Mixed Integer Programming (MIP) Approaches for Adaptive Choice-Based Conjoint Analysis

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Joint work with Denis Saure

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Adaptive Choice-Based Conjoint Analysis

Feature	SX530	RX100
Zoom	50x	3.6x
Prize	$249.99	$399.99
Weight	15.68 ounces	7.5 ounces
Prefer	☑	☐

Feature	TG-4	G9
Waterproof	Yes	No
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Estimate of preference **parameter**

- Today: Minimize **variance** of **parameter** estimates

Parametric Model = Logistic Regression

Feature	Chewbacca	BB-8
Wookiee	Yes	No
Droid	No	Yes
Blaster	Yes	No
Prefer?	☑	☐
	x^1	x^2

Product profile

$$\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = x^2$$

MNL Random **Linear** Utility

$$U_j = \underbrace{\beta \cdot x^j}_{\sum_{j=1}^{d} \beta_i x_i^j} + \epsilon_j$$

$$\Leftrightarrow \quad z = x^1 - x^2$$

Question:

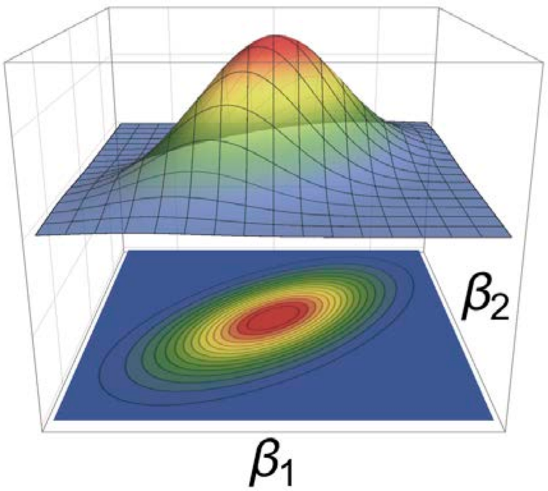
$$x^1 \succeq x^2 \Leftrightarrow \quad U_1 \text{“}\geq\text{”} U_2$$

$$\Leftrightarrow \quad \beta \cdot z \text{“}\geq\text{”} 0$$

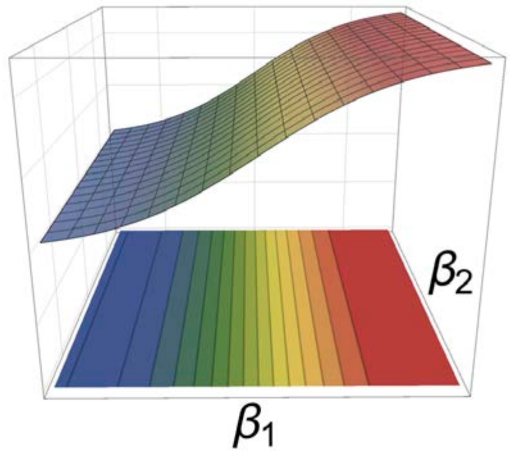
$$\mathbb{P}\left(x^1 \succeq x^2 \mid \beta\right) = \frac{1}{1 + e^{-\beta \cdot z}}$$

Bayesian Model with Normal Prior

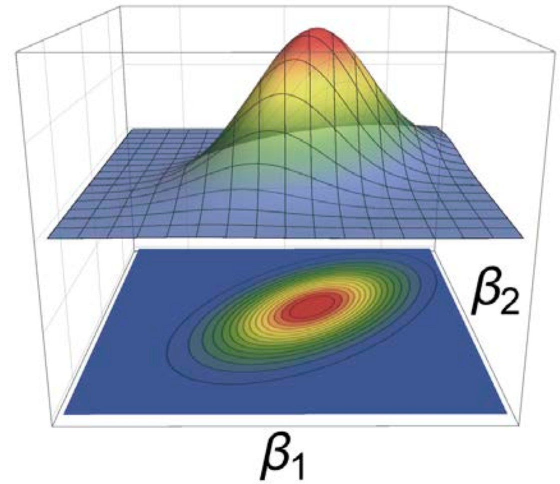
Prior distribution



Answer likelihood



Posterior distribution

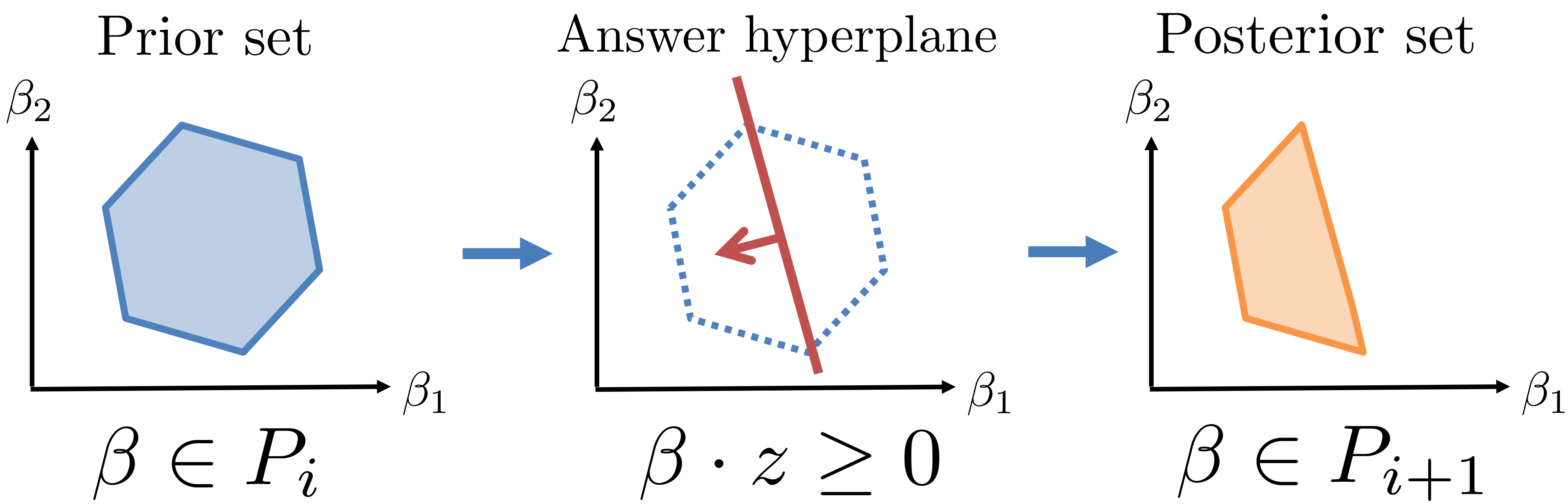


$$\beta \sim N(\mu, \Sigma) \qquad L\left(y \mid \beta, z\right) \qquad g\left(\beta \mid y, z\right)$$

$$y = \mathrm{sign}(\beta \cdot z) \qquad L\left(y \mid \beta, z\right) = \left(1 + e^{-y\beta \cdot z}\right)^{-1}$$

$$g\left(\beta \mid y, z\right) \propto \phi\left(\beta\, ;\ \mu, \Sigma\right) L\left(y \mid \beta, z\right)$$

Geometric Models ≈ Bayesian Model



Method	Response Error
Polyhedral Method (Toubia et al. ‘03,’04)	No
Probabilistic Polyhedral Method (T. et al. ‘07)	Yes, ≈ Bayesian
Robust Method (Bertsimas and O’Hair ‘13)	Yes, Robust
Ellipsoidal Method (Saure and Vielma ‘16)	Yes, = Bayesian

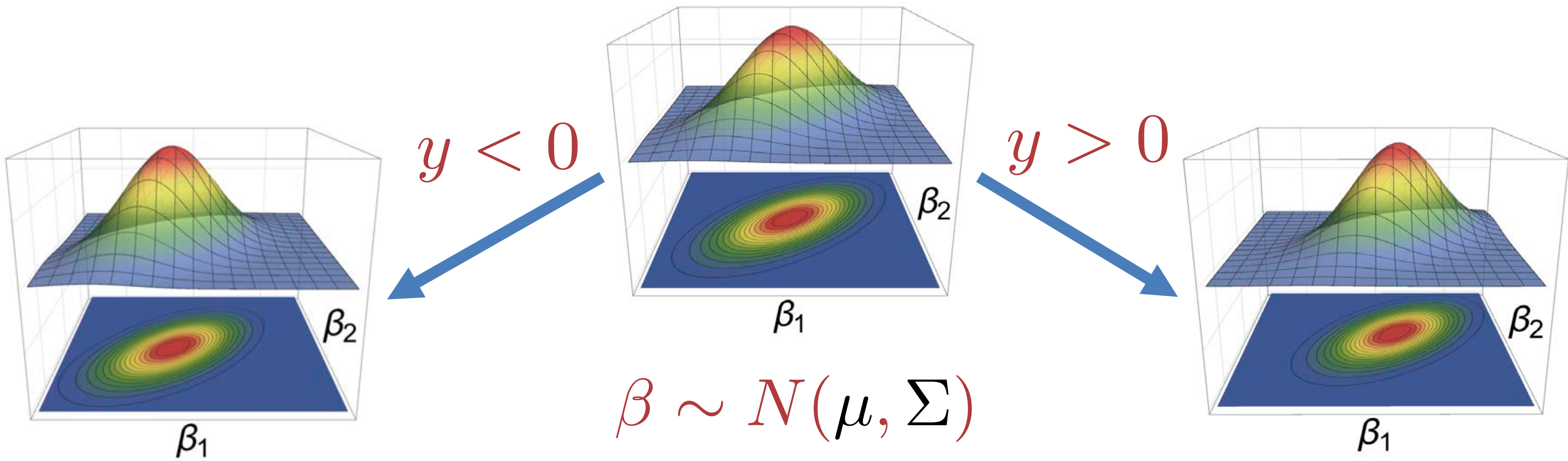
Bayesian v/s Geometric

	Bayesian	Geometric
Response Error	**MNL**	**None** / **Non-MNL or ≈ MNL**
Update	**Integration or MCMC**	**Simple Linear Algebra**
Question Selection	**Integration + Enumeration**	**MIP**

- **Ellipsoidal Method**:
  – MIP, julia and JuMP bridges the GAP (**Question Selection**)

D-Efficiency and Expected Posterior Variance

$$f(z, \mu, \Sigma) := \mathbb{E}_{y,\beta} \left\{ \left(\det \operatorname{cov}(\beta \mid y, z) \right)^{1/m} \right\}$$



$y < 0$ $\qquad$ $y > 0$

$\beta \sim N(\mu, \Sigma)$

$\operatorname{cov}(\beta) = \Sigma_1$ $\qquad$ $\operatorname{cov}(\beta) = \Sigma_2$

$$\max_{z \in \{-1,0,1\}^n} f(z, \mu, \Sigma)$$

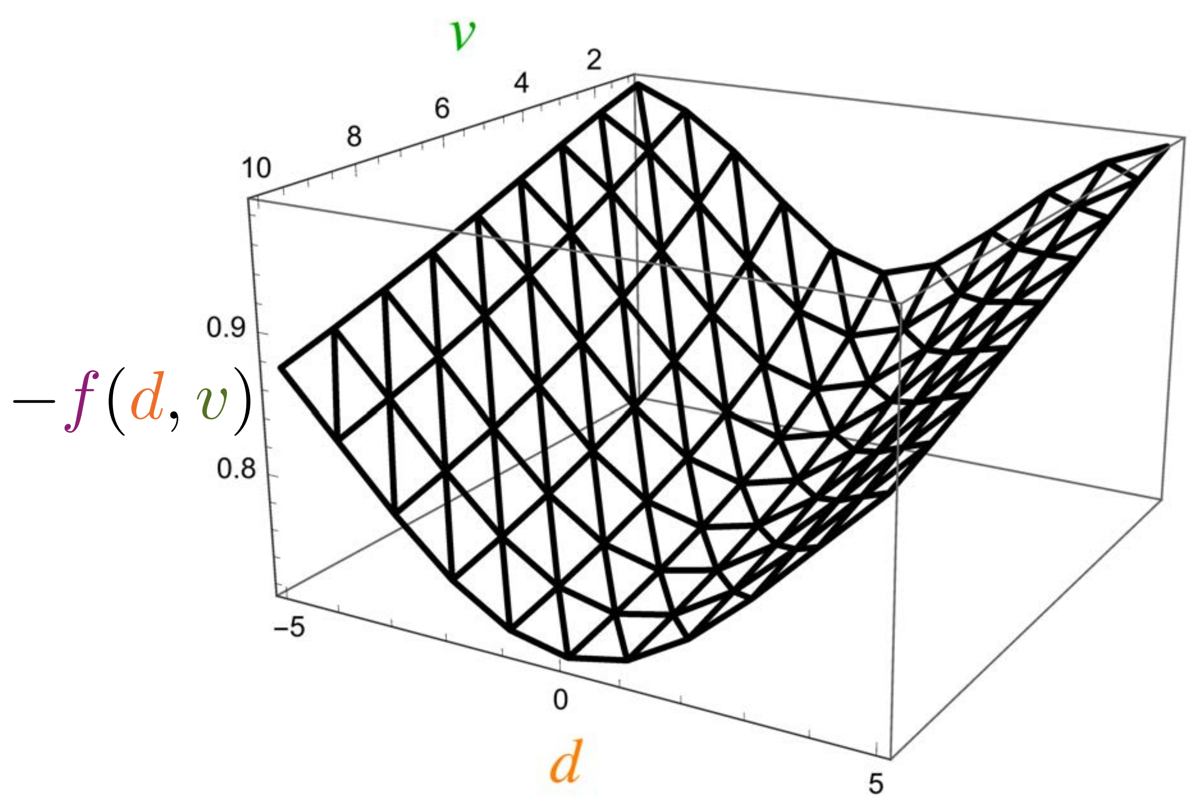
- $f(z, \mu, \Sigma)$ is hard to evaluate, non-convex and n large

Reformulation from V. and Saure '16

- D-efficiency $f(z)$ = Non-convex function $f(d,v)$ of

$$\text{mean:} \quad d := \mu \cdot z$$

$$\text{variance:} \quad v := z' \cdot \Sigma \cdot z$$



Can evaluate $f(d,v)$ with 1-dim integral 🙂

Piecewise Linear Interpolation

Linear MIP formulation (standard linearization)

Aligns with selection criteria from Toubia et al. '04: minimize mean and maximize variance

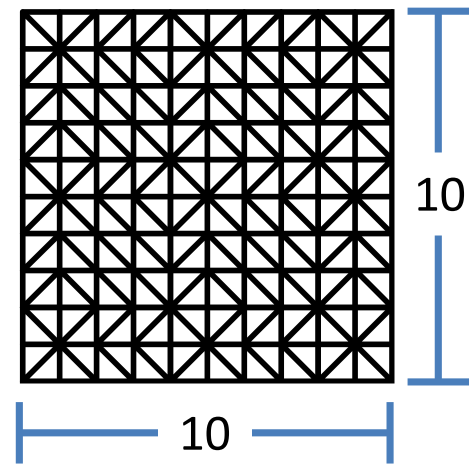
Easy to Build through julia & JuMP

- PiecewiseLinearOpt.jl (Huchette and V. 2017)

$$\begin{aligned} \min \quad & \exp(x+y) \\ s.t. \quad & \\ & x, y \in [0,1] \end{aligned}$$

Automatically select Δ

Automatically construct formulation (easily chosen)




```julia
using JuMP, PiecewiseLinearOpt
m = Model()
@variable(m, x)
@variable(m, y)

z = piecewiselinear(m, x, y, 0:0.1:1, 0:0.1:1, (u,v) -> exp(u+v))
@objective(m, Min, z)
```

MIP-based Adaptive Questionnaires

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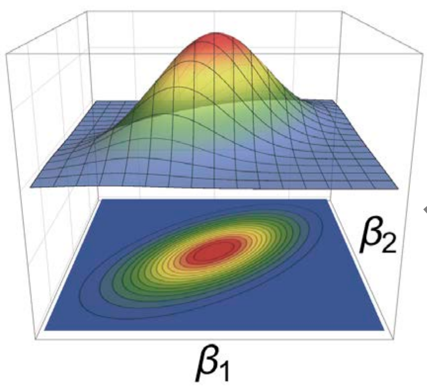
$$\mathbb{E}\left(\beta \mid Y, X^1, X^2\right)$$

$$\operatorname{cov}\left(\beta \mid Y, X^1, X^2\right)$$

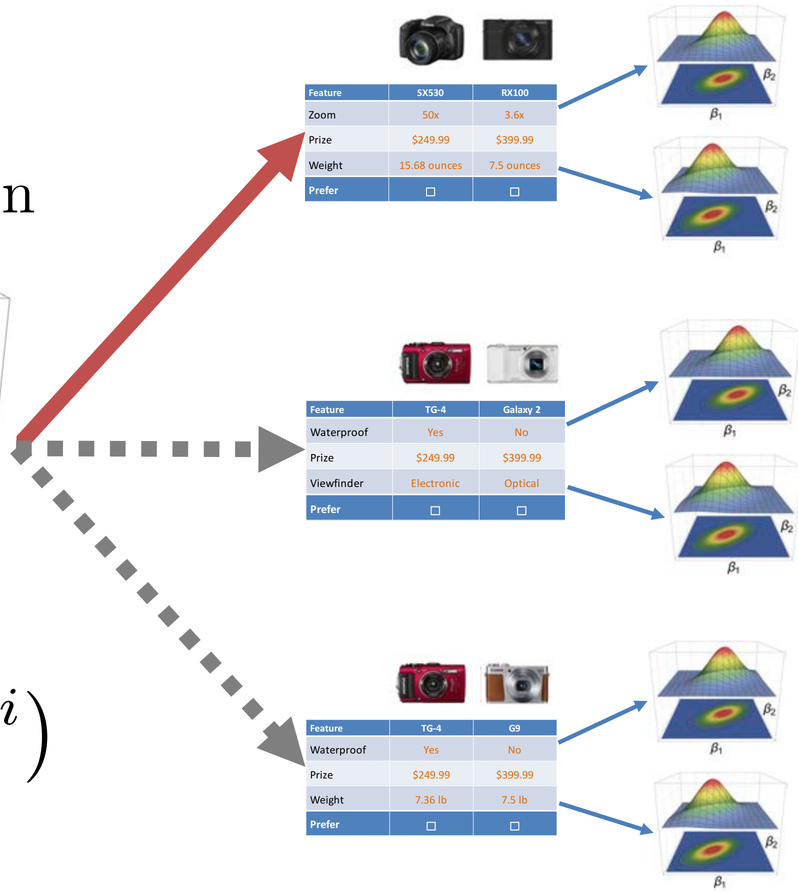
- **Optimal one-step look-ahead moment-matching approximate Bayesian approach = Ellipsoidal Method**

Optimal One-Step Look-Ahead = MIP

Prior distribution



$$\beta \sim N\left(\mu^i, \Sigma^i\right)$$



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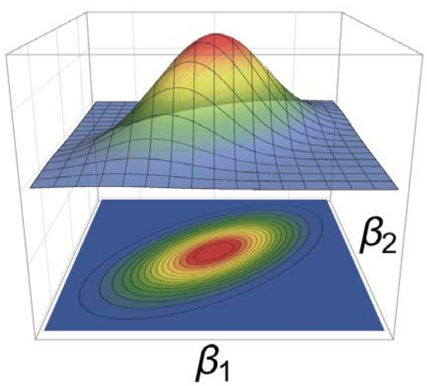
$$\min_{x^1, x^2} f\left(x^1, x^2\right)$$

$$\min_{x^1, x^2, d, v \in Q} f(d, v)$$

- Solve with MIP formulation
- Sampling : all precomputed (2-dim grid)

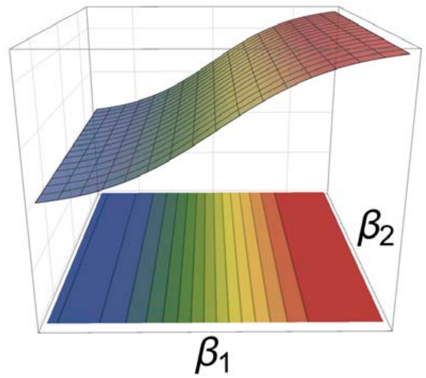
Moment-Matching Approximate Bayesian Update

Prior distribution



$$\beta \sim N\left(\mu^i, \Sigma^i\right)$$

Answer likelihood



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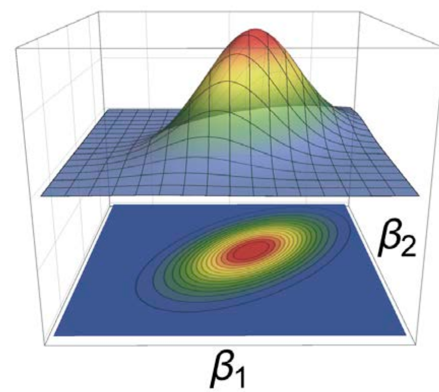
Posterior distribution



$$\beta \overset{approx.}{\sim} N\left(\mu^{i+1}, \Sigma^{i+1}\right)$$

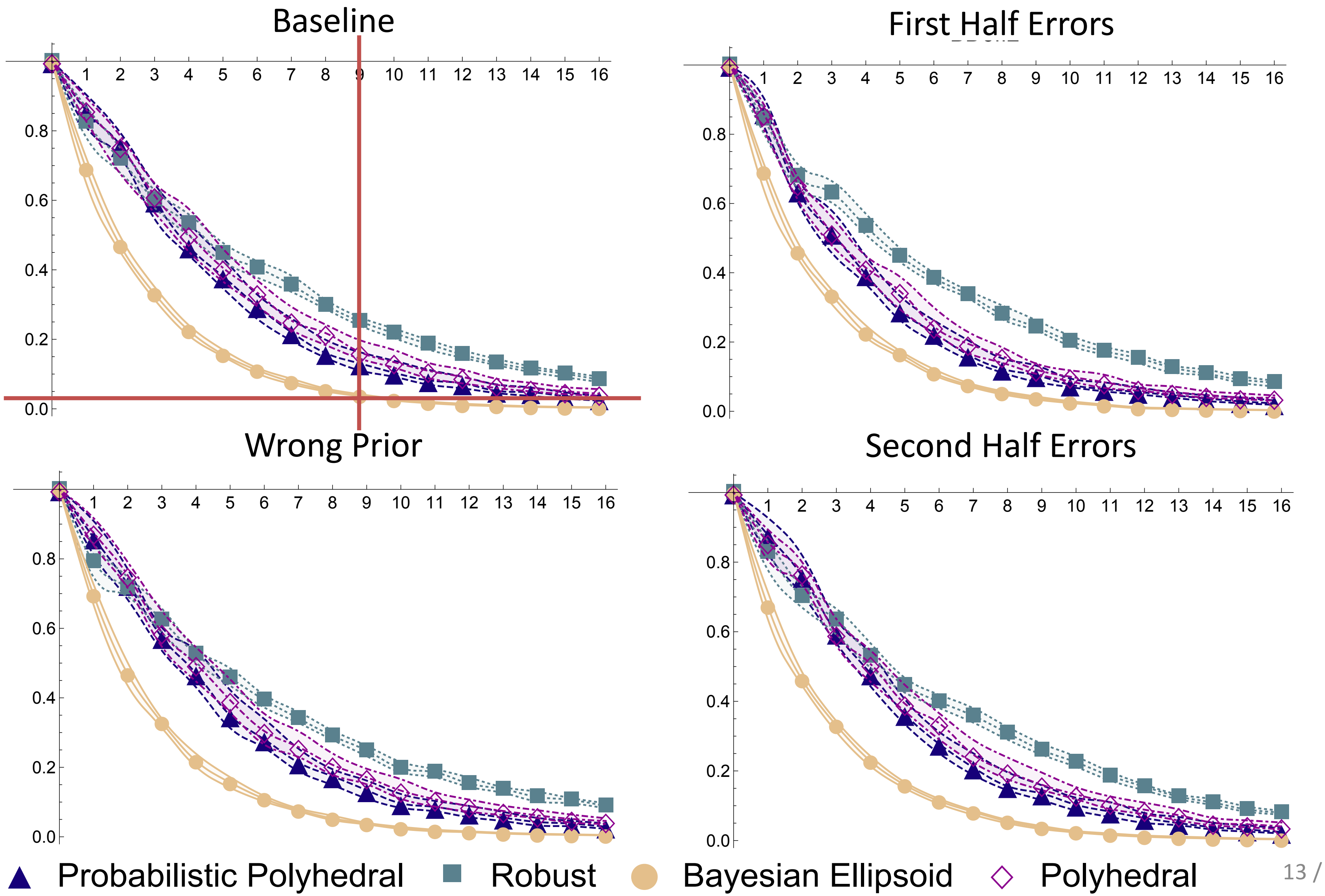
- $\mu^{i+1} = \mathbb{E}\left(\beta \,|\, y, x^1, x^2\right)$
- $\Sigma^{i+1} = \mathrm{cov}\left(\beta \,|\, y, x^1, x^2\right)$

- 1-d integral : $I(d, v)$
- Sampling : all precomputed

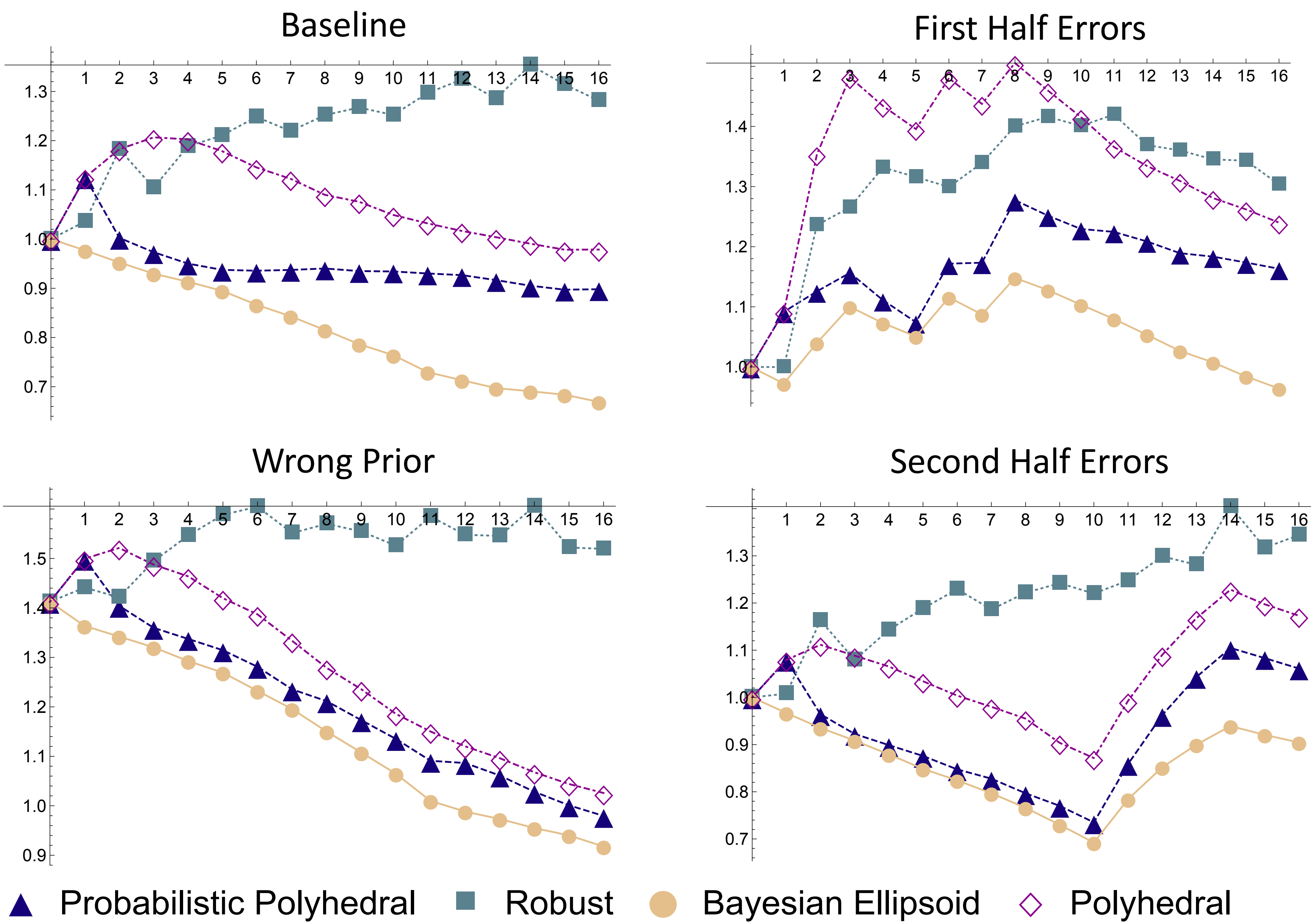
Simulation Experiments

- 16 questions, 2 options, 12 features
- Simulate MNL responses with known β^*
- 100 individual β^* sampled from N(μ,Σ) prior
- Methods:
  - Polyhedral, Prob. Polyhedral, Robust and Ellipsoidal
  - All get same ellipsoidal prior
  - All < 30’’ inter-question (except robust < 90’’)
- Metrics:
  - RMSE of β estimator, error in market share and D-eff.
  - Normalized values = smaller better
  - Versions: Method, Individual and Hierarchical Bayesian
  - Sensitivity: Wrong prior μ, all errors in first/second half

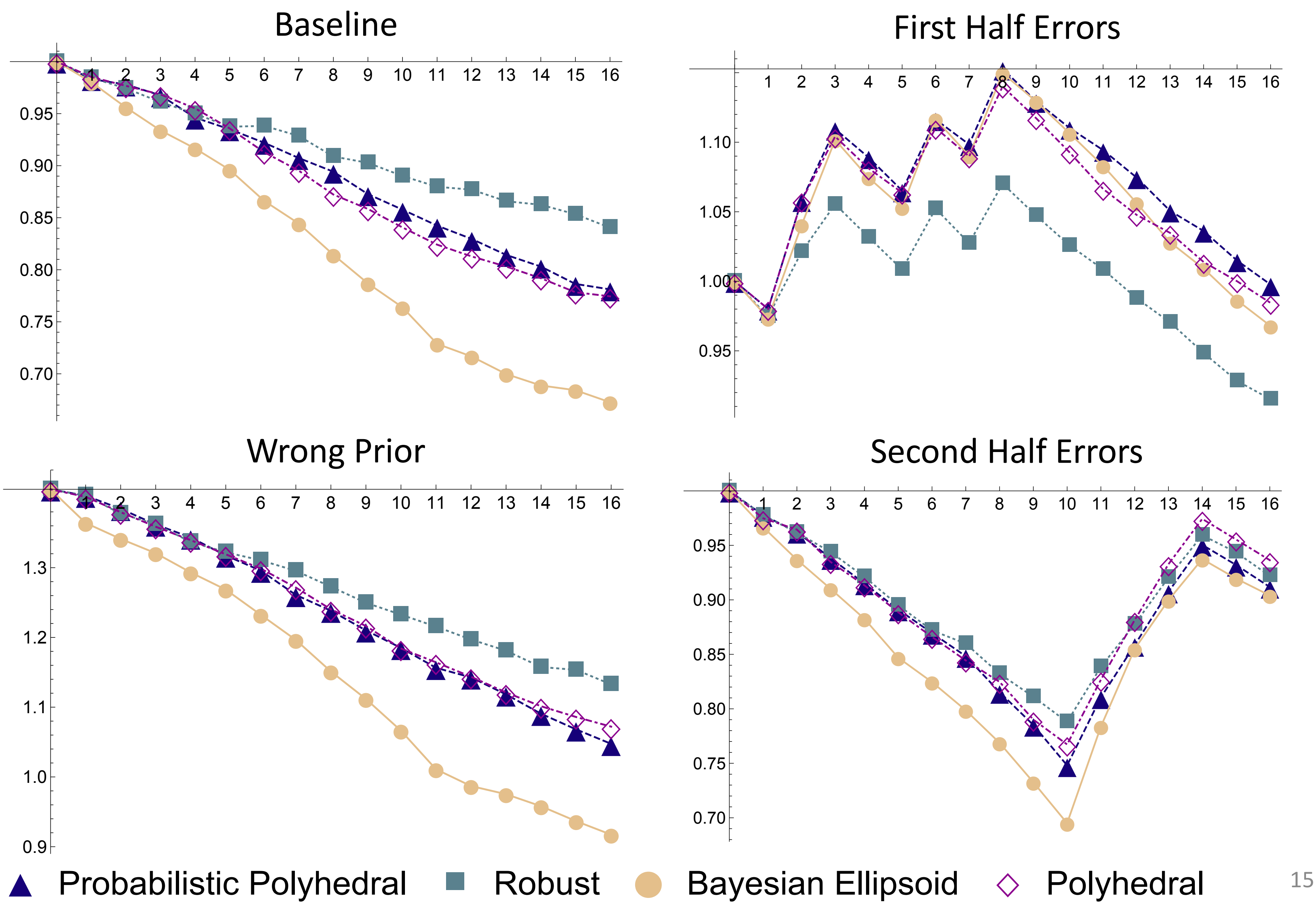
D-Efficiency for Individual Bayesian



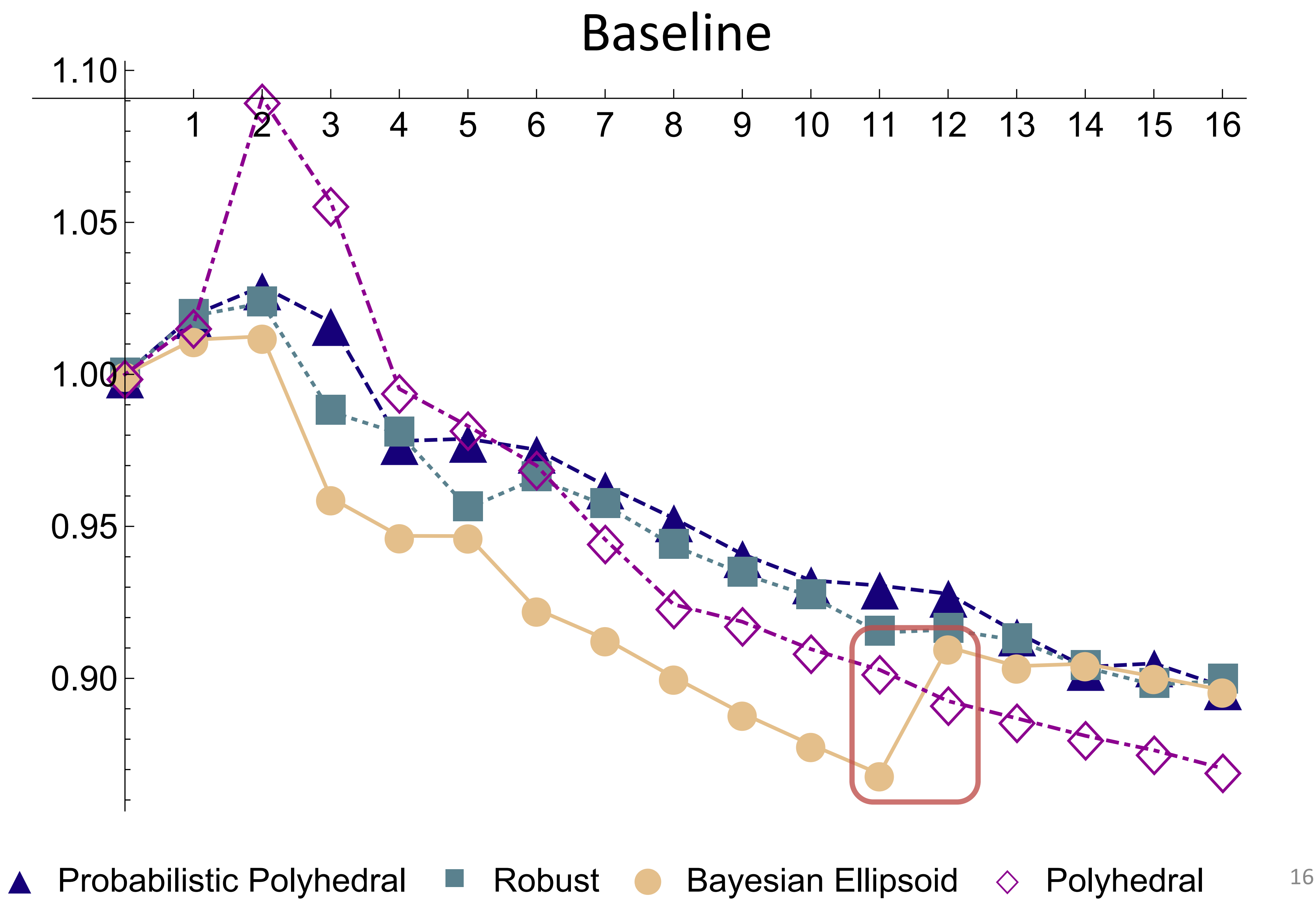
RMSE for Methods Estimator



RMSE for Individual Bayesian Estimator

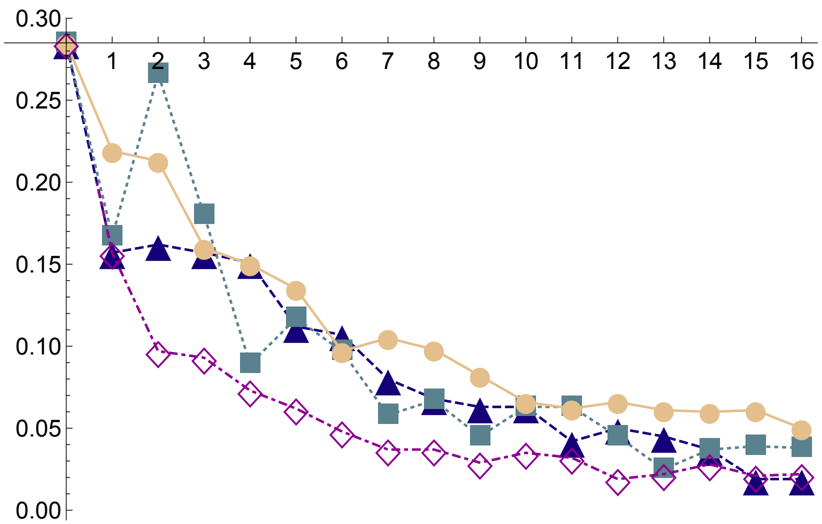


RMSE for Hierarchical Bayesian Estimator

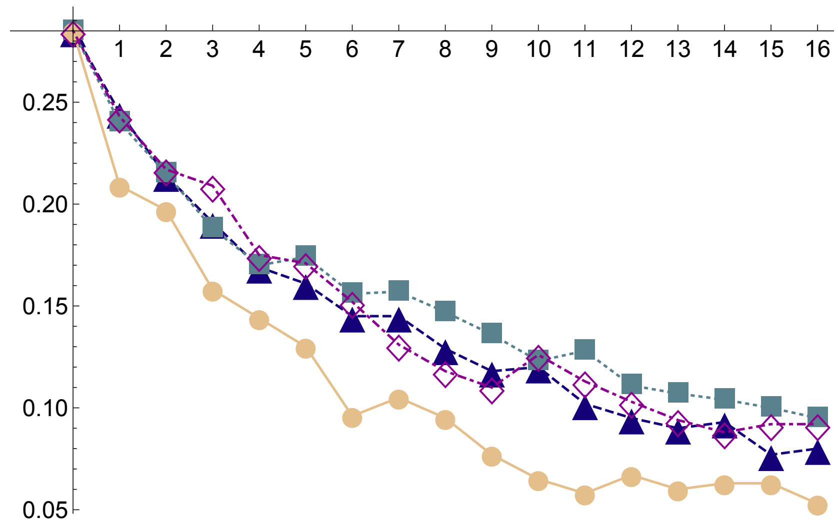


Market share for Baseline

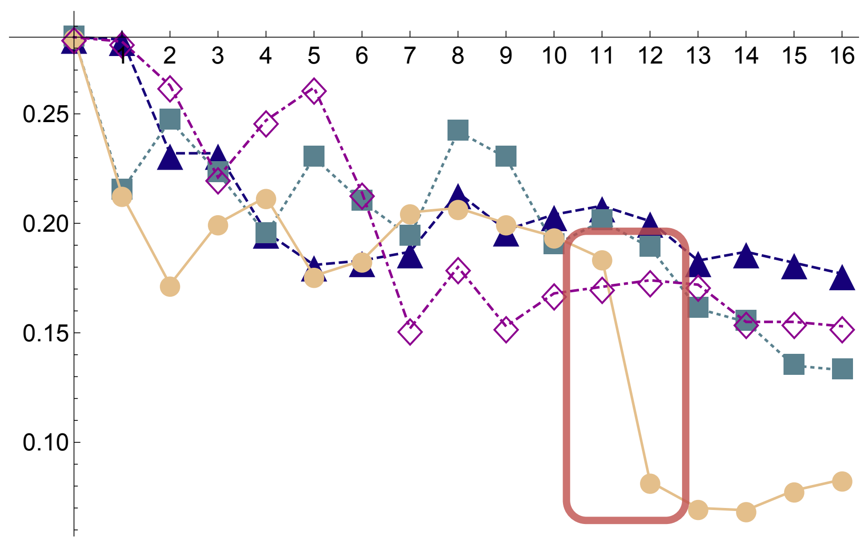
Method



Individual Bayesian



Hierarchical Bayesian



▲ Probabilistic Polyhedral ■ Robust ● Bayesian Ellipsoid ◇ Polyhedral

Summary

- Mixed Integer Programming for ACBCA
  - n-variate function to 2-variate function + MIP
  - Precomputed 2-variate PWL function
  - Advanced MIP formulation + solver
  - Easy to access with JuMP!
    - Ask me and get a sticker 🙂
- Also for other estimator variance / linear models
- Significantly faster reduction of estimator variance
  - Maybe too fast for HB?
- Future: MIP flexibility → Managerial Objective