

# Imposing Old-growth Patch Constraints in Forest Harvest Scheduling Models

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# Forest Harvest Scheduling



1. Maximize profits
2. Protect Environment

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1. Maximize profits

2. Protect Environment

● Schedule “stands” for harvest

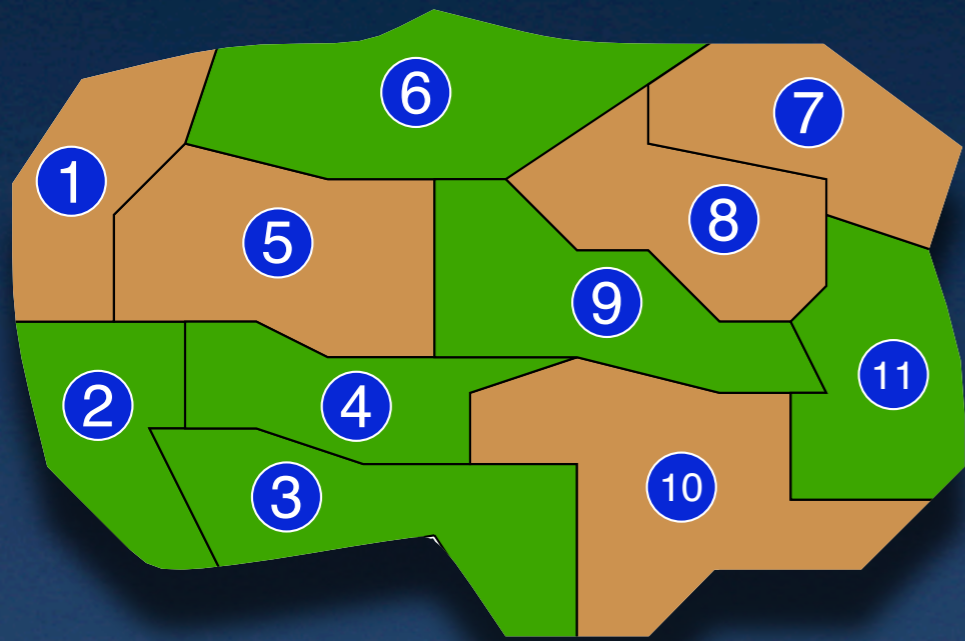
# Protect Environment: Connectivity

- Forbid large clear-cut areas:
  - Area Restriction Model (ARM)
  - Good IP models (6000 stands)
  - Goycoolea, Murray, V. and Weintraub, 2009.
- Protect some contiguous areas:
  - Harder problems (400 stands)
  - Old-growth, reserve selection, wildlife corridors

# Outline

- Introduction
- Connectivity Constraints
  - ARM
  - Area Protection
- Computational Example
- Conclusions and Future Work

# IP Models for Harvest Scheduling

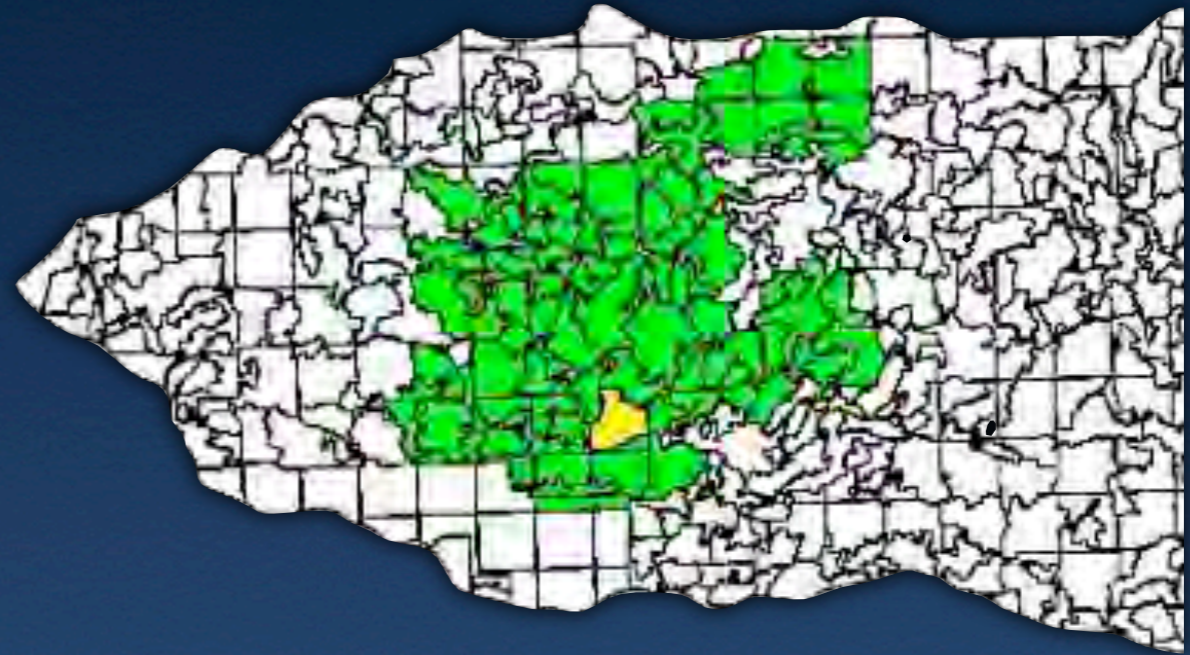


$$y_{v,t} = \begin{cases} 1 & \text{if stand } v \text{ is harvested} \\ & \text{in period } t. \\ 0 & \text{otherwise} \end{cases}$$

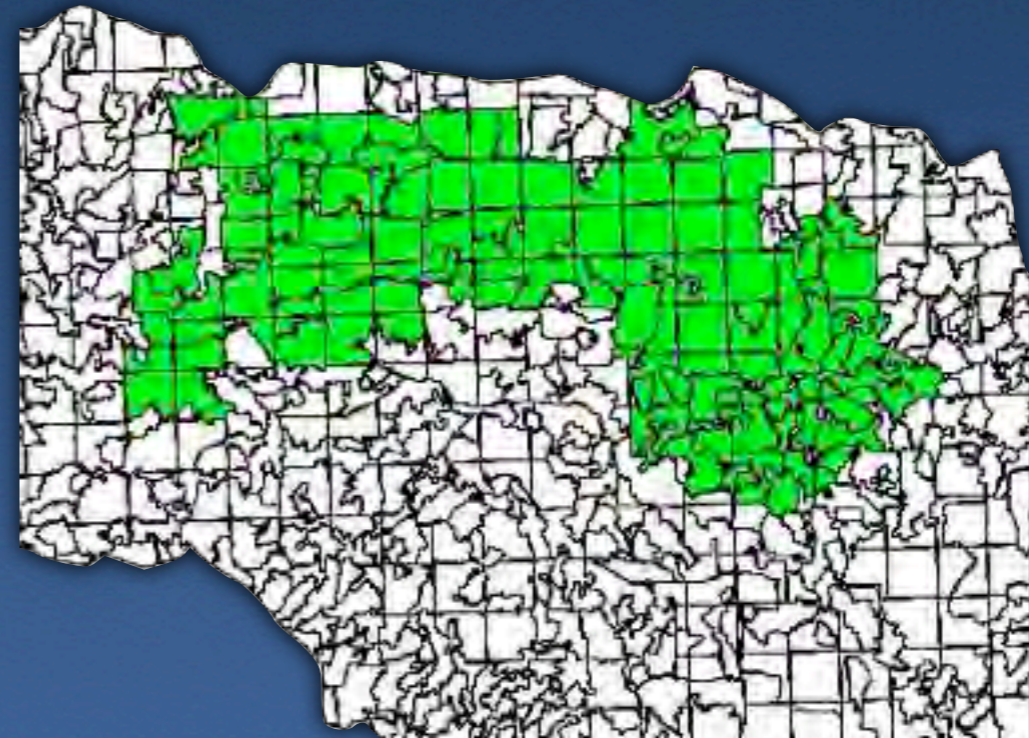
- Linear Constraints/Objective:
  - Profits, timber flow, ending age of forest, etc.
- Combinatorial Constraints:
  - Protect Environment

# Connectivity: Single Patch

- Rooted



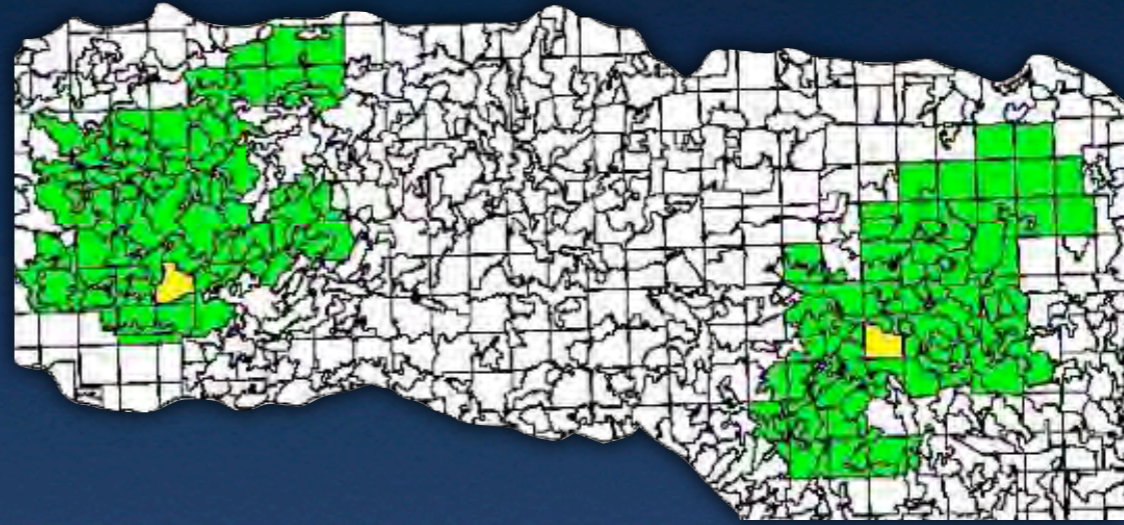
- Unrooted



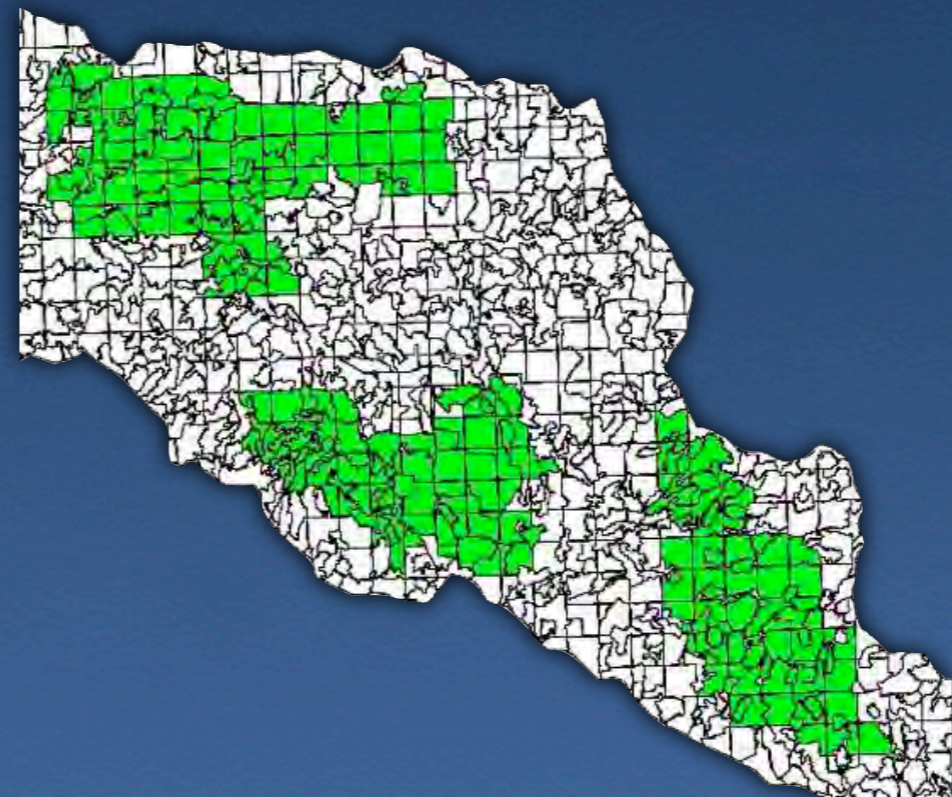


# Connectivity: Multiple Patches

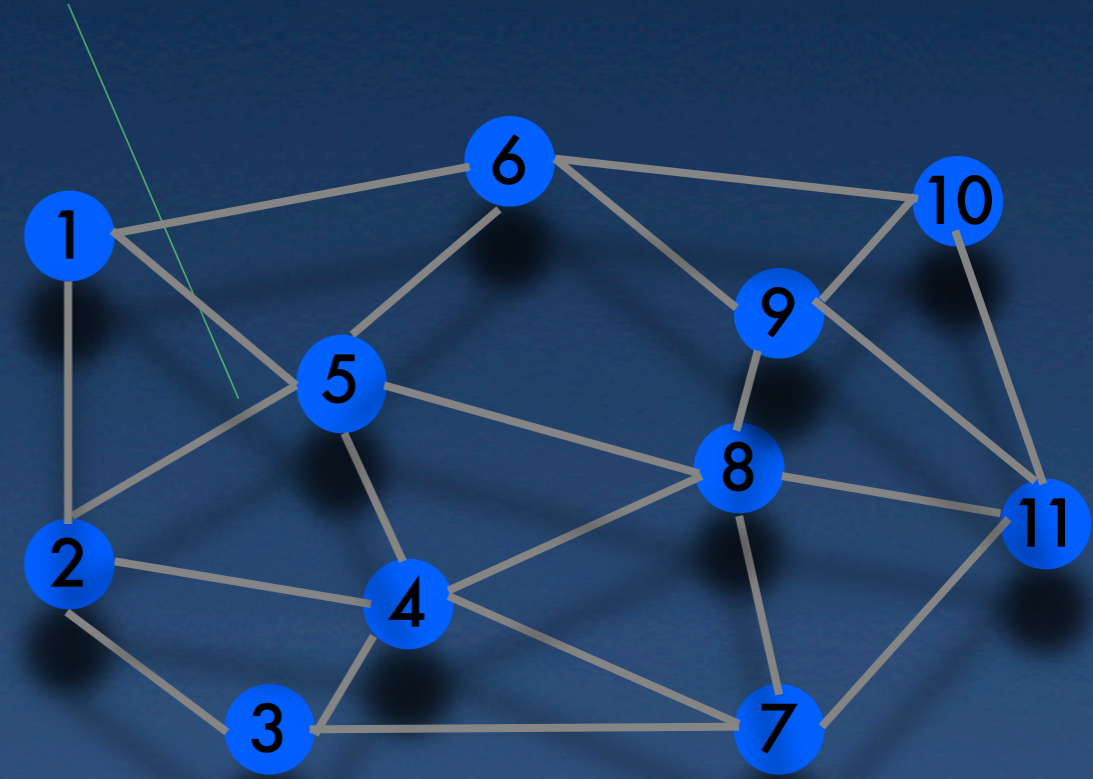
- Rooted



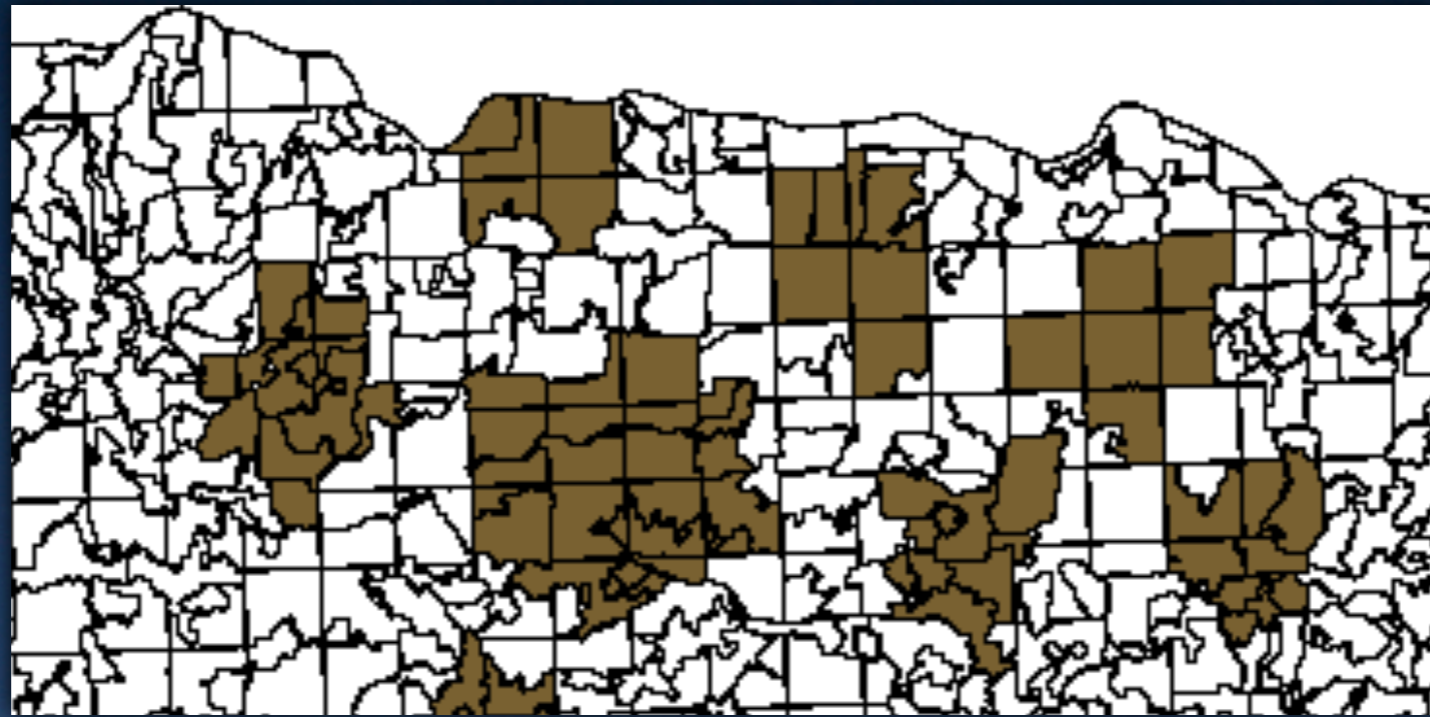
- Unrooted



# Graph Representation of Forest

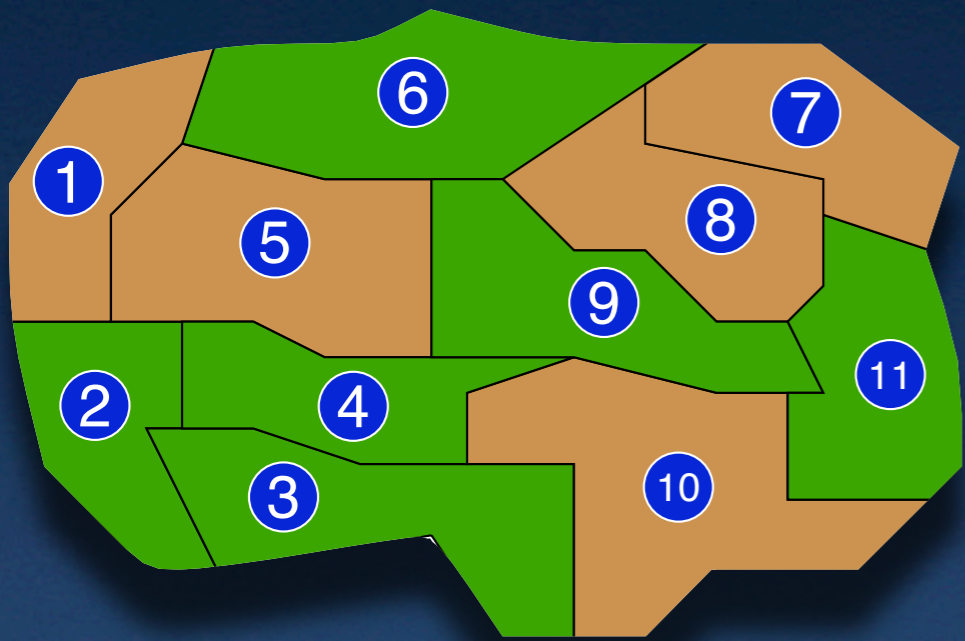


# Area Restriction Model (ARM)



- Limit area of contiguous clear-cut region
- Unrooted multi-patch model:
  - Limit *maximum area* of patches

# Assumptions and Notation

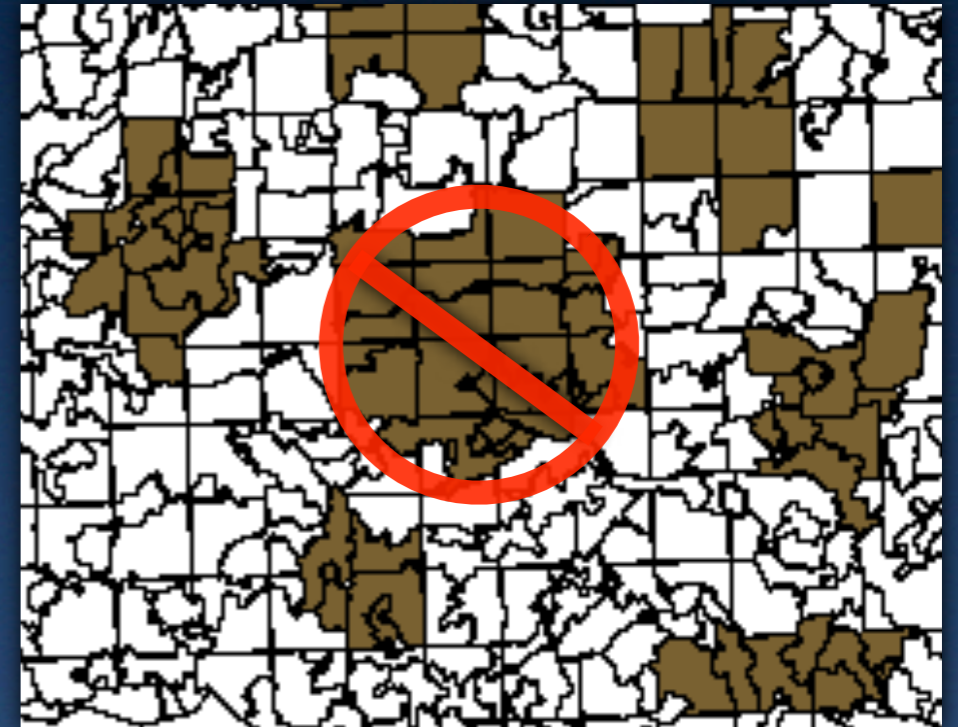


$$y_{v,t} = \begin{cases} 1 & \text{if stand } v \text{ is harvested} \\ & \text{in period } t. \\ 0 & \text{otherwise} \end{cases}$$

- Harvested stands are clear-cut and replanted
  - Stand harvested in  $t$  is clear-cut only in  $t$
  - ARM constraints span only one period
- Stands can only be harvested once

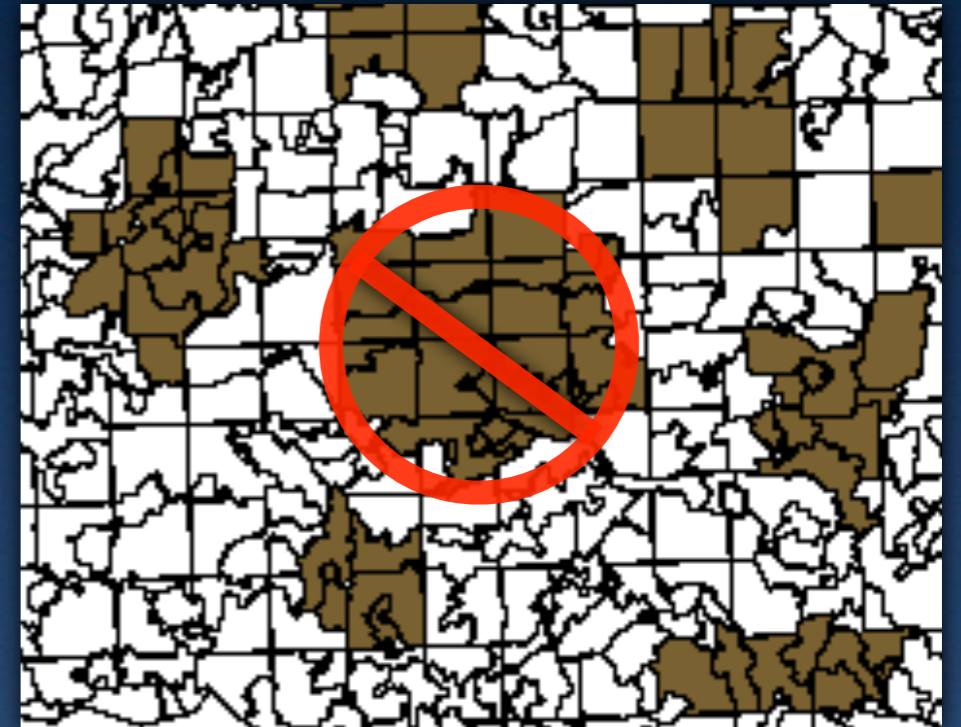
# ARM Constraints: Forbid Sets

- Connected set of stands  $C$ :
  - Area is strictly greater than *maximum area*
  - Minimal with respect to inclusion



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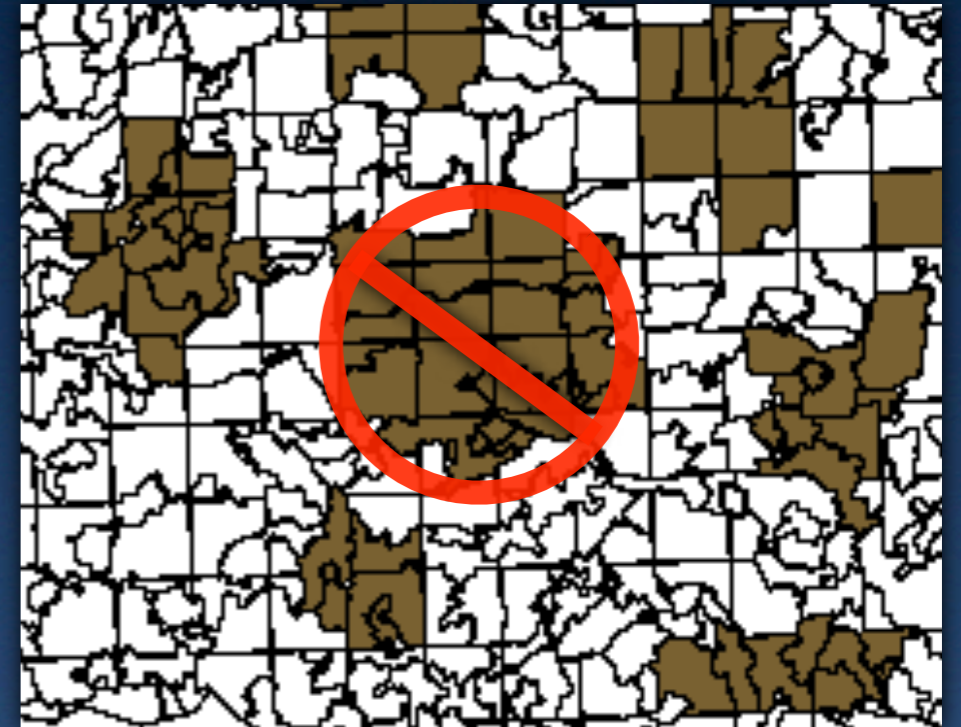
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$$\sum_{v \in C} y_{v,t} \leq |C| - 1$$

# ARM Constraints: Forbid Sets

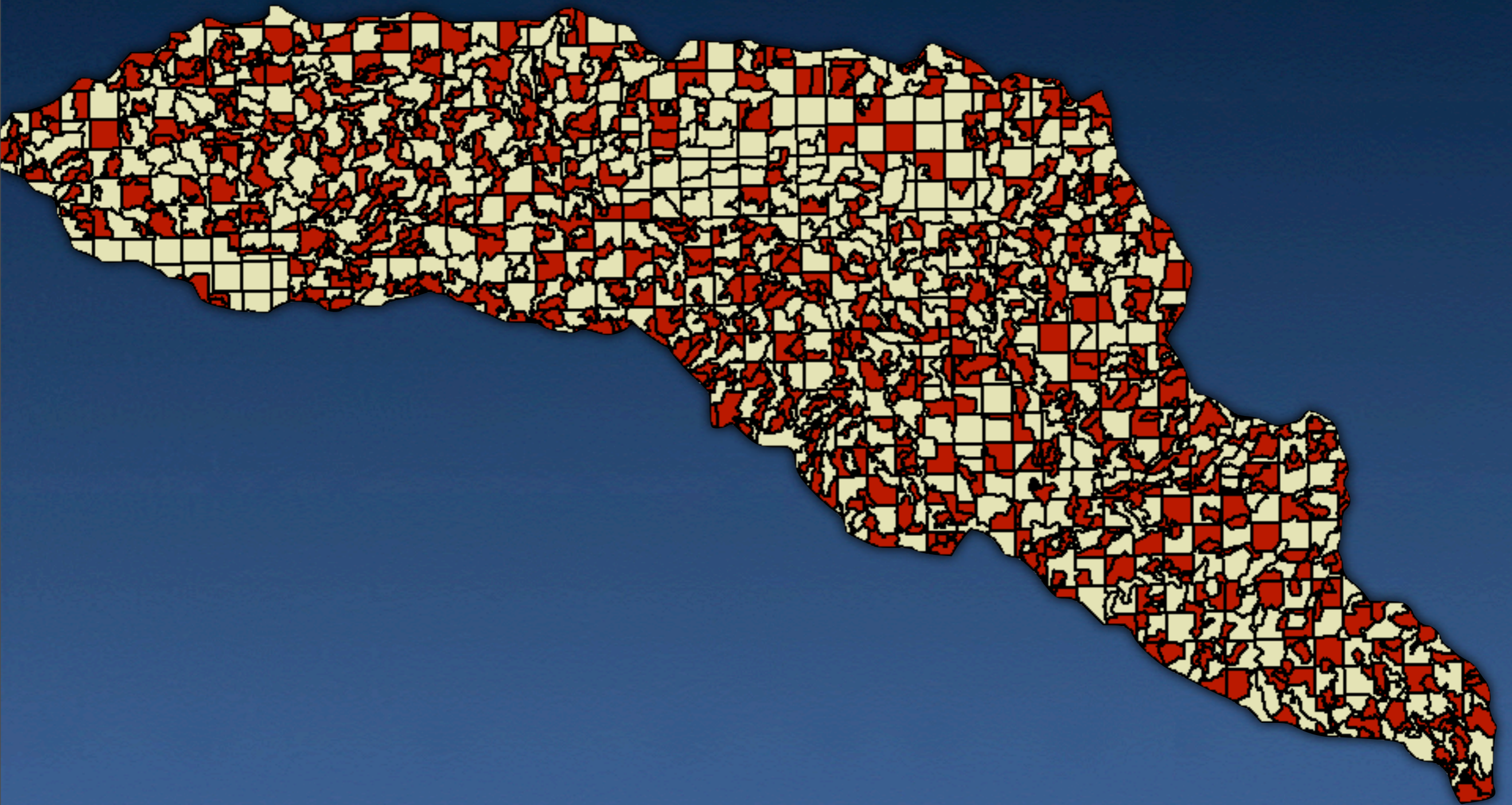
- Connected set of stands  $C$ :
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$$\sum_{v \in C} y_{v,t} \leq |C| - 1$$

- ☑ Usually few of these sets exist

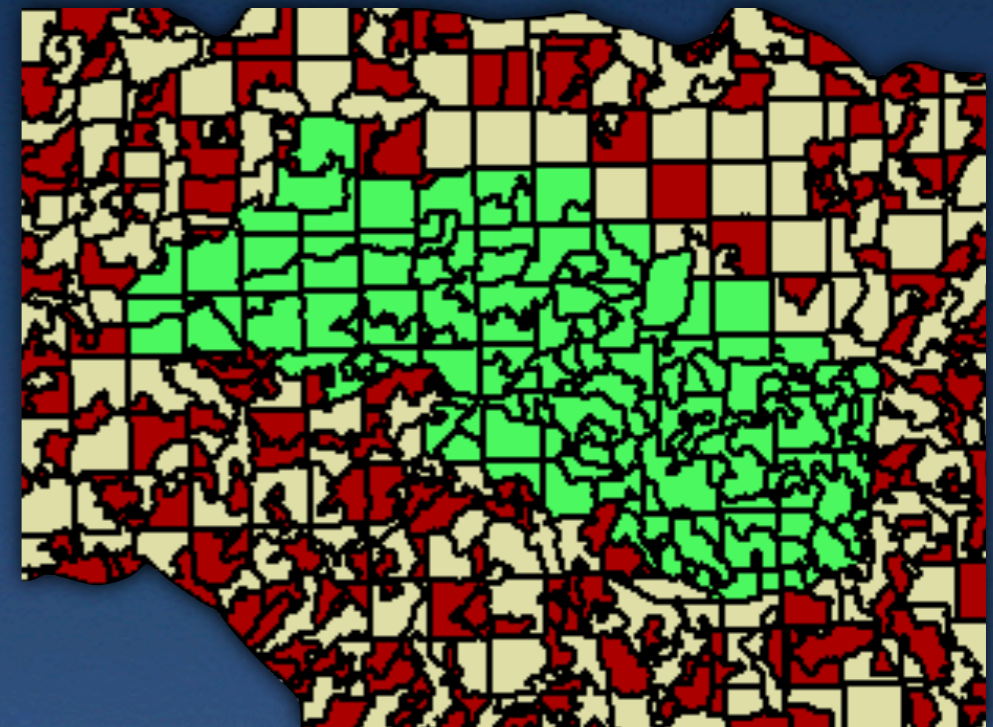
# ARM solution = Fragmentation



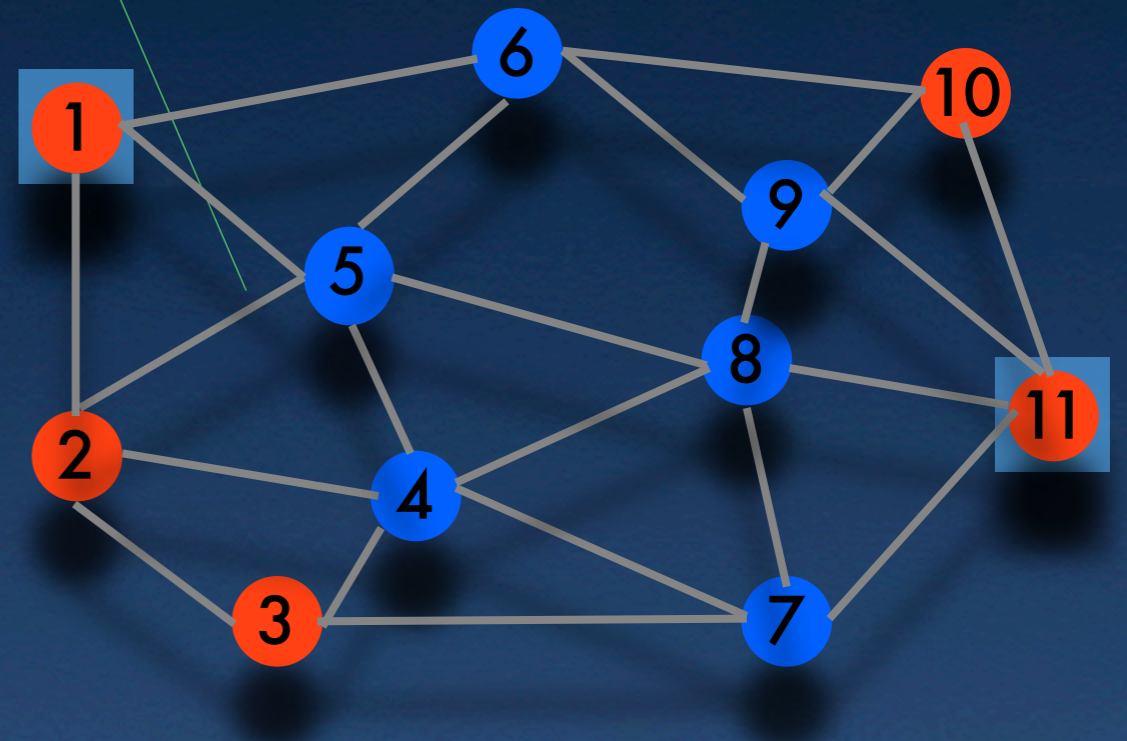
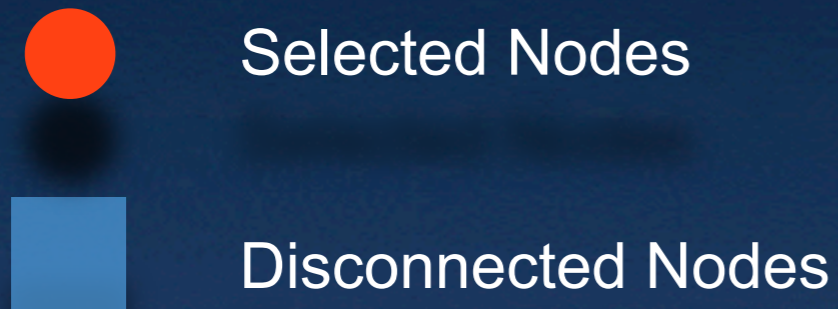


## Need Min Area Connected Patch

- 1+ connected regions:
  - *minimum* (average) area
  - is old-growth, contains animal population, contain water source, etc.
  - Other: shape, edge, etc.
- Force connectivity and add other constraints

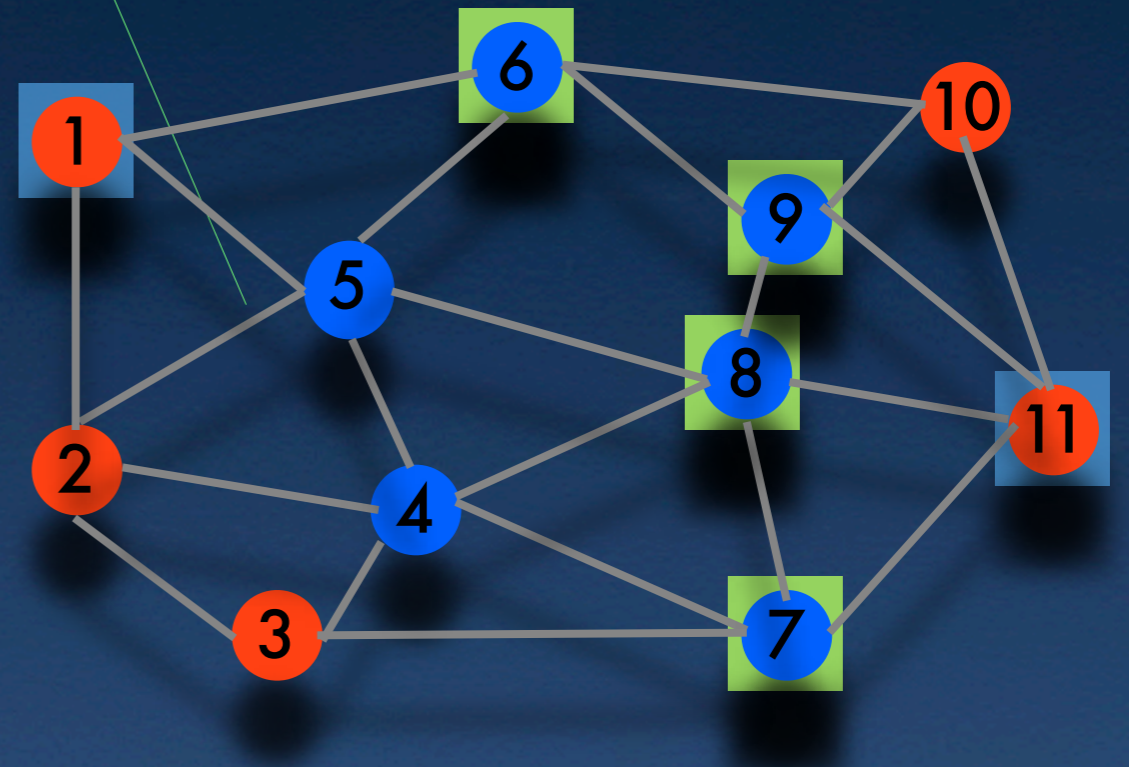
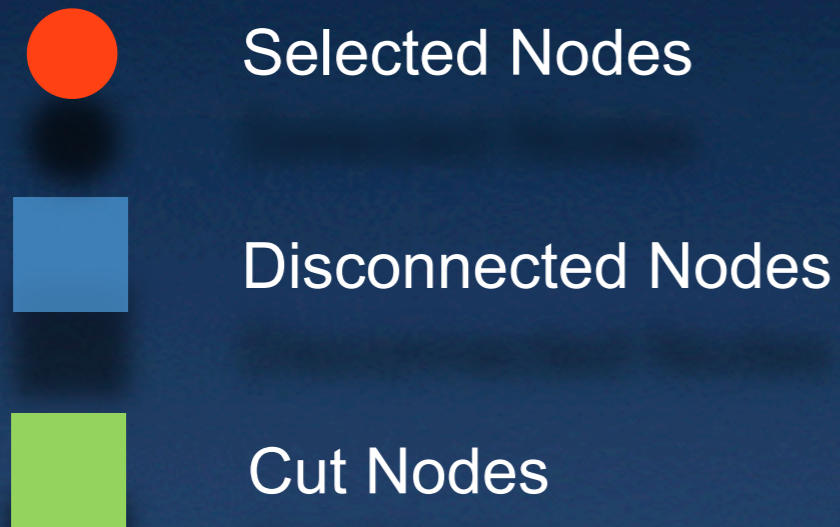


# Unrooted (Lack of) Connectivity



- Select red nodes for old-growth/reserve
- Red nodes are disconnected because:

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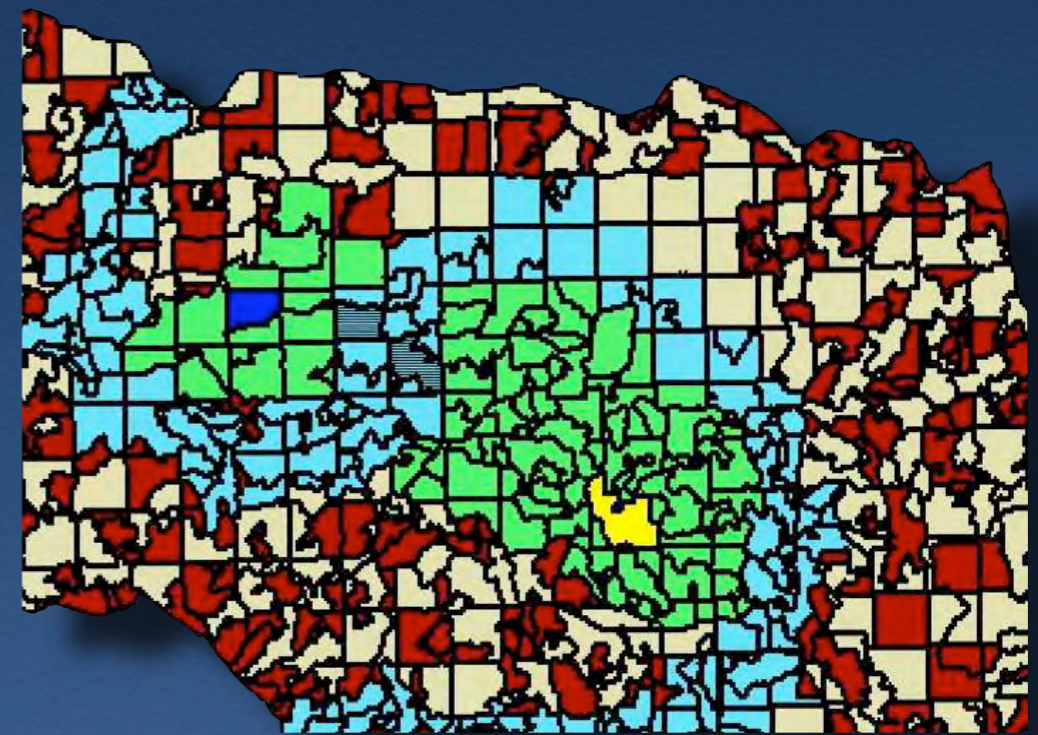
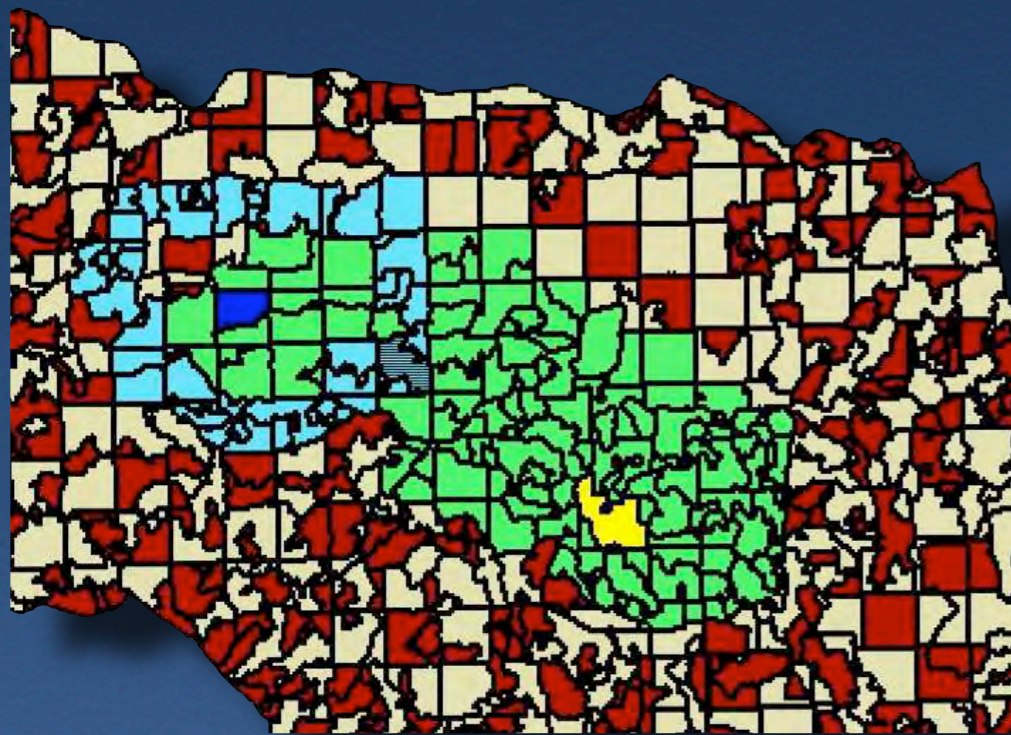
- Select red nodes for old-growth/reserve
- Red nodes are disconnected because:
  - There is a node-cut separating 1 and 11 with no selected nodes

## Selected Set Is Connected if ...

- Set is connected  $\Leftrightarrow$  pairs of nodes are connected
- Pairs are connected  $\Leftrightarrow$  every cut separating them intersects selected nodes

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■ Selected pair of nodes

■ Other selected nodes

■ Separating cut

■ Separating cut intersects selected nodes

# Force Connectivity Constraints

$$z_v = \begin{cases} 1 & \text{if stand } v \text{ is selected to be old-growth/reserve} \\ 0 & \text{otherwise} \end{cases}$$

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For every cut  $S$   
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- Rooted: All selected stands connected to root  $r$

$$\sum_{w \in S} z_w \geq z_v \quad \forall v$$

For every cut  $S$   
separating  $r$  and  $v$



# Advantages and Disadvantages

- ☑ Can easily add extra requirements
  - e.g. minimum area

$$\sum_v a_v z_v \geq A_{\text{Min}} \quad a_v = \text{area of stand } v$$

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- Too many separating-cut constraints
  - Separating the constraints is easy

# Cutting Plane Procedure

$$\sum_{w \in S} z_w \geq z_u + z_v - 1$$

$$z^* = \text{current solution}$$

Find cut  $S^*$  that separates  $u, v$  and minimizes  $\sum_{w \in S^*} z_w^*$

For every pair  $u, v$   
check if  $z_u^* + z_v^* - 1 > 0$

Use max-flow solver

If  $\sum_{w \in S^*} z_w^* < z_u^* + z_v^* - 1$   
add cut

# Problem Specification

1. Maximize NPV of harvest schedule s.t.:

- ARM Constraints: maximum clear-cut
- Volume flow constraints
- Bound on average ending age of forest

2. Additionally:

- Reserve 10% of forest area as a contiguous old-growth path (unrooted model)

# Instances and Solvers

- 5-period instances from FMOS repository:

Instance	Stands	Total area	Max CC Area
El Dorado	1363	52,255.5	120
Shulkell	1039	11,116.65	40
NBCL5A	5581	149,235	80
FLG9A	850	24,708.1	80

- CPLEX 11 on a Quad-core Xeon with 32Gb RAM

## Results: Time limit of 4 hours

### 1. ARM:

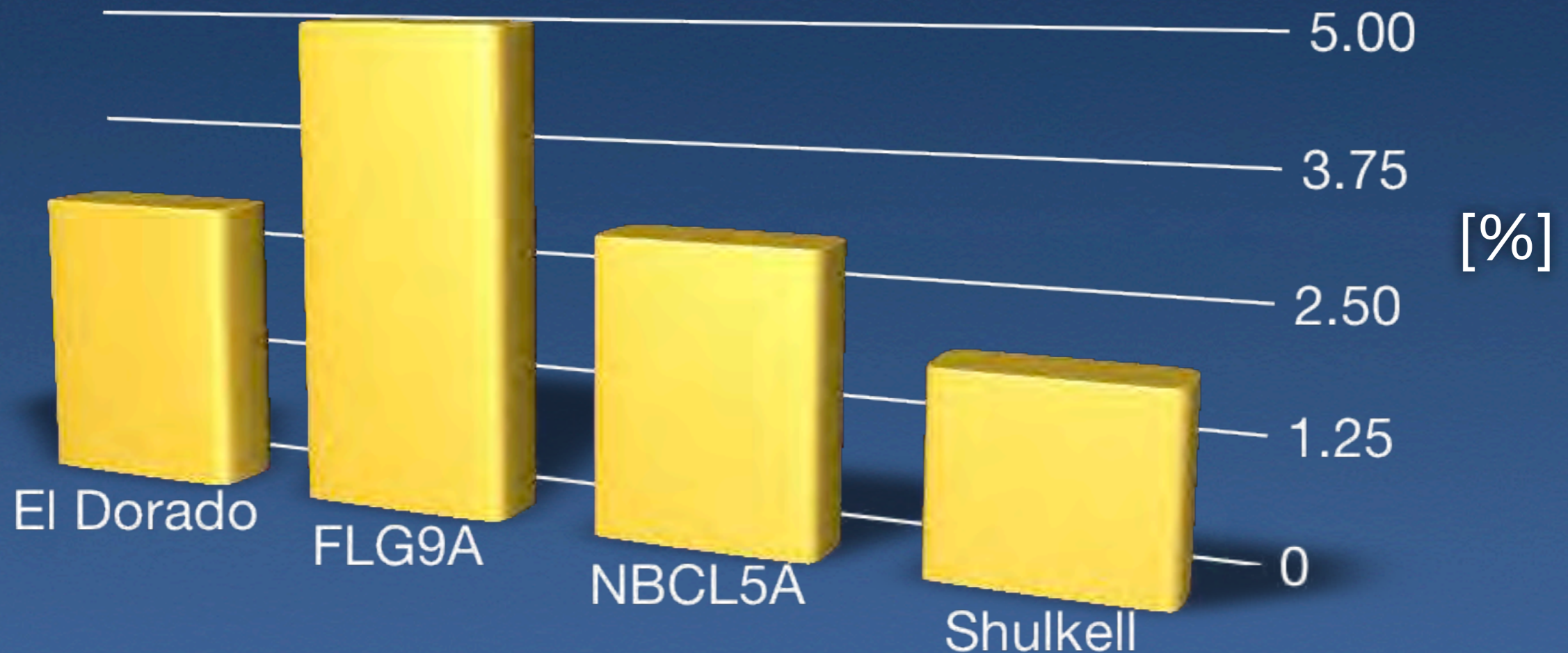
- Directly solved by CPLEX
- 3 optimal in <400s, 1 reaches 0.03% GAP

### 2. ARM+old-growth

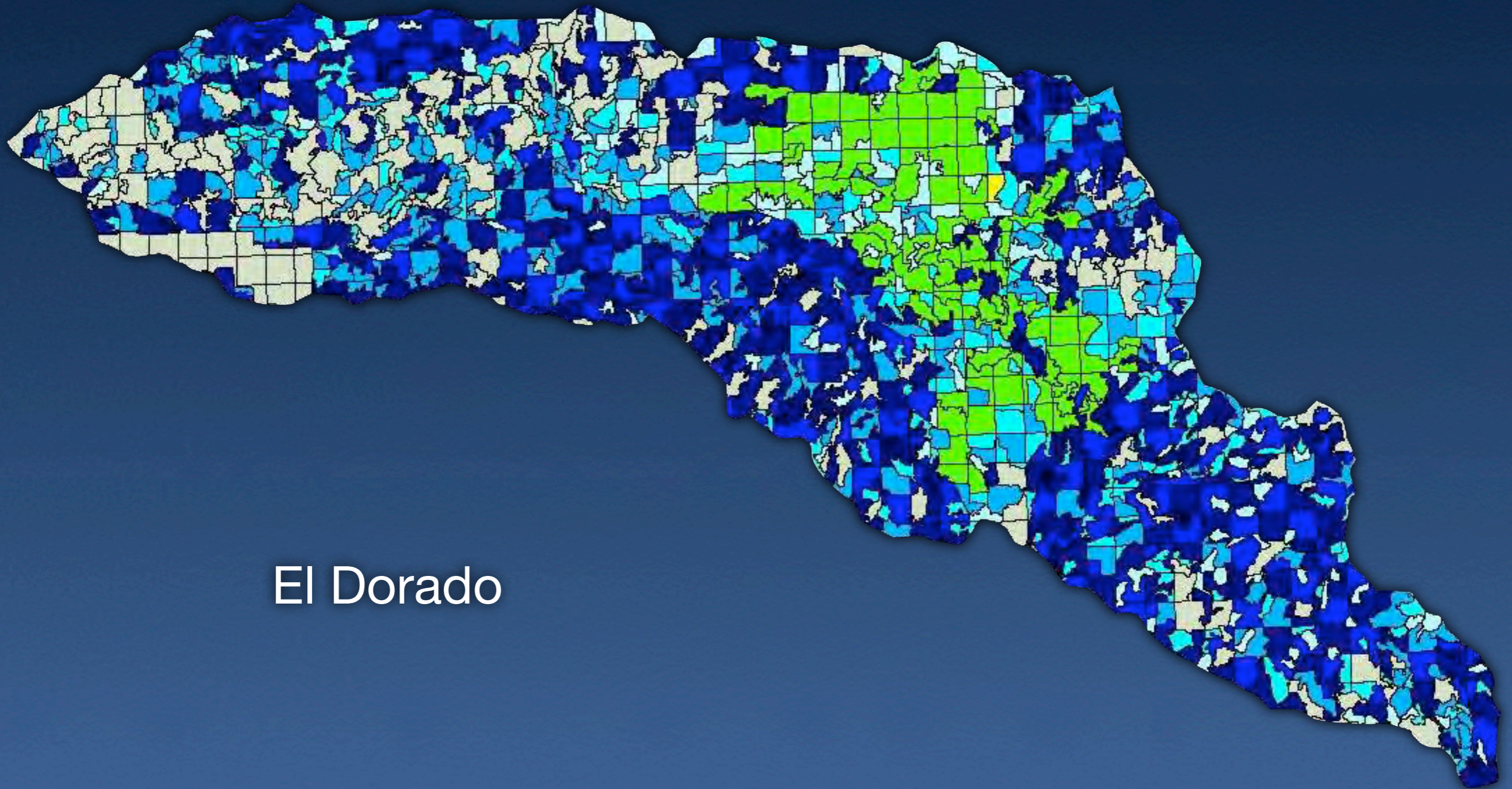
- CPLEX based branch-and-cut: need heuristics, use of rooted formulations, “ring” cuts, etc.
- 3 with <1% GAP, 1 with 2.2% GAP

# Economic Effect

- ARM: 2-5% loss in NPV
- ARM+old-growth: additional loss of:



# Solutions Sometime Look Good

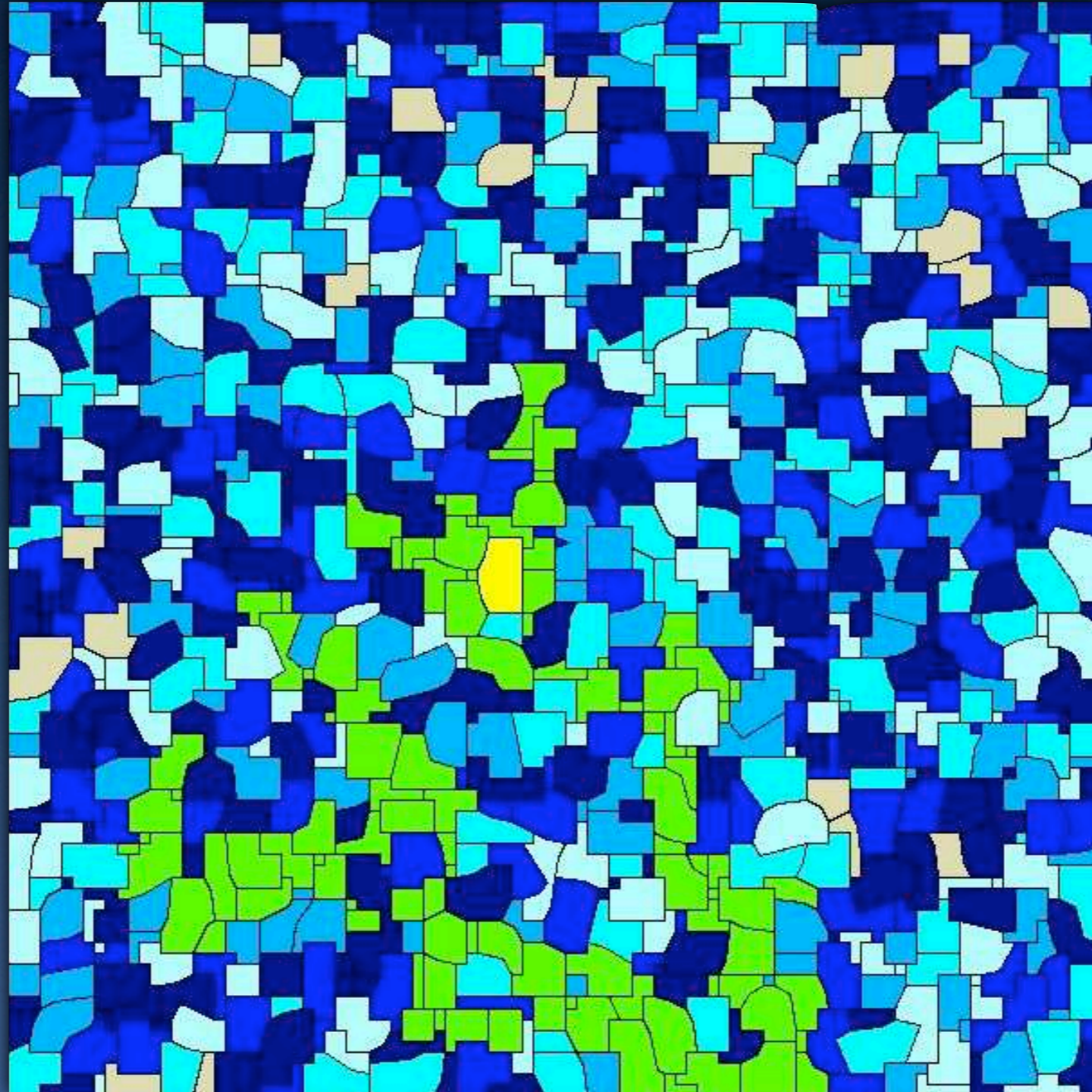


El Dorado



# Solutions Sometime Don't Look Good

FLG9A



# Conclusions and Future Work

- Done:
  - Connectivity for environmental protection
  - Can obtain good solutions for old-growth
  - Optimization: Cost is moderate
- To do:
  - Optimization too “clever”: snake like patches
  - Some challenges in branch-and-cut