Extended Formulations for Quadratic Mixed Integer Programming

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Nonlinear MIP B&B Algorithms

\[
\max \sum_{i=1}^{n} c_i x_i \\
\text{s.t.} \quad g_i(x) \leq 0, \ i \in I, \quad x \in \mathbb{Z}^n \\
x \in \mathbb{Z}^{n_1} \times \mathbb{R}^{n_2}
\]
Nonlinear MIP B&B Algorithms

• NLP (QCP) Based B&B

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• NLP (QCP) Based B&B
  • Few cuts = high speed.
  • Possible slow convergence.

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Extended Formulations for MIQCP
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- (Dynamic) LP Based B&B
  
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Nonlinear MIP B&B Algorithms

- **NLP (QCP) Based B&B**
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- **(Dynamic) LP Based B&B**
  - Dynamic relaxation
  - Standard LP B&B

- **LiTed LP B&B**
  - Extended or Lifted relaxation.
  - Static relaxation
    - Mimic NLP B&B.

Extended Formulations for MIQCP
Nonlinear MIP B&B Algorithms

- NLP (QCP) Based B&B
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- Lifted LP B&B
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    - Mimic NLP B&B.
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\begin{align*}
\text{max} & \quad \sum_{i=1}^{n} c_i x_i \\
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& \quad x \in \mathbb{Z}^{n_1} \times \mathbb{R}^{n_2}
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\]
• Approximation of Second Order Cone of dimension $n$ by Ben-Tal and Nemirovski (Glineur).

• $O(n \log(1/\varepsilon))$ variables and constraints for quality $\varepsilon$.
  – Exponential increment in # of constraints through projection.

• Problem:
  – Fixed a-priori quality: no dynamic improvement (e.g. $\varepsilon = 0.01$ best for some portfolio optimization problems)
Correct Quality = Significant Improvement

- Results from V., Ahmed and Nemhauser ’08.
Dynamic Lifted LP for Separable Problems

- Motivating example from Hijazi et al. ‘14

\[ F^n := \left\{ x \in \mathbb{R}^n : \sum_{i=1}^{n} \left( x_i - \frac{1}{2} \right)^2 \leq \frac{n-1}{4} \right\} \]

Showing \( F^n \cap \mathbb{Z}^n = \emptyset \) requires \( 2^n \) cuts.
Dynamic Lifted LP for Separable Problems

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Extended formulation of \( F^n \):

\[
\left( x_i - \frac{1}{2} \right)^2 \leq z_i \quad \forall i \in [n]
\]

\[
\sum_{i=1}^{n} z_i \leq \frac{n - 1}{4}
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\[ \left(x_i - \frac{1}{2}\right)^2 \leq z_i \quad \forall i \in [n] \]

\[ \sum_{i=1}^{n} z_i \leq \frac{n - 1}{4} \]

\( B^n \cap \mathbb{Z}^n = \emptyset \) with only \( 2n \) cuts on extended formulation.
Significant Improvement For Many Problems

• Tawarmalani and Sahinidis ’05:

\[ f_j : \mathbb{R} \to \mathbb{R} \text{ convex differentiable} \]

Lifted Relaxation of \( F := \left\{ (y_0, y) \in \mathbb{R}^{n+1} : \sum_{j=1}^{n} f_j(y_j) \leq y_0 \right\} : \]

\[ f_j(\gamma) + f'_j(\gamma)(y_j - \gamma) \leq w_j \quad \forall \gamma \in \Gamma_j, \quad j \in [n] \]

\[ \sum_{j=1}^{n} w_j \leq y_0 \]
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Projection to \( (y_0, y) = \text{ up to } \prod_{j=1}^{n} |\Gamma_j| \text{ constraints} \)
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    f_j(\gamma) + f_j'(\gamma)(y_j - \gamma) &\leq w_j \quad \forall \gamma \in \Gamma_j, \quad j \in [n] \\
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Projection to \((y_0, y) = \text{up to } \prod_{j=1}^{n} |\Gamma_j| \text{ constraints} \)

• Polynomial (degree \( n \)) increment in # of constraints through projection
Separable Approach for Conic Quadratic?

• “Separable Sets” include many quadratics:
  – Euclidean Ball
    \[ B^n := \left\{ y \in \mathbb{R}^n : \sum_{j=1}^{n} y_j^2 \leq 1 \right\} \]
  – Paraboloids
    \[ Q^n := \left\{ (y_0, y) \in \mathbb{R}^{n+1} : \sum_{j=1}^{n} y_j^2 \leq y_0 \right\} \]

• Does not include Lorentz/SOCP cone:
  \[ L^n := \left\{ (y_0, y) \in \mathbb{R}^{n+1} : \sqrt{\sum_{j=1}^{n} y_j^2} \leq y_0 \right\} \]
Separable to Conic: Homogenize

- From Euclid to Lorentz

\[ B^n := \left\{ y \in \mathbb{R}^n : \sum_{j=1}^{n} y_j^2 \leq 1 \right\} \]

\[ L^n := \left\{ (y_0, y) \in \mathbb{R}^{n+1} : \sqrt{\sum_{j=1}^{n} y_j^2} \leq y_0 \right\} \]

\[ L^n = \text{cone} \left( \{1\} \times B^n \right) \]
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\[ B^n \subseteq P \]
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\[ L^n = \text{cone} (\{1\} \times B^n) \]

\[ B^n \subseteq P \Rightarrow L^n \subseteq \text{cone} (\{1\} \times P) \]
Lifted Relaxation for Separable Conic Sets

\[ f_j : \mathbb{R} \to \mathbb{R} \text{ convex, differentiable and } 1\text{-coercive} \]

\[ C := \left\{ y \in \mathbb{R}^n : \sum_{j=1}^{n} f_j(y_j) \leq 1 \right\} \]

Lifted Relaxation of \( \text{cone} \left( \{1\} \times C \right) \):

\[
(f(\gamma) - \gamma f'(\gamma))y_0 + f'(\gamma)y \leq w_j \quad \forall \gamma \in \Gamma_j, \ j \in [n]
\]

\[
\sum_{j=1}^{n} w_j \leq y_0
\]

\[
0 \leq y_0
\]
Lifted Reformulation for Conic Quadratic Sets

- Lifted Reformulation of $L^n := \{(y_0, y) \in \mathbb{R}^{n+1} : \|y\|_2 \leq y_0\}$:

\[
y_i^2 \leq z_i \cdot y_0 \quad \forall i \in [n]
\]

\[
\sum_{i=1}^{n} z_i \leq y_0
\]
Lifted Reformulation for Conic Quadratic Sets

- Lifted Reformulation of $L^n := \{(y_0, y) \in \mathbb{R}^{n+1} : \|y\|_2 \leq y_0\}$:

  - $y_i^2 \leq z_i \cdot y_0$
  - $\sum_{i=1}^{n} z_i \leq y_0$  \hspace{0.5cm} \forall i \in [n]$

  Rotated SOCP cone
Lifted Reformulation for Conic Quadratic Sets

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  y_i^2 &\leq z_i \cdot y_0 \\
\sum_{i=1}^{n} z_i &\leq y_0
\end{align*}
\]

\[\forall i \in [n] \]

Rotated SOCP cone

• Lifted polyhedral relaxation automatic from standard polyhedral approximation of (rotated) SOCP cone:
  – Dynamic Lifted LP-based algorithm:
    1. Replace every SOCP cone with lifted reformulation
    2. Solve with standard LP-based algorithm
Computational Experiments 1: Solvers
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• NLP-based Branch-and-Bound:
  – CPLEXCP : MIQCPSTRAT = 1
  – GurobiCP : MIQCPMethod = 0
Computational Experiments 1: Solvers

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• Traditional LP-based Branch-and-Bound:
  – CPLEXLP : MIQCPSTRAT = 2
  – GurobiLP : MIQCPMethod = 1
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• Dynamic Lifted LP-based Branch-and-Bound:
  – CPLEXSepLP : CPLEXLP on lifted reformulation
  – GurobiSepLP : GurobiLP on lifted reformulation
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- CPLEX v 12.6 and Gurobi v 5.6.3
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• CPLEX v 12.6 and Gurobi v 5.6.3

• Time limit of 3,600 s on i7-3770 3.40GHz
Computational Experiments 1: Instances

• Portfolio optimization problems:

Classical:

\[ \text{max} \quad \tilde{\mathbf{a}} \mathbf{x} \]

\[ \text{s.t.} \]

\[ \left\| Q^{1/2} \mathbf{x} \right\|_2 \leq \sigma \]

\[ \sum_{j=1}^{n} x_j = 1, \quad x \in \mathbb{R}_+^n \]

\[ x_j \leq z_j \quad \forall j \in [n] \]

\[ \sum_{j=1}^{n} z_j \leq K, \quad z \in \{0, 1\}^n \]

• $\tilde{\mathbf{a}}$ expected returns.

• $Q^{1/2}$ square root of covariance matrix.

• $K$ maximum number of assets.

• $\sigma$ maximum risk.
Results for CPLEX: 100 instances per n

Extended Formulations for MIQCP
Results for Gurobi: 100 instances per n

Time [s]

20 30 40 50 60

GurobiLP  GurobiCP  GurobiSepLP

Extended Formulations for MIQCP
Computational Experiments 2: More Solvers

• Static Lifted LP-based Branch-and-Bound:
  – **LiftedLP**: from V., Ahmed and Nemhauser ’08
  – Fixed approximation by Ben-Tal and Nemirovski (Glineur)
  – No refinement: integer nodes = solve NLP and process
  – Heuristic: Correct integral solutions (fix integers, solve NLP)
  – CPLEX Branch, heuristic and incumbent callbacks in JuMP
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• Static/Dynamic Lifted LP-based Branch-and-Bound:
  – *CPLEXSepLazy / GurobiSepLazy*: LiftedLP + Refinement through Separable extended formulation
  – Solver independent implementation in JuMP
Computational Experiments 2: More Solvers

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- ~400 lines of JuMP code in about a week
JuMP : Julia for Mathematical Programming

- Developed by ORC students: Iain Dunning, Joey Huchette and Miles Lubin.
- “As easy as AMPL and faster than C++” (JPV 2014).
- Linear/Quadratic MIP and general nonlinear
  - Cbc/Clp, CPLEX, ECOS, GLPK, Gurobi, Ipopt, KNITRO, MOSEK, and Nlopt.
- Automatic differentiation, solver independent MIP callbacks, etc.
- https://github.com/JuliaOpt/JuMP.jl
Computational Experiments 2: More Instances

Portfolio optimization problems:

Classical:

\[
\begin{align*}
\text{max} & \quad \bar{a}x \\
\text{s.t.} & \quad \left\| Q^{1/2}x \right\|_2 \leq \sigma \\
& \quad \sum_{j=1}^{n} x_j = 1, \quad x \in \mathbb{R}_+^n \\
& \quad x_j \leq z_j \quad \forall j \in [n] \\
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- \(\bar{a}\) expected returns.
- \(Q^{1/2}\) square root of covariance matrix.
- \(K\) maximum number of assets.
- \(\sigma\) maximum risk.
Computational Experiments 2 : More Instances

- Portfolio optimization problems:

**Shortfall:**

\[
\begin{align*}
\text{max} & \quad \bar{a} x \\
\text{s.t.} & \quad \Phi^{-1}(\eta_i) \left\| Q^{1/2} y \right\|_2 \leq \bar{a} y - W_i^{low} \quad i \in \{1, 2\} \\
& \quad \sum_{j=1}^{\infty} x_j = 1, \quad x \in \mathbb{R}_+^n \\
& \quad x_j \leq z_j \quad \forall j \in [n] \\
& \quad \sum_{j=1}^{n} z_j \leq K, \quad z \in \{0, 1\}^n
\end{align*}
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- $\bar{a}$ expected returns.
- $Q^{1/2}$ square root of covariance matrix.
- $K$ maximum number of assets.
- $\approx P(\bar{a} x \geq W_i^{low}) \geq \eta_i$
Computational Experiments 2: More Instances

- Portfolio optimization problems:

Robust:

\[
\begin{align*}
\max & \quad \bar{a}x - \alpha \left\| R^{1/2}y \right\|_2 \\
\text{s.t.} & \quad \left\| Q^{1/2}x \right\|_2 \leq \sigma \\
& \quad \sum_{j=1}^{n} x_j = 1, \quad x \in \mathbb{R}_+^n \\
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- \( \bar{a} \) expected returns.
- \( Q^{1/2} \) square root of covariance matrix.
- \( K \) maximum number of assets.
- \( \sigma \) maximum risk.
- Robust objective.
Performance Profile for n “=” 20-60, 100 and 200
Performance Profile for n "=" 20-60, 100 and 200

Performance Ratio

Problems solved

10^0 10^1 10^2 10^3 10^4 10^5

Extended Formulations for MIQCP
Performance Profile for n “=“ 20-60, 100 and 200
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Summary
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• Extended Formulations can help in
  – LP-based B&B: Both in theory and practice.
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  – LP-based B&B: Both in theory and practice.

• Most ideas can be extended beyond quadratic
  – p-order cones almost directly
  – General nonlinear through perspective functions
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• Extended Formulations can help in
  – LP-based B&B: Both in theory and practice.
• Most ideas can be extended beyond quadratic
  – p-order cones almost directly
  – General nonlinear through perspective functions
• You should definitely try: