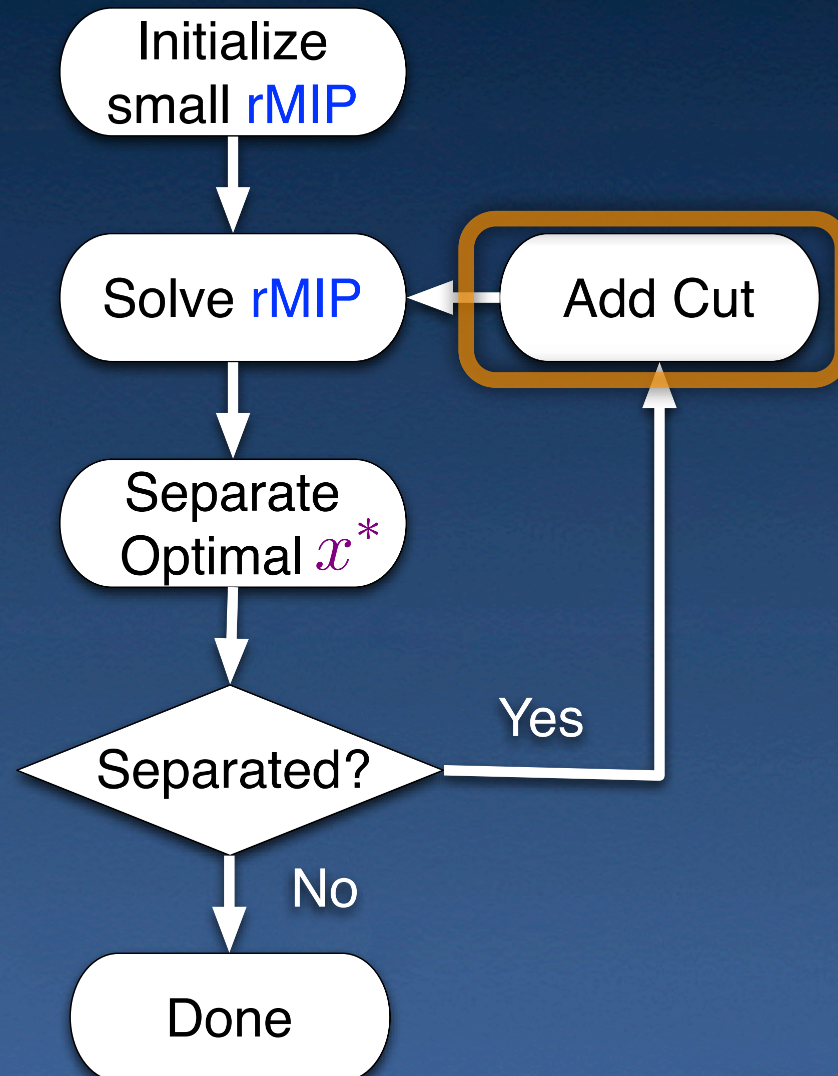

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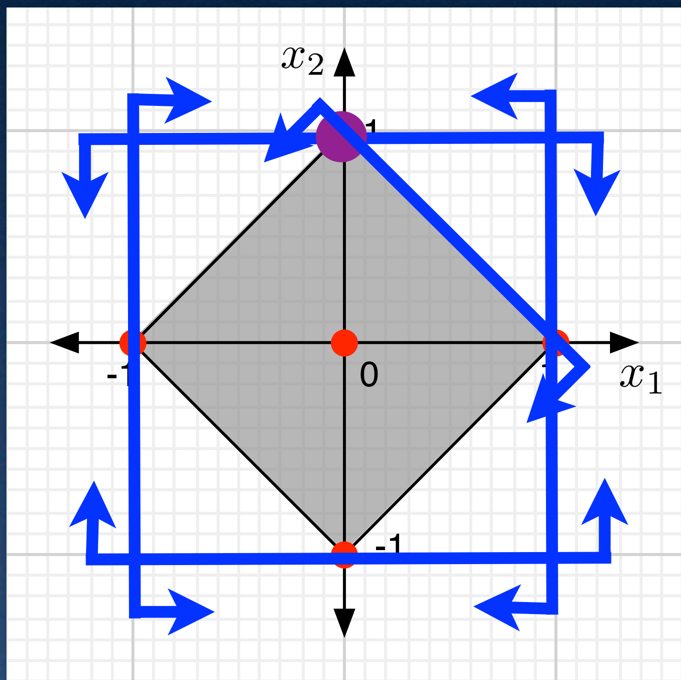

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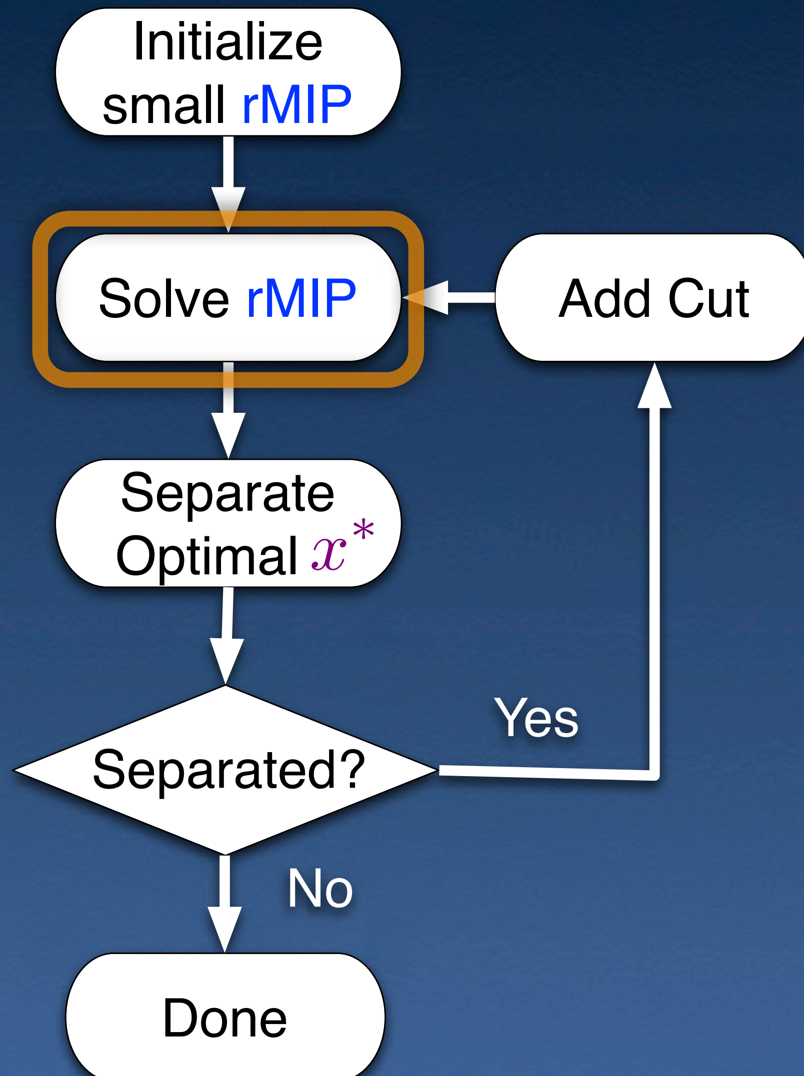

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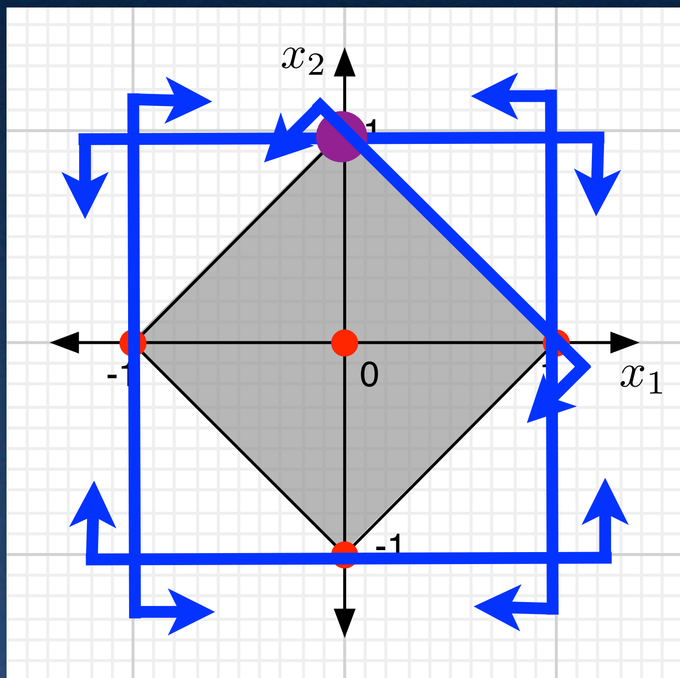

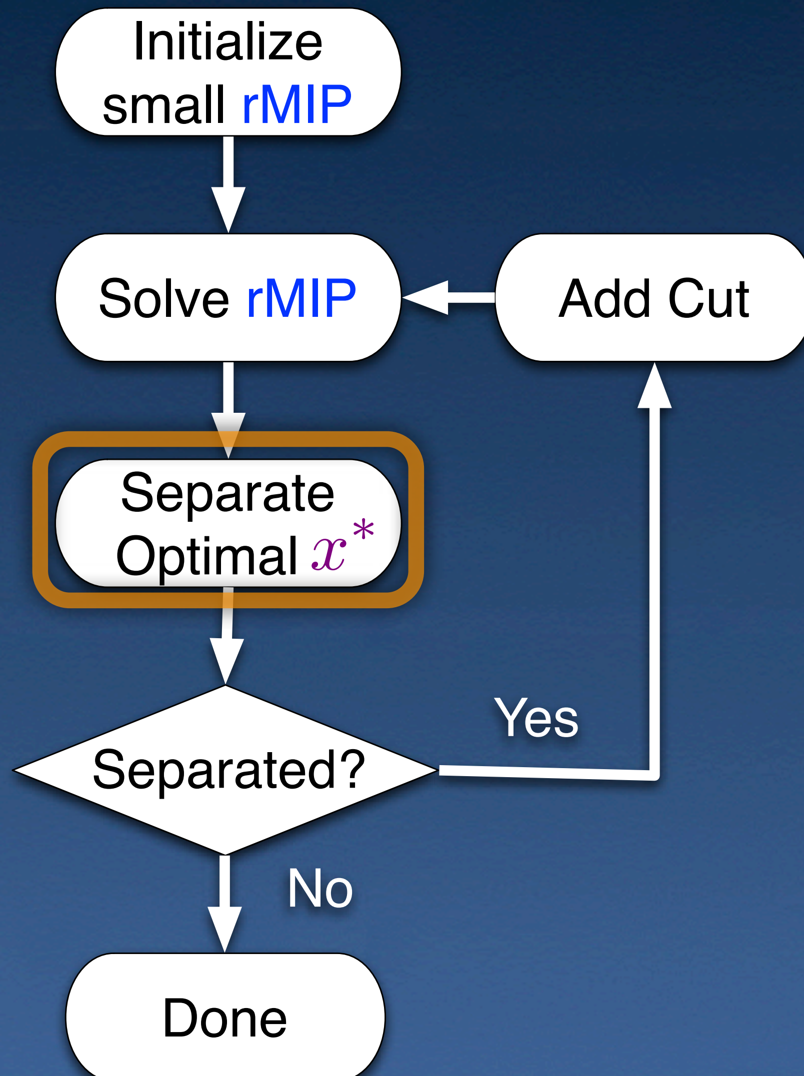

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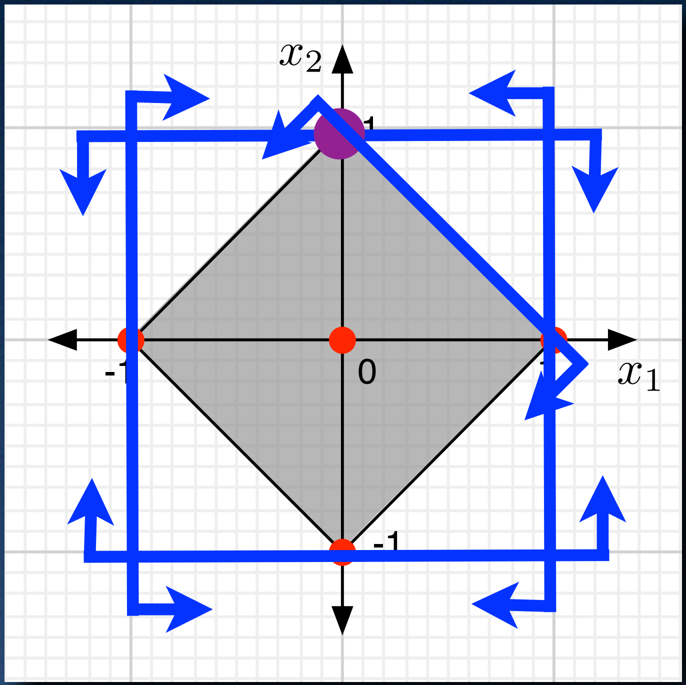

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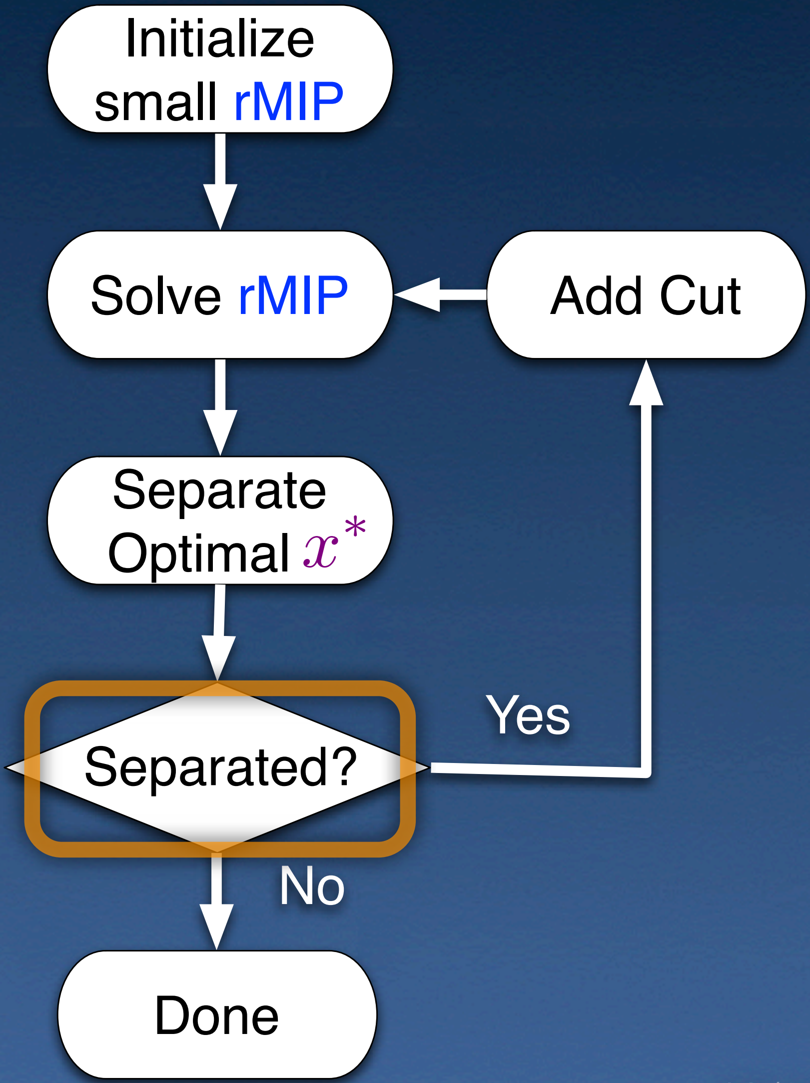

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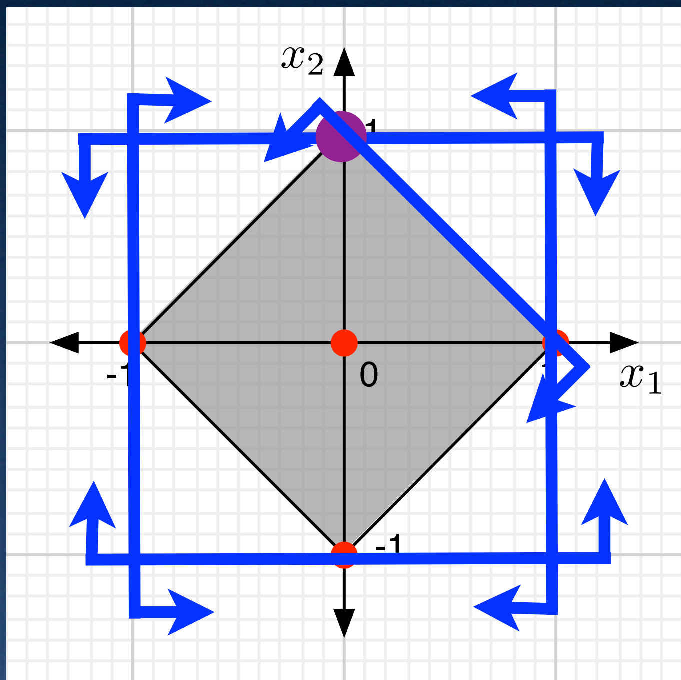

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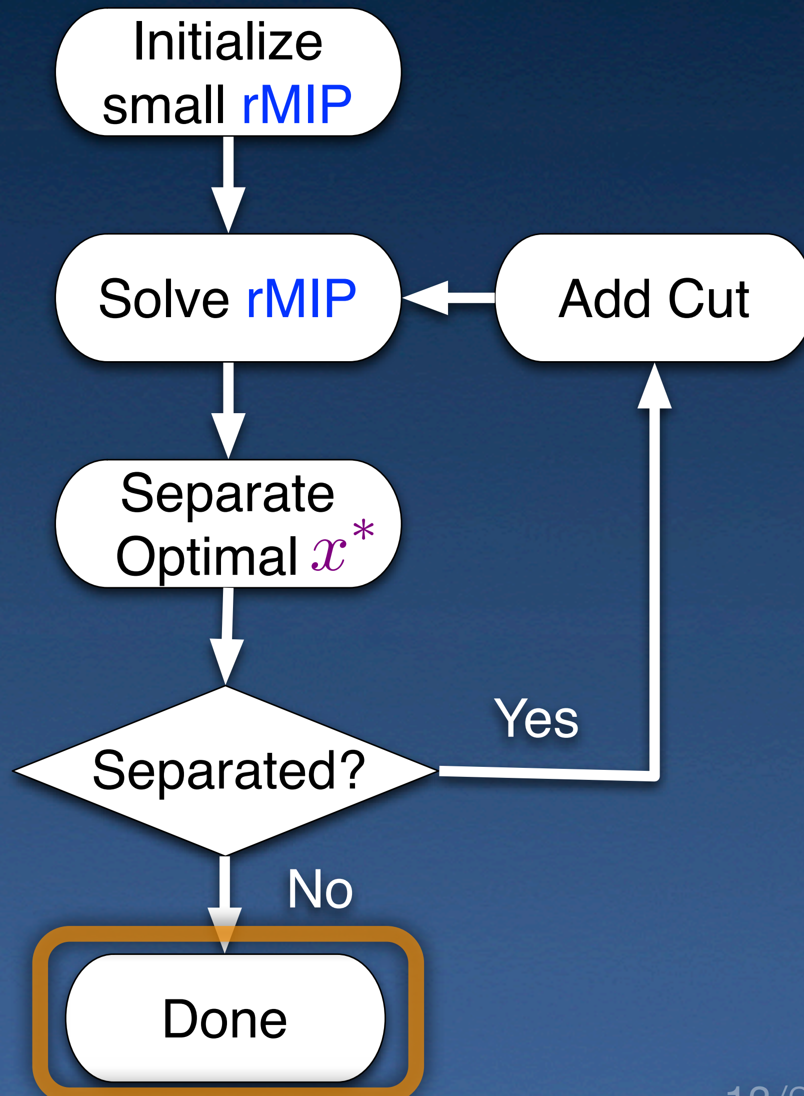
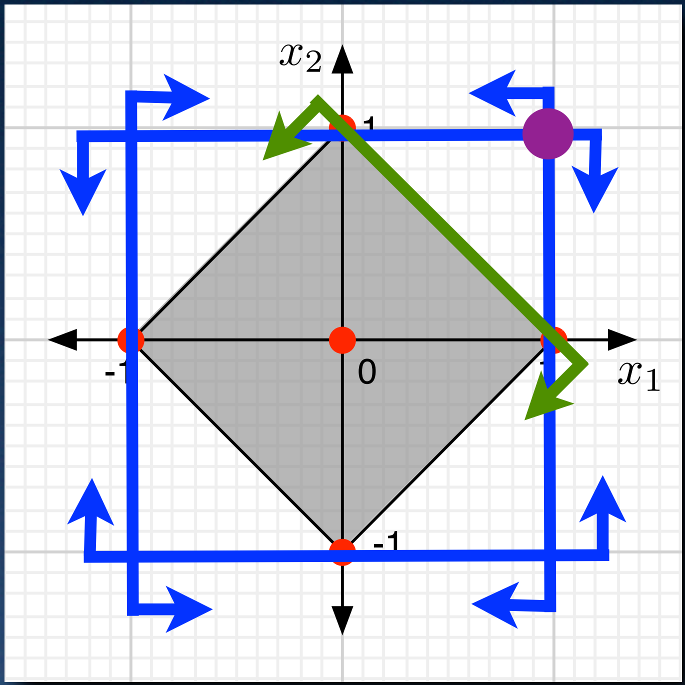


Key is Fast Separation



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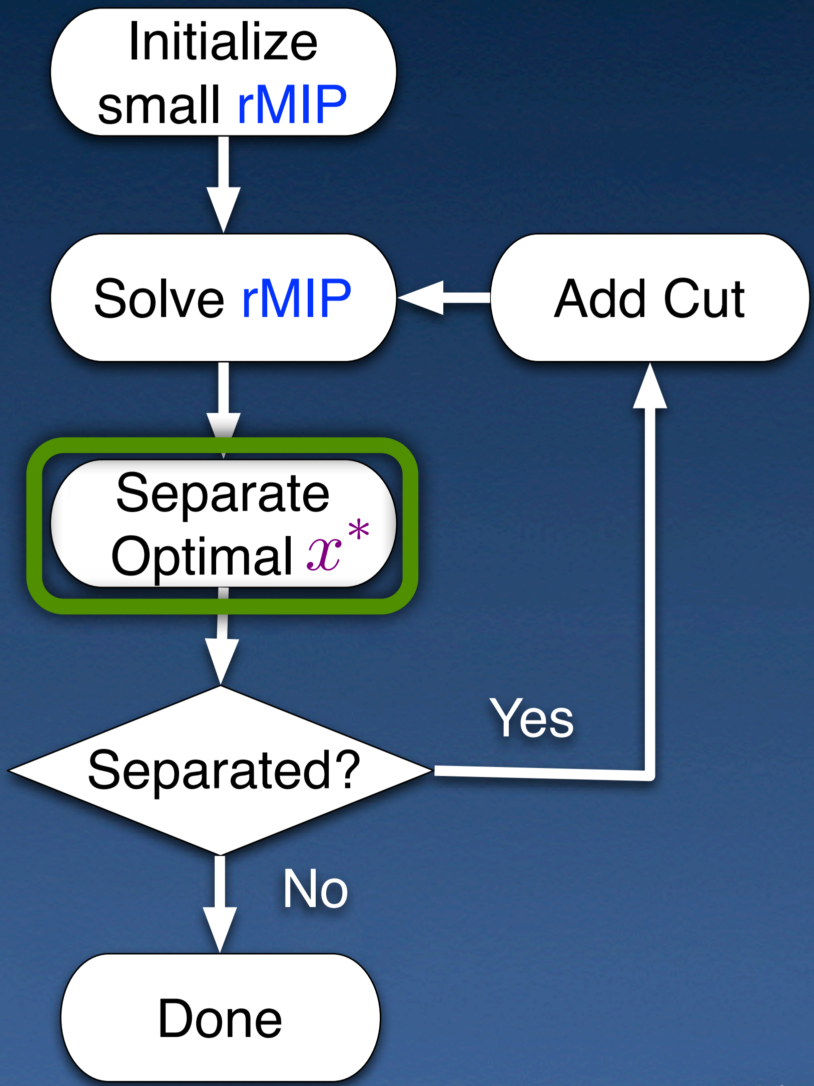
$$\sum_{i=1}^{n} \operatorname{sign}(x_i^*) x_i \le 1$$



In Practice ...

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- Modern IP for many applications:
  - Traveling Salesman, Vehicle Routing, etc.
- Still (Cool) compact extended formulations have applications

Back to Connected Forests, i.e. Trees

Connectivity Formulations in Forestry

- Compact Extended and Large Formulations:
  - Önal and Briers (2006), Önal and Wang (2008), Rebain and McDill (2003), Martins et al. (2005), Carvajal et al. (2010), etc

Connectivity Formulations in Forestry

- Compact Extended and Large Formulations:
  - Önal and Briers (2006), Önal and Wang (2008), Rebain and McDill (2003), Martins et al. (2005), Carvajal et al. (2010), etc

- Today = Carvajal et al. Large Formulation

Graph Representation of Forest

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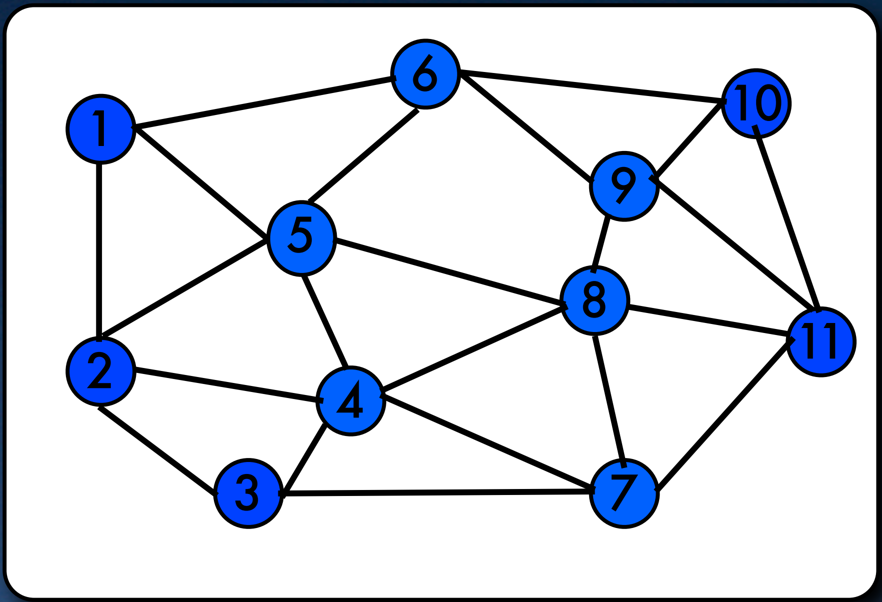

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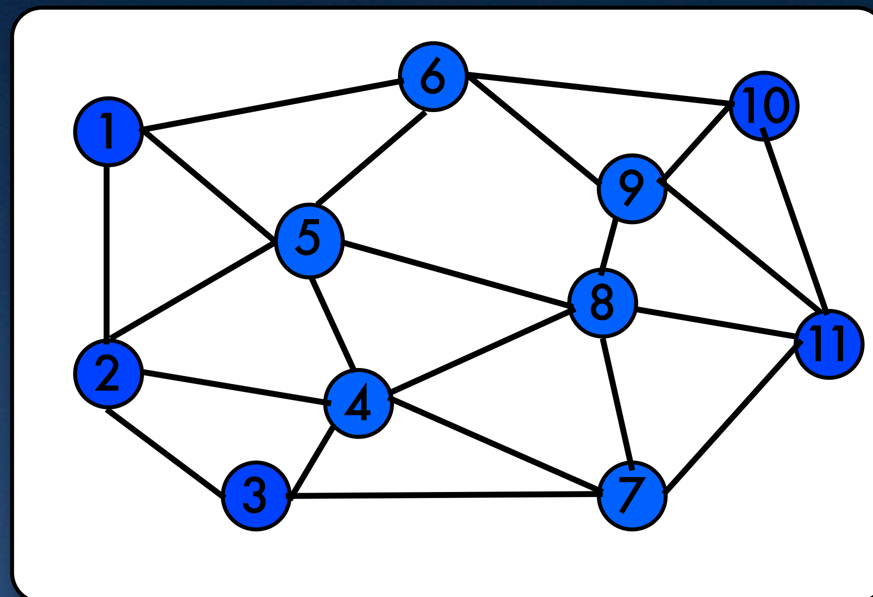
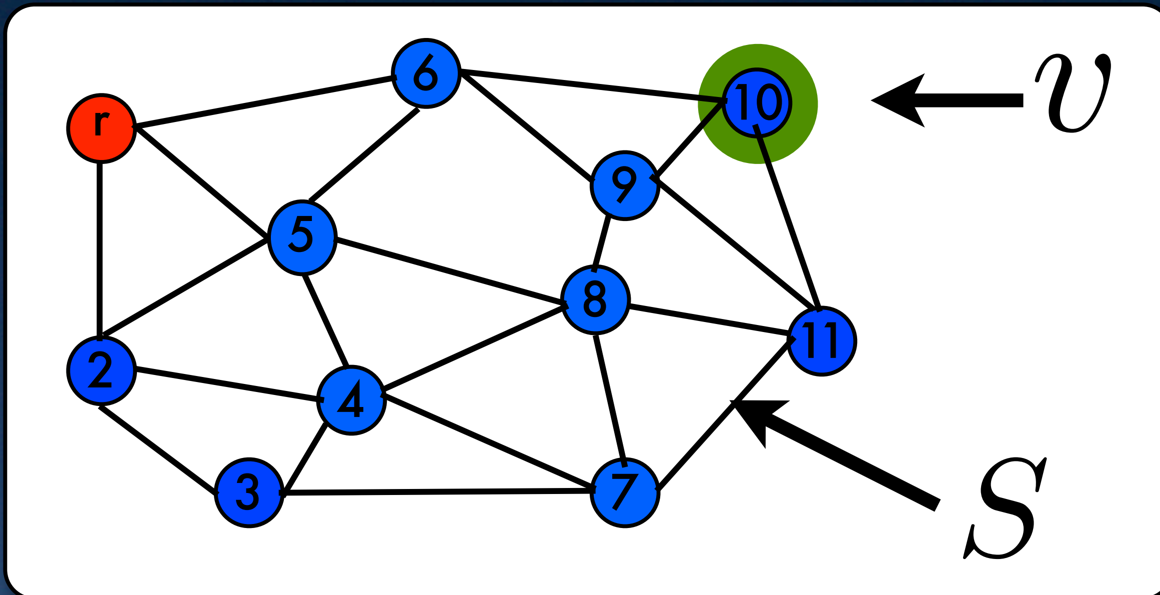


$$z_{v,t} = \begin{cases} 1 & \text{if stand } v \text{ is old-growth} \\ & \text{or reserve in period } t \\ 0 & \text{otherwise} \end{cases}$$

Rooted (Lack of) Connectivity



- Solution:

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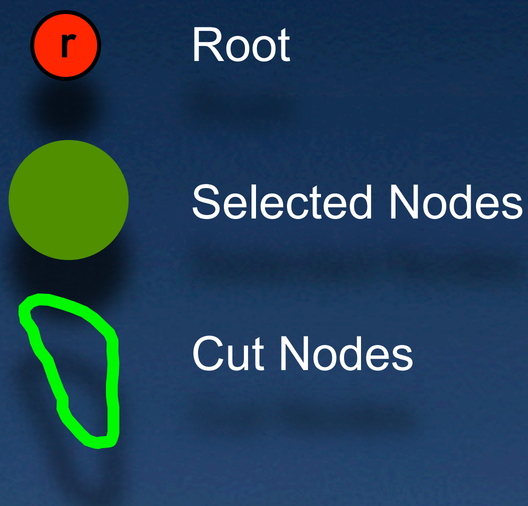

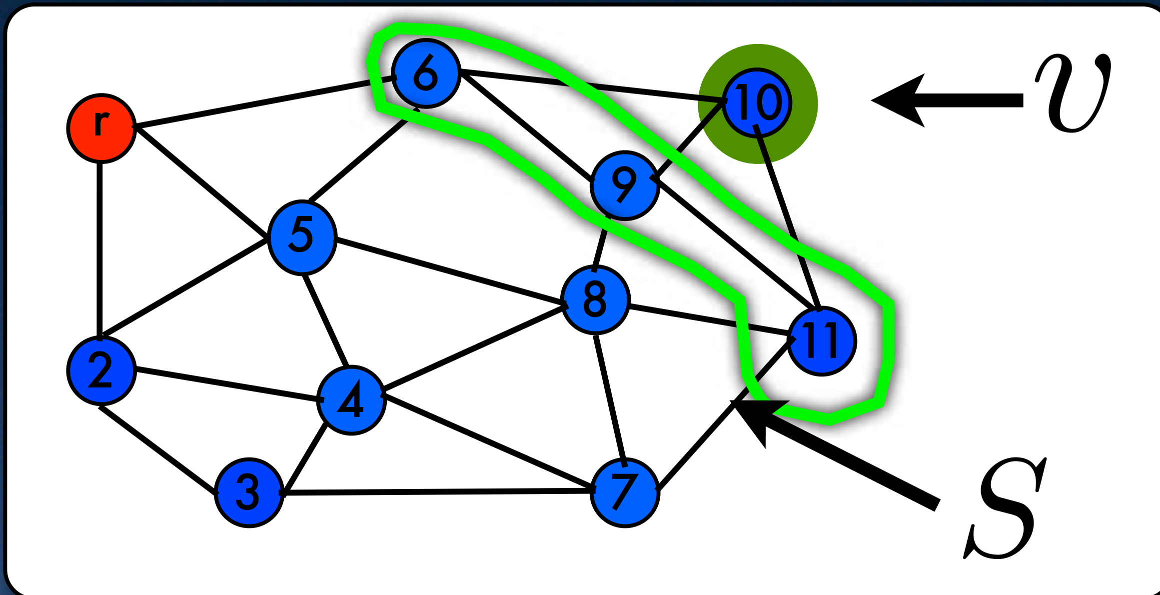

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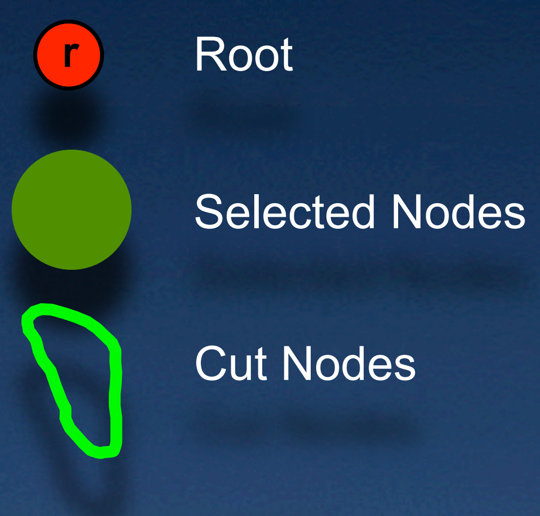

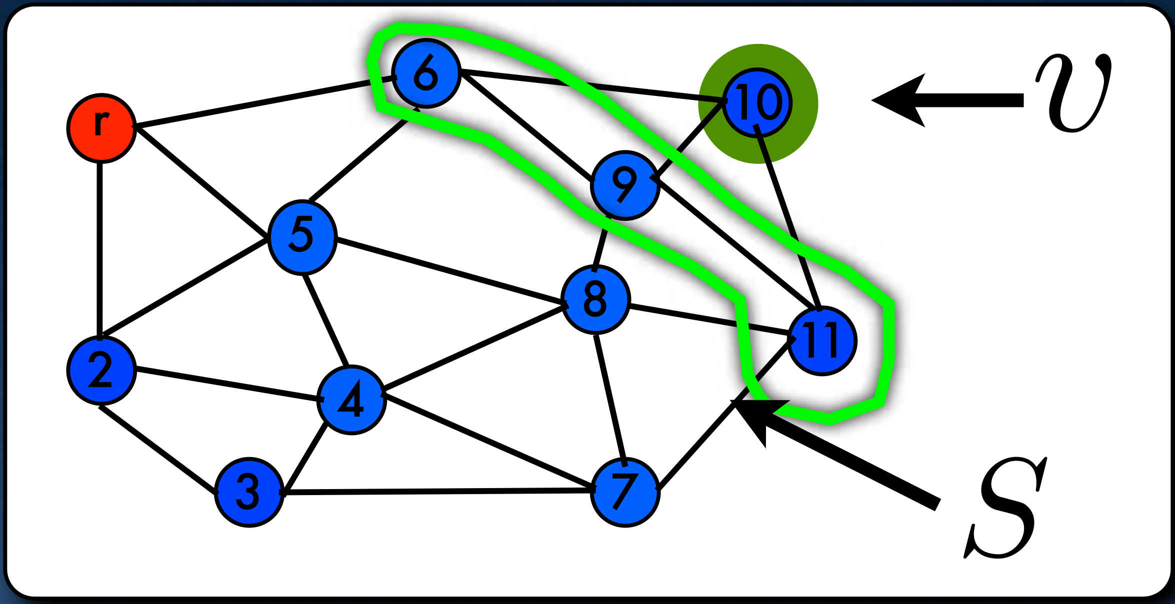

- Solution:

$$z_6 + z_9 + z_{11} \geq z_{10}$$

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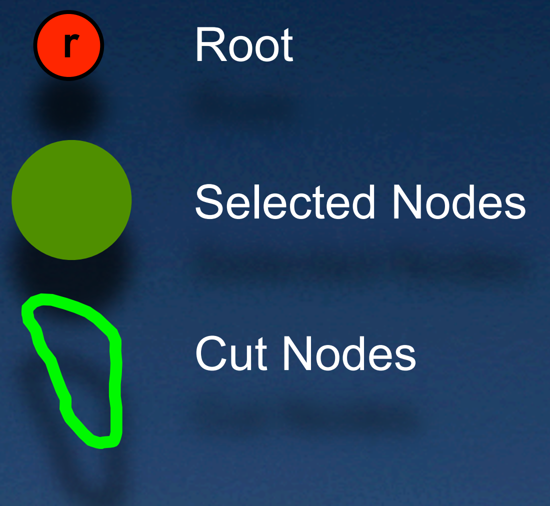

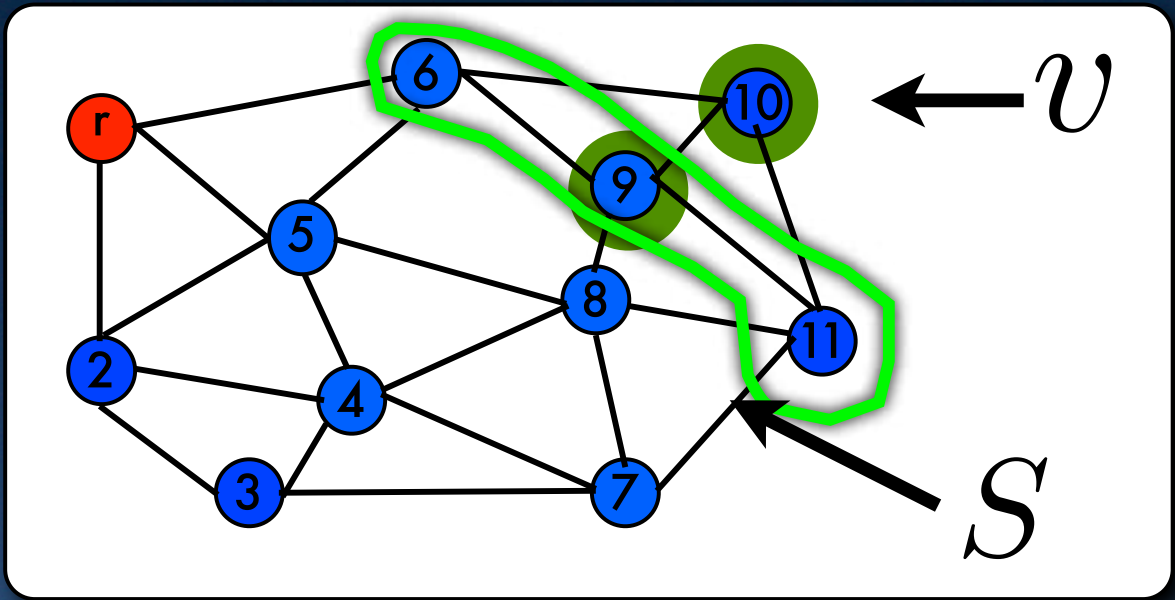

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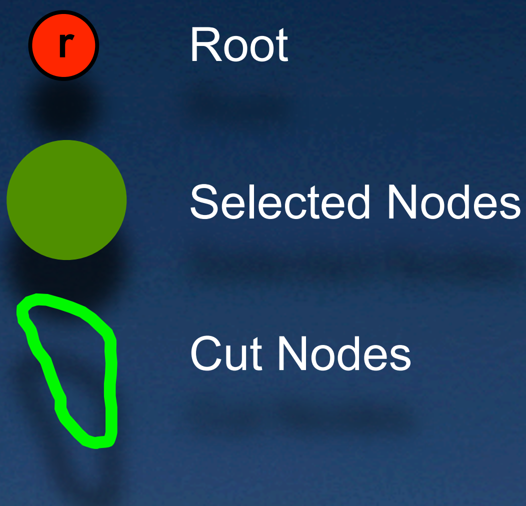

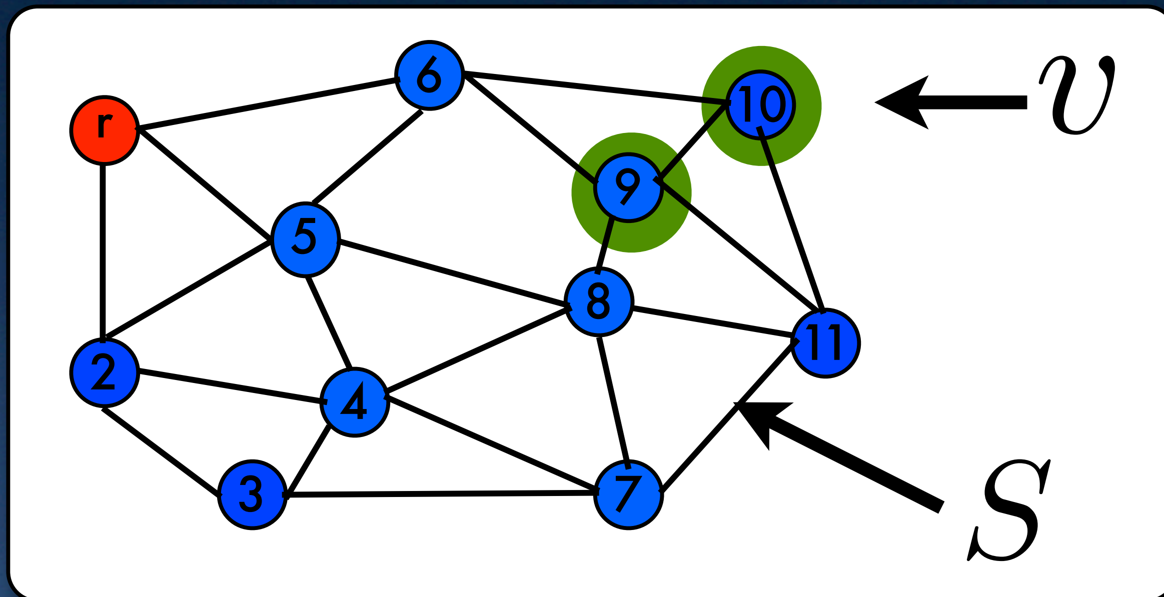

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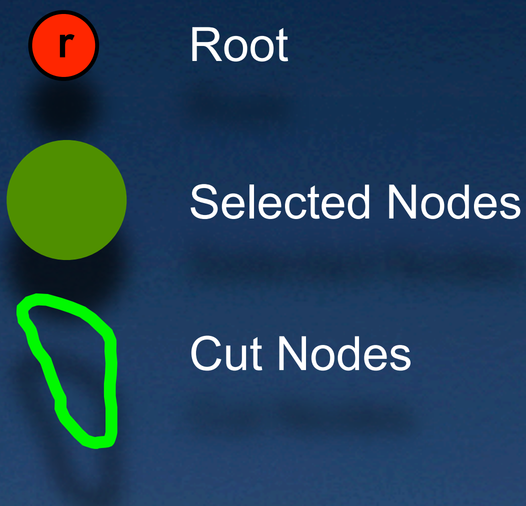

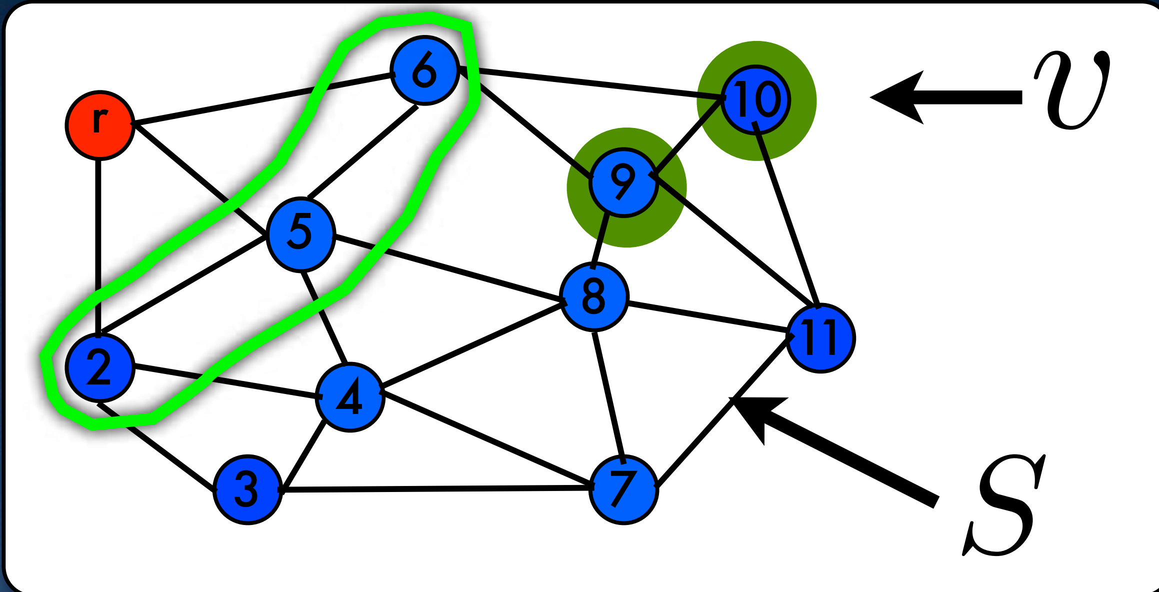

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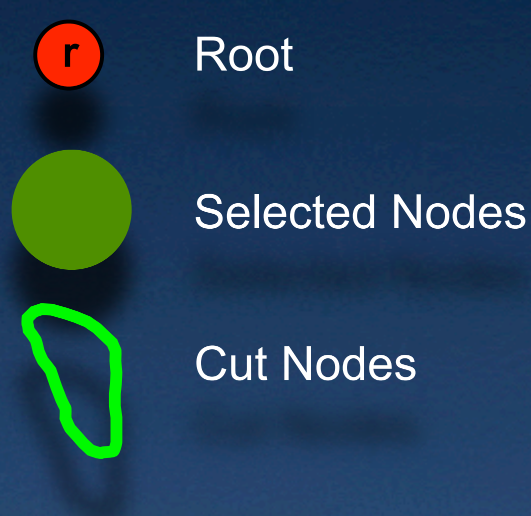

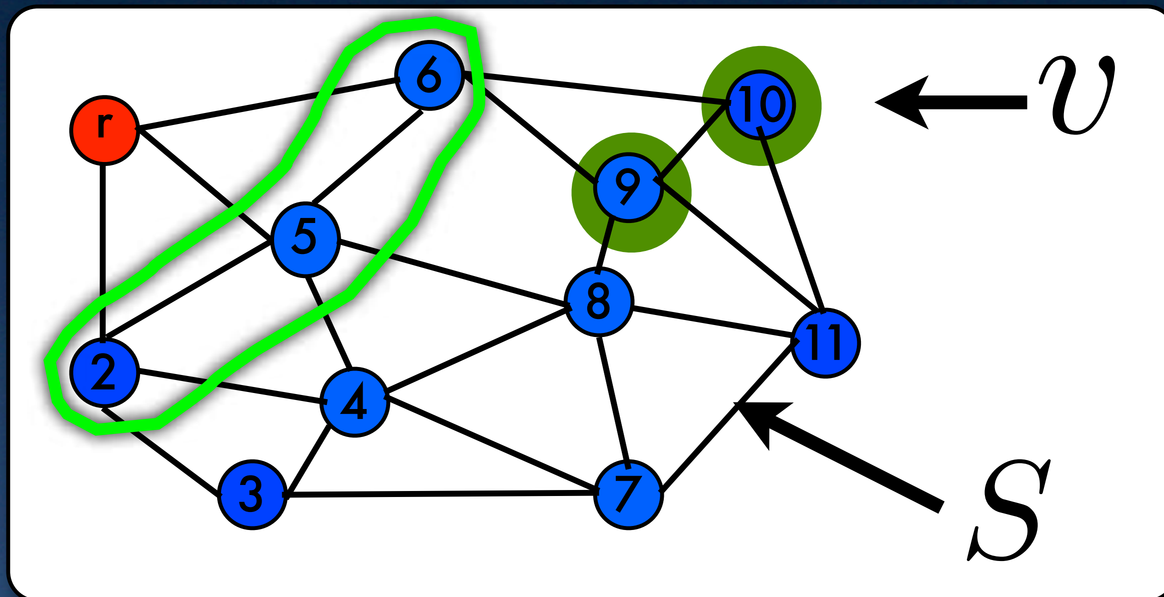

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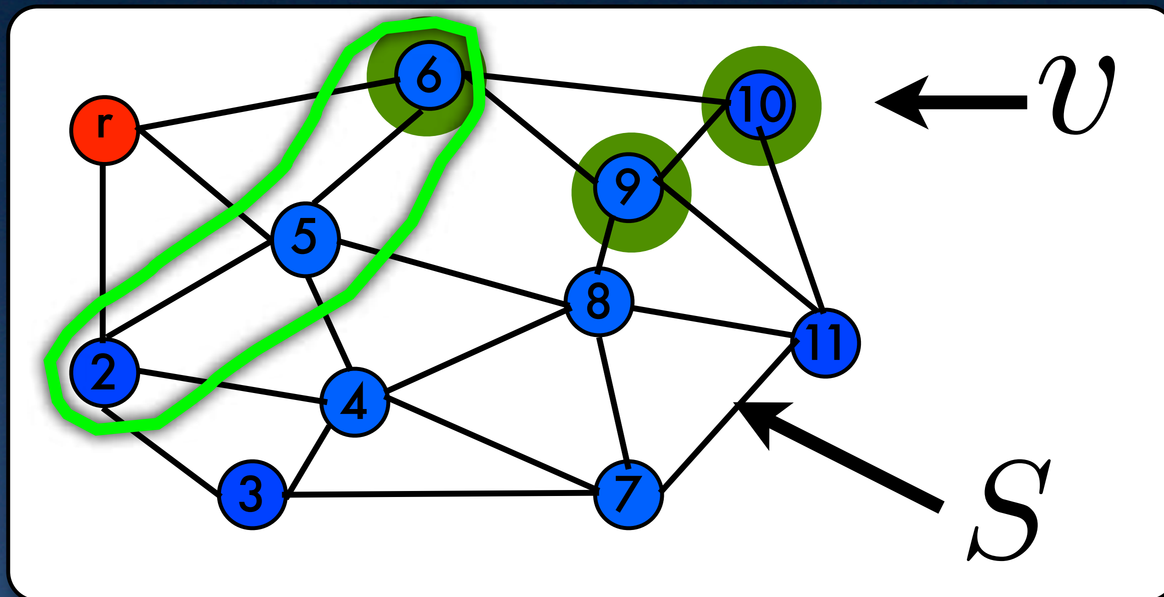

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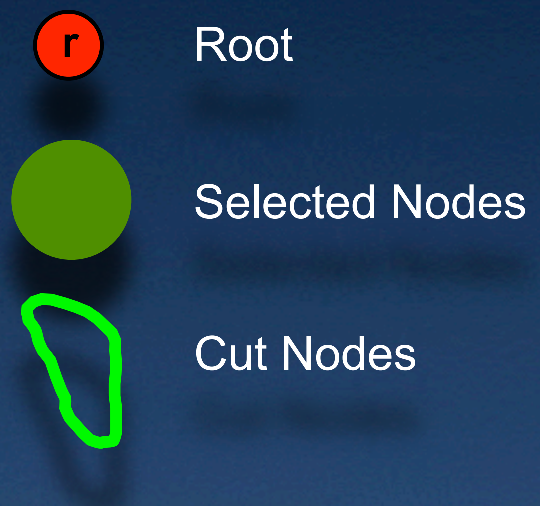

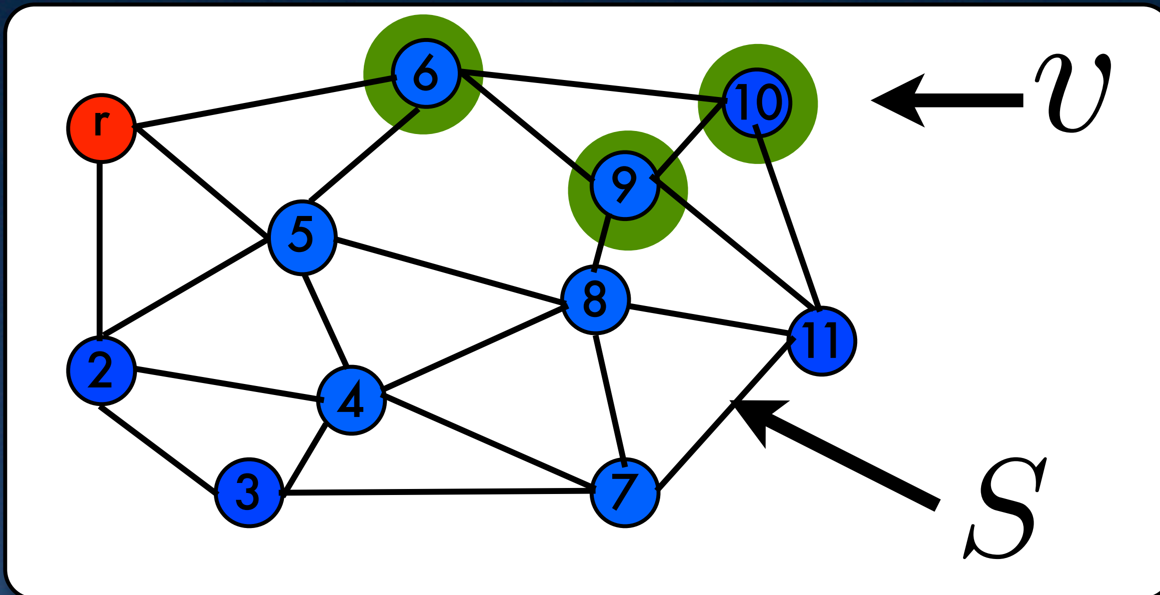

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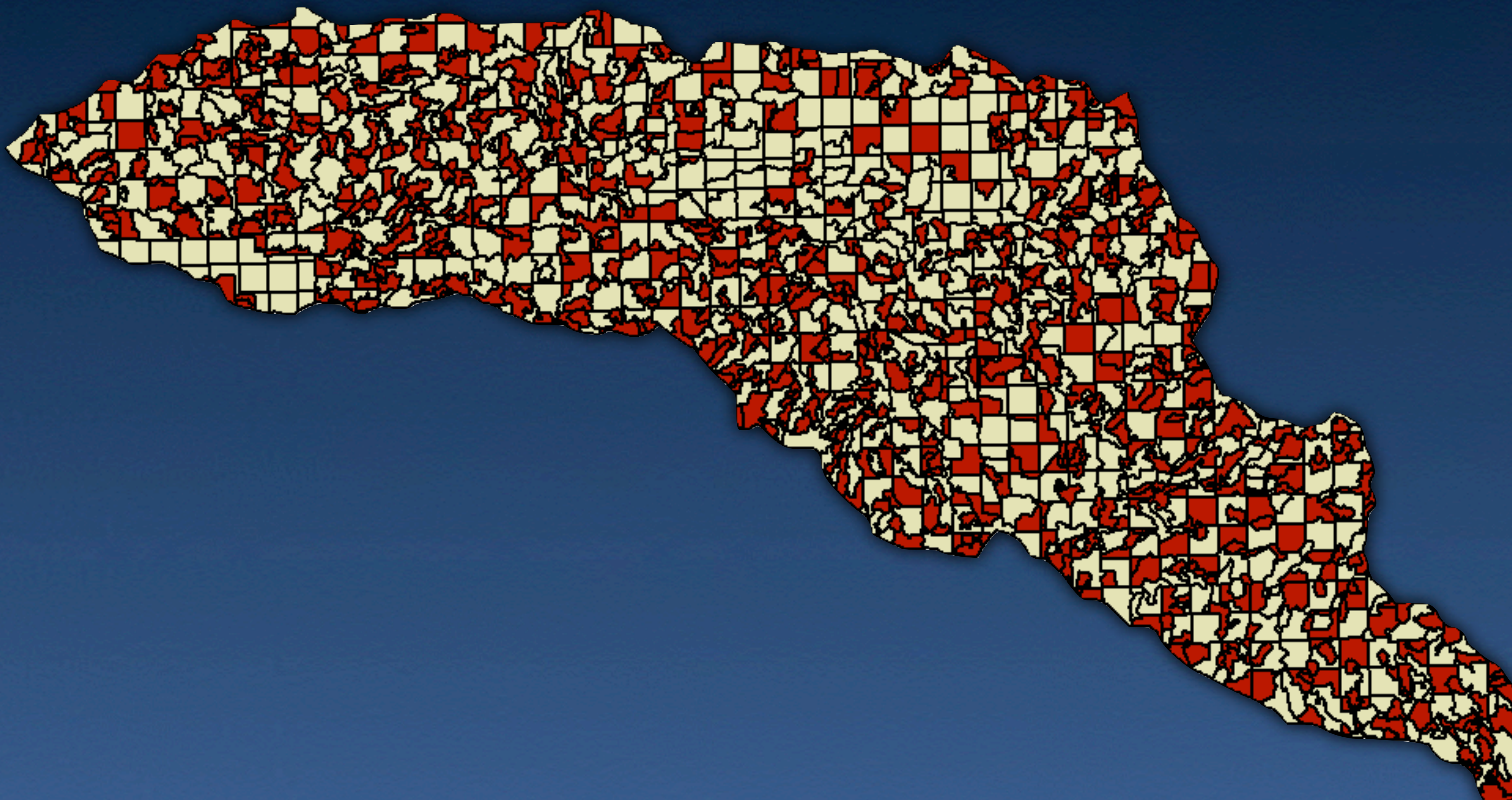
Maximum Clearcut Area (ARM)



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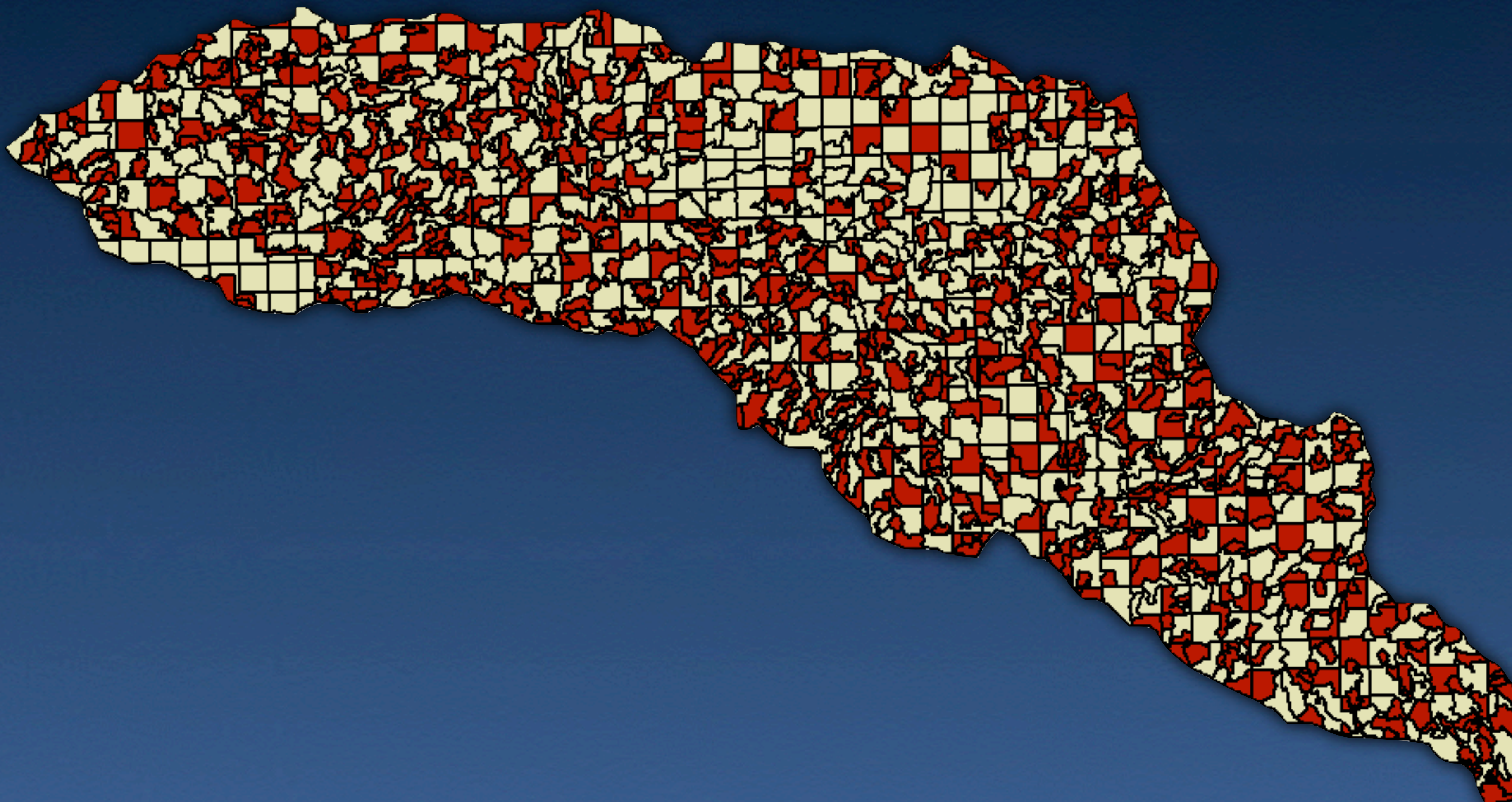

- Partial Solution: **Connected** Old Growth Patch

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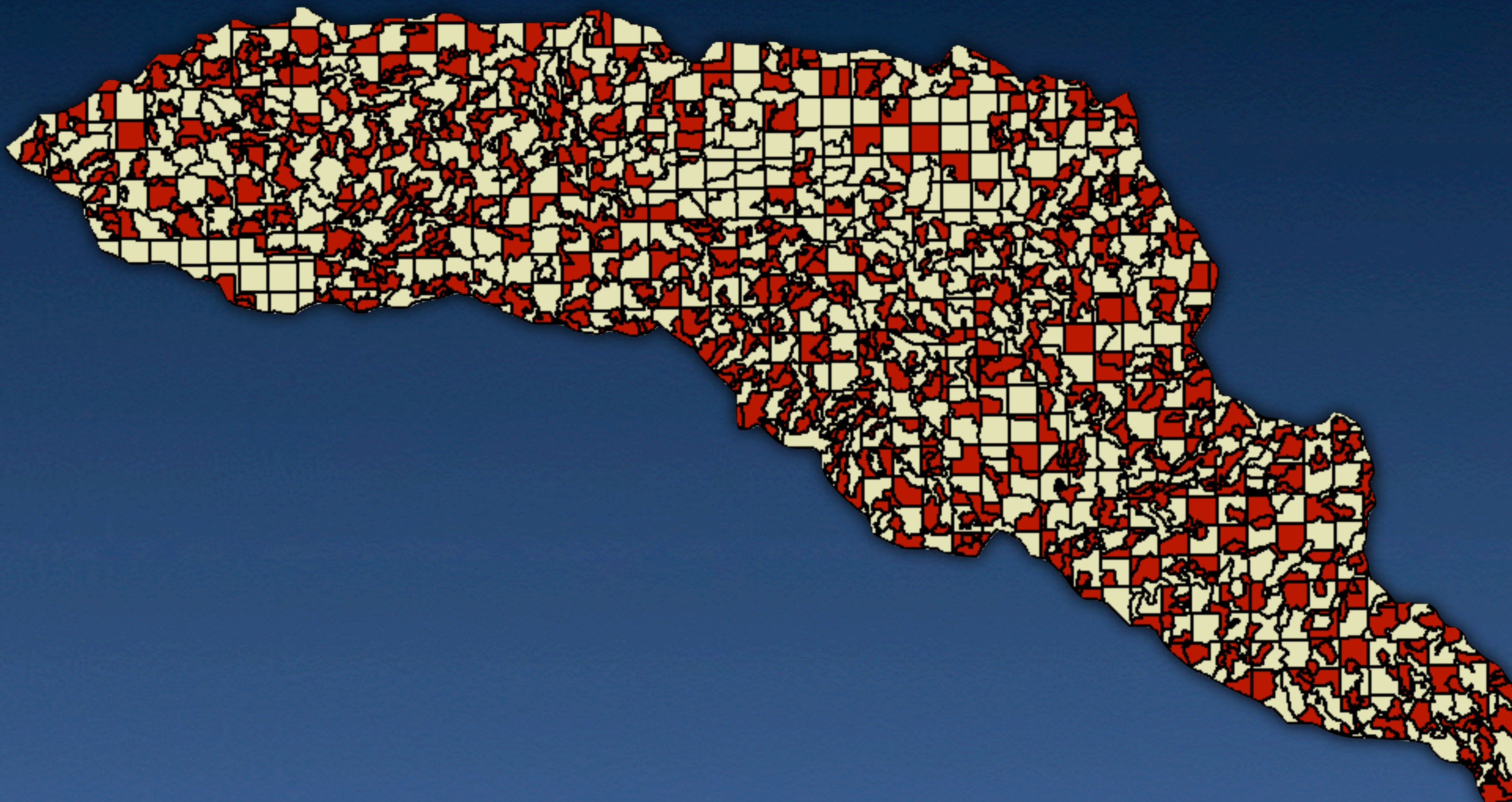

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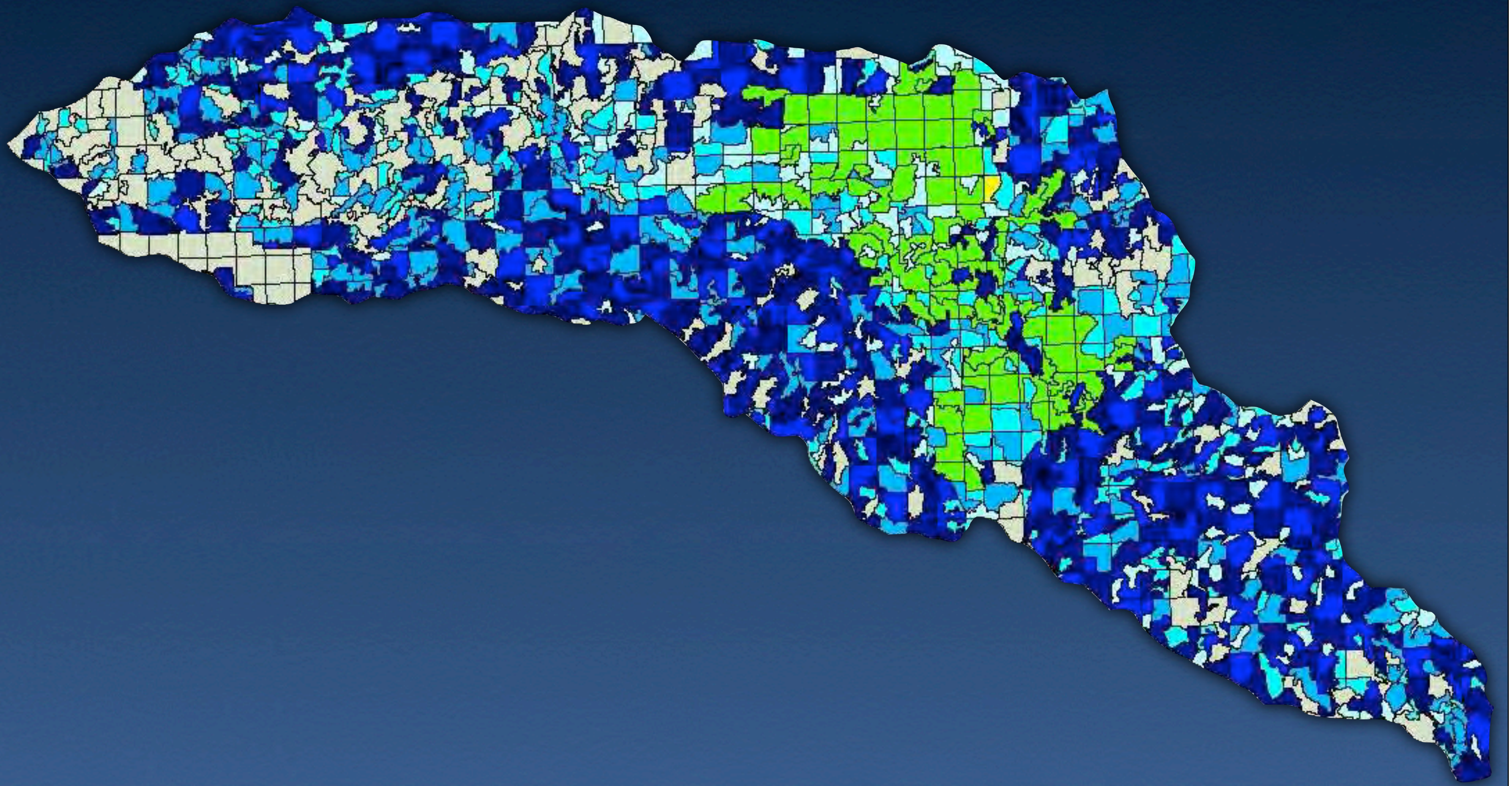
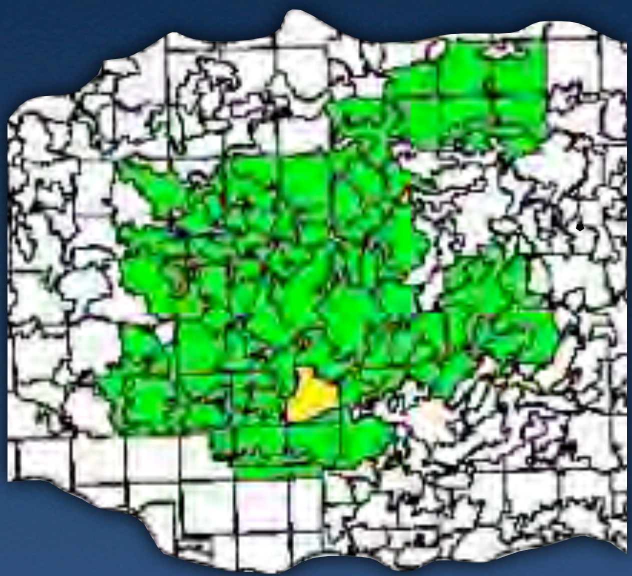


- Partial Solution: **Connected** Old Growth Patch

Test Problem: Harvest Scheduling

- 3 Periods
- Maximize NPV of schedule
- Maximum clear-cut
- Volume flow
- Average ending age of forest
- **Connected** old-growth patch (rooted model)



Instances = FMOS, Solver=CPLEX 11

Instance	Stands
El Dorado	1363
Shulkell	1039
NBCL5A	5581

Static Set

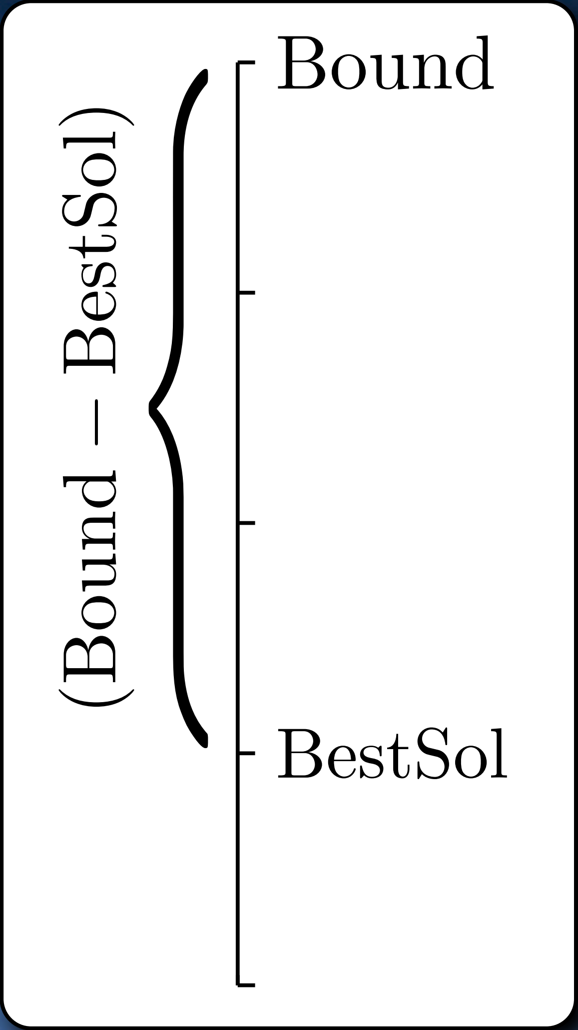
Instance	Stands
Gavin	352
Hardwicke	423

Dynamic Set

- Time Limit = 4 hours

Modern IP = Quality Bounds = GAP

- For Max,
  $$\text{GAP} = 100 \times \frac{(\text{Bound} - \text{BestSol})}{\text{BestSol}}$$
- $\text{Bound} =$
  - Root = LP relaxation + cuts
  - Final = Best B&B node left
- $\text{BestSol} =$ best known solution



Results for Static Patch

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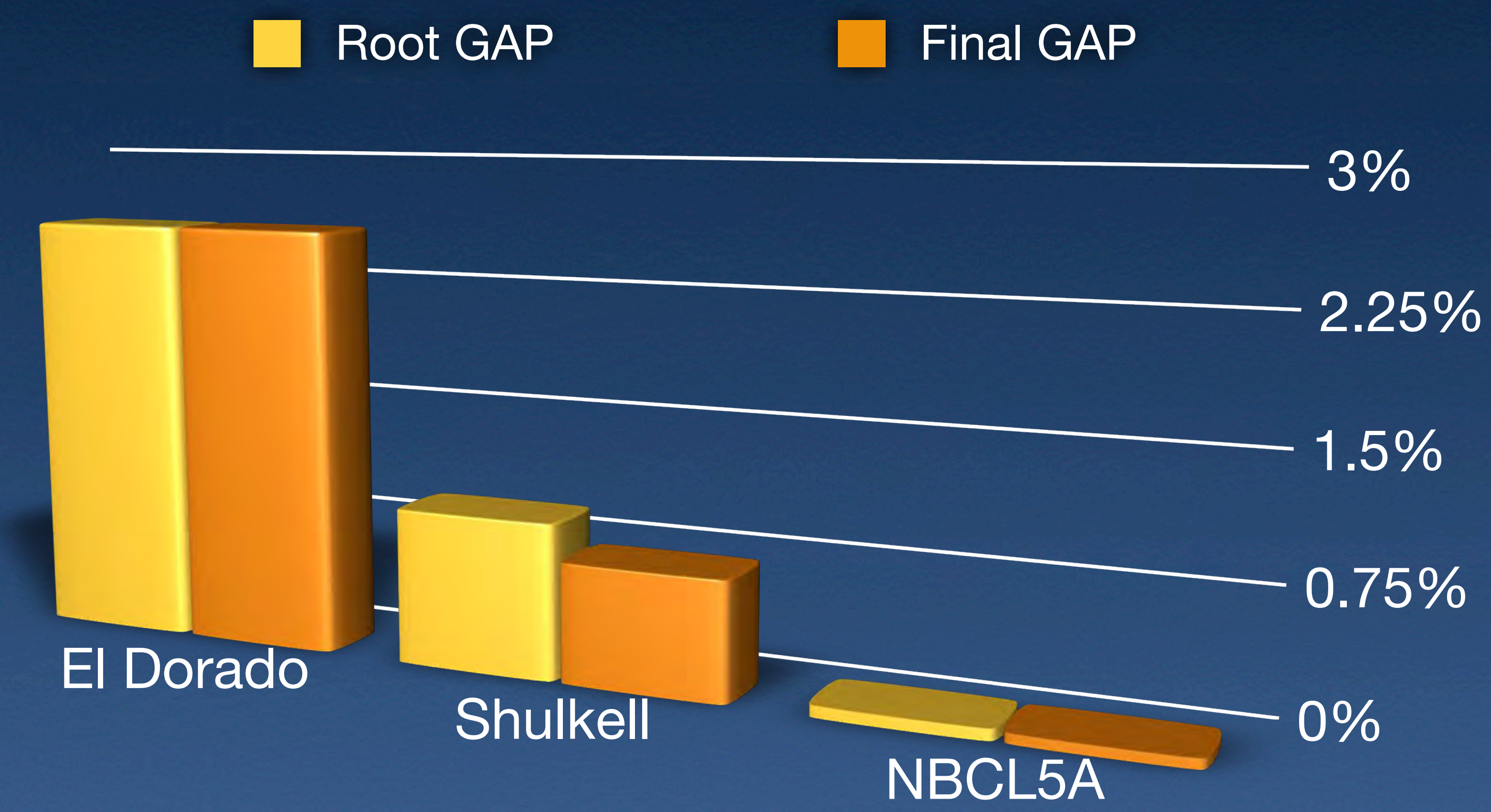

- Root Solve Time < 3 min

Results for Dynamic Patch

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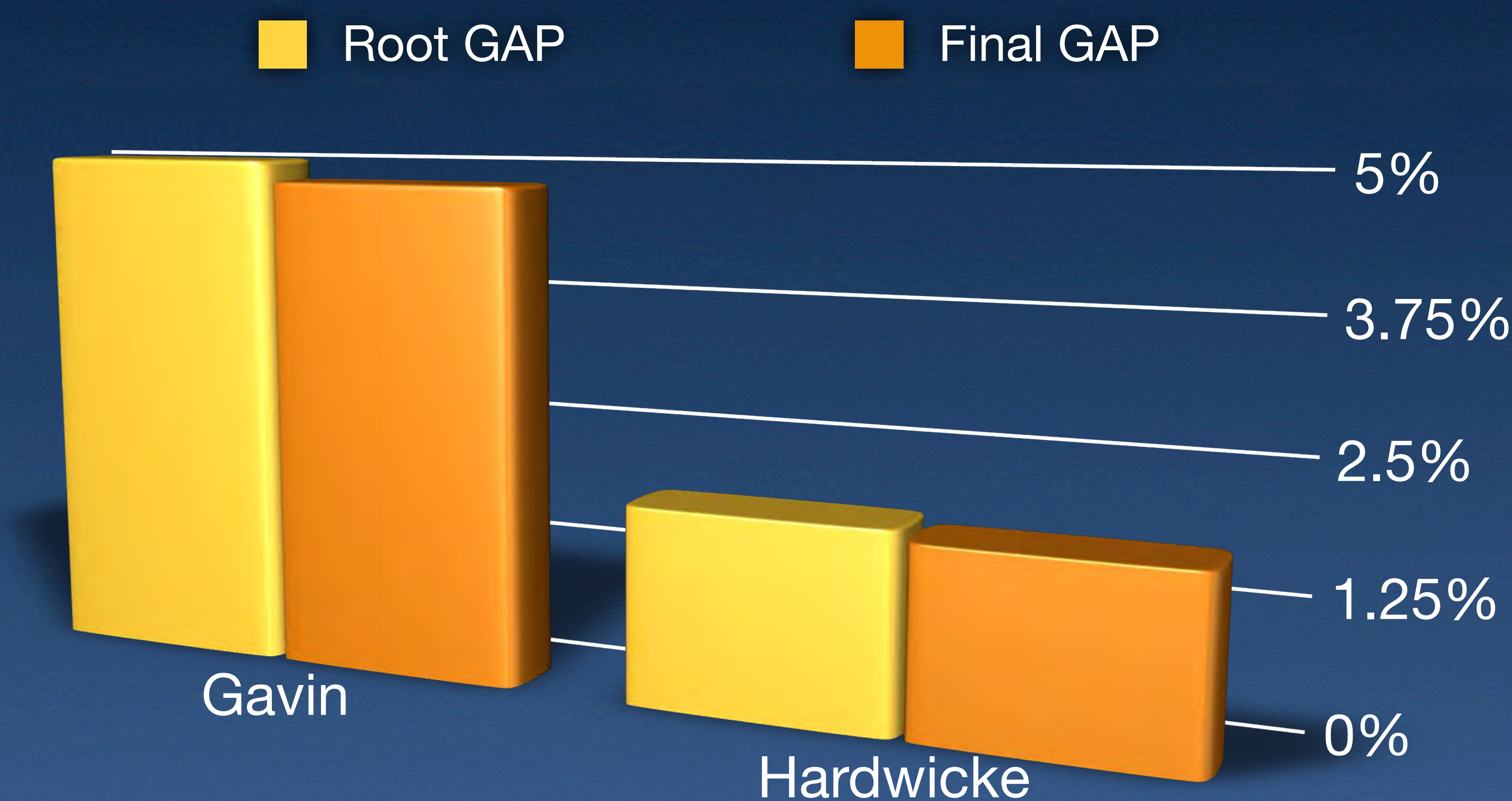
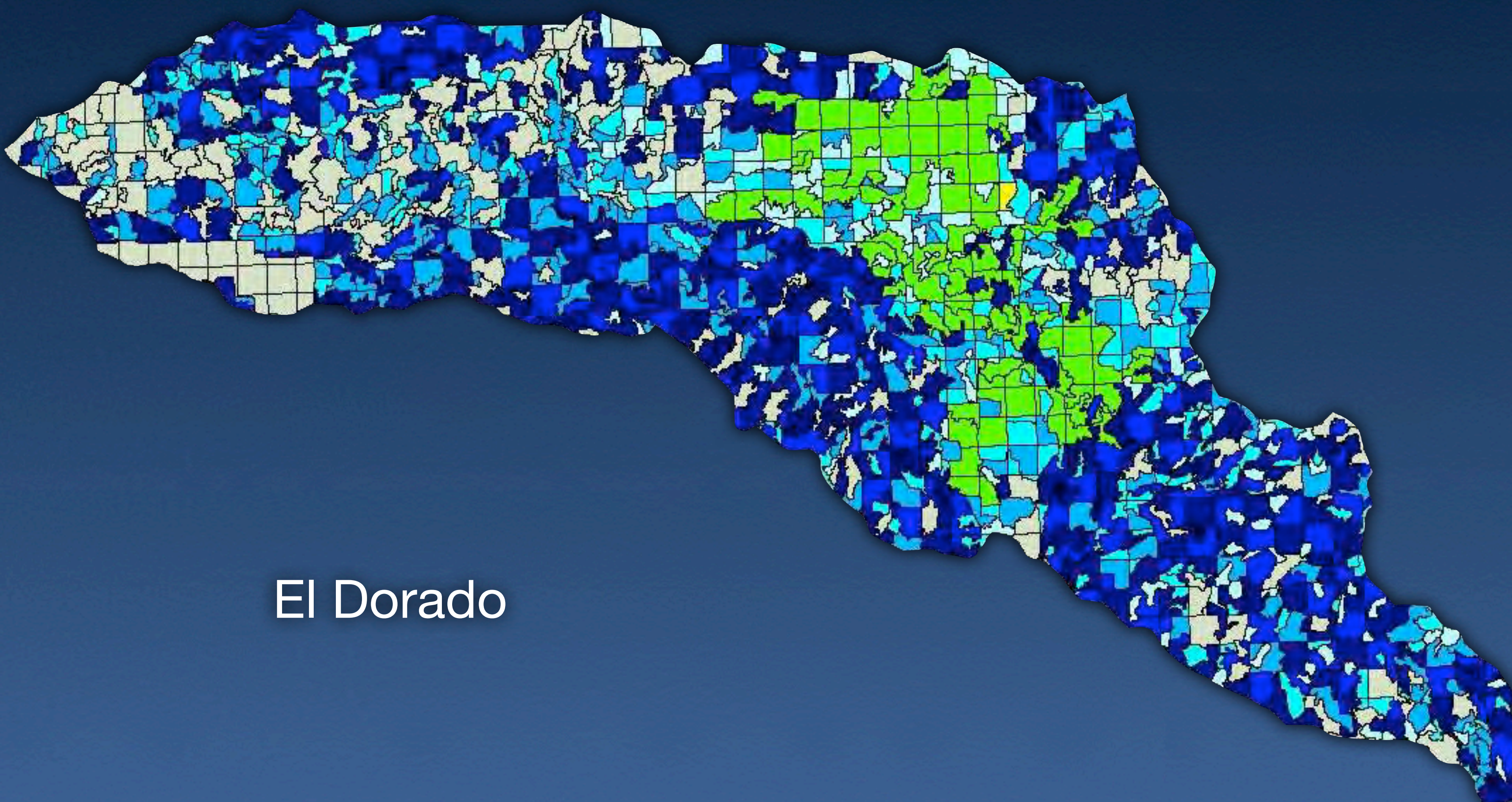


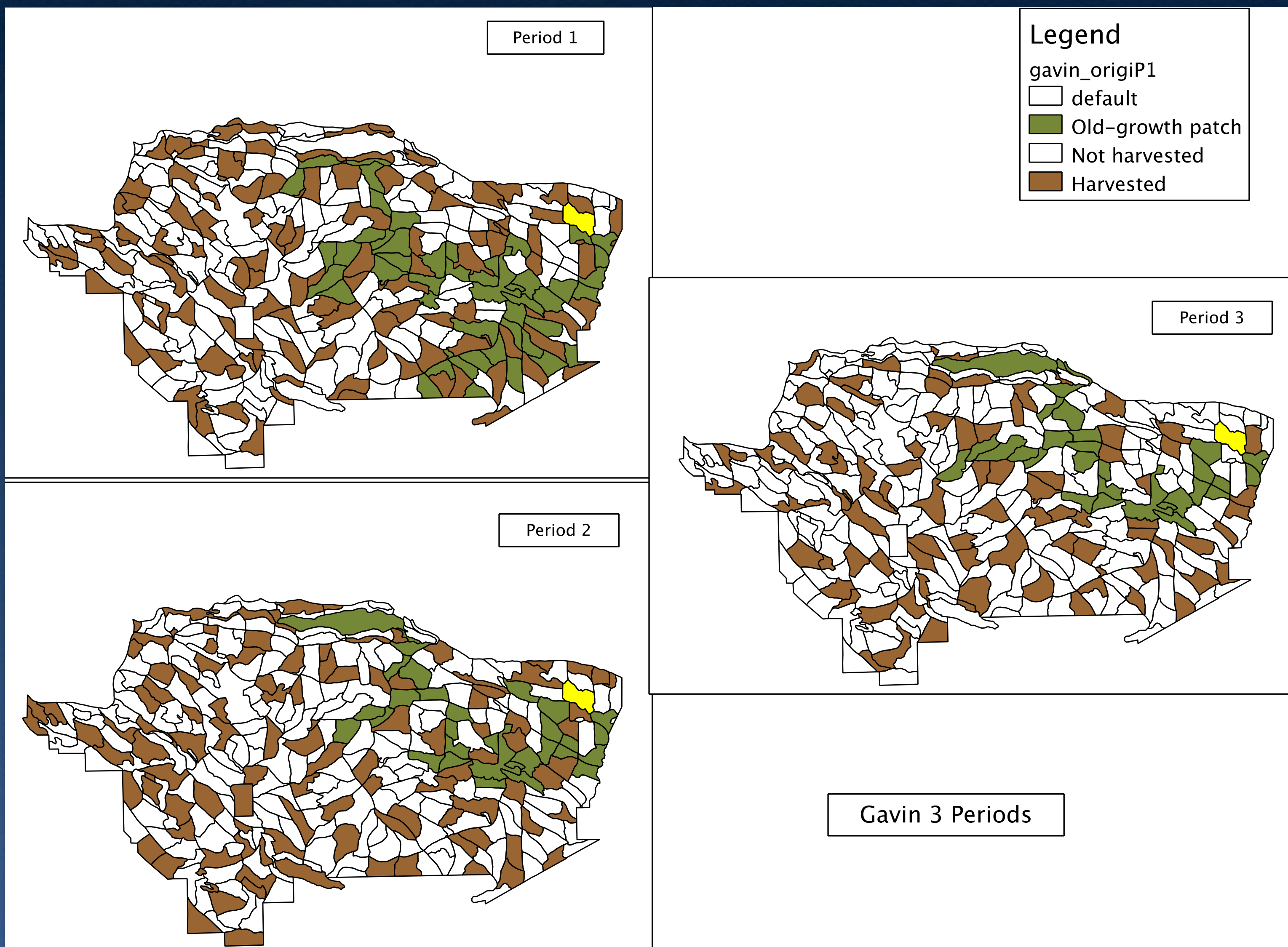
- Root Solve Time < 4 min

Solutions Sometime Look Good



El Dorado

Solutions Sometime Don't Look Good



Final Thoughts...

- Don’t blindly use IP:
  - Optimization can be “too” clever
  - Don’t fear “Large” formulations
  - Do use special purpose heuristics
- BIG open problem in IP: Compact extended formulation (Strongest) for general matching
  - Known large formulation is similar to Önal and Wang (2008)