The Chvátal-Gomory Closure of a Strictly Convex Body is a Rational Polyhedron

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Joint work with

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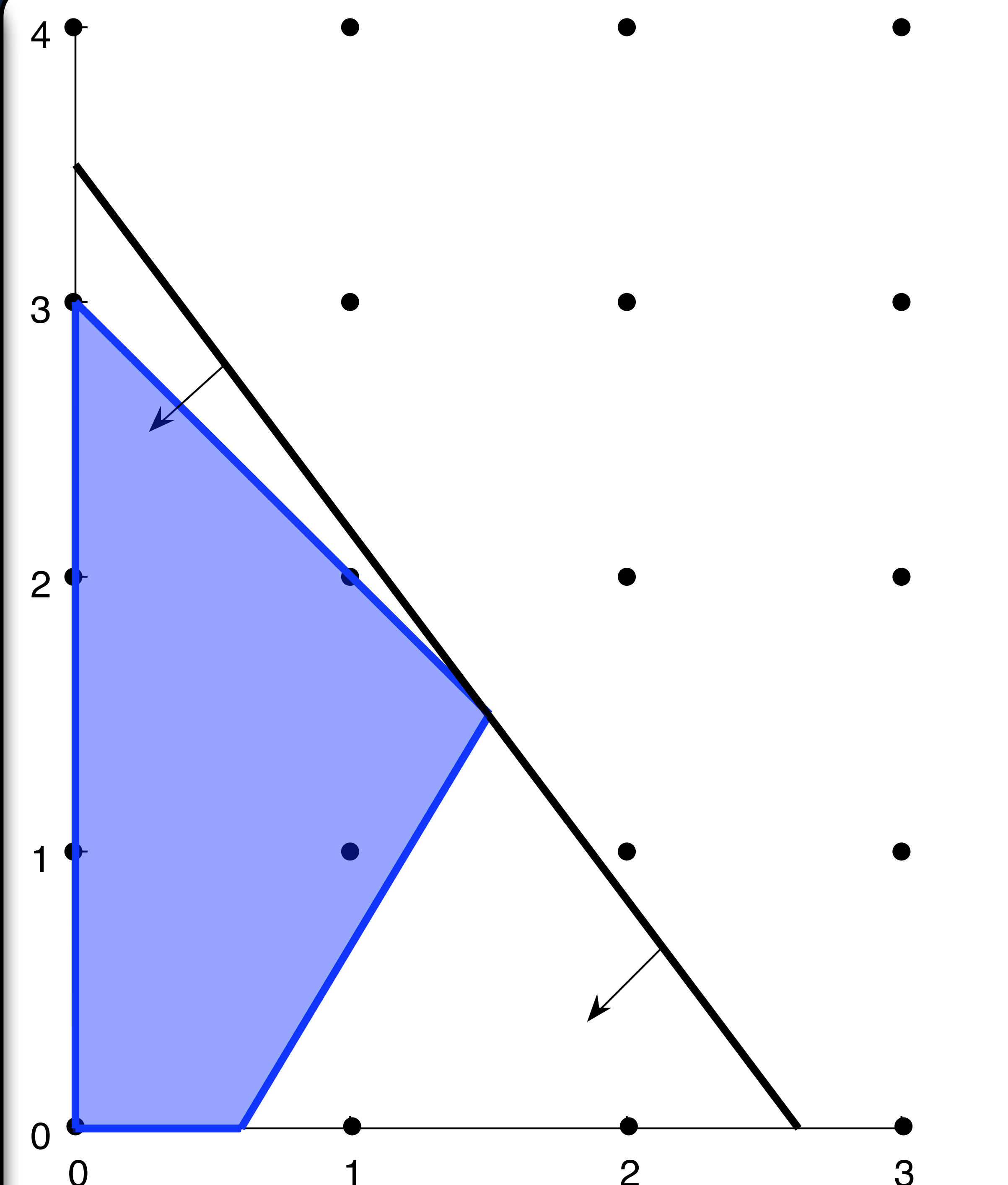
Outline

- Introduction
- Proof:
  - Step 1
  - Step 2
- Intersection with Rational Polyhedra
- Example of Non-Polyhedral Closure.
- Conclusions and Future Work

CG Cuts for Rational Polyhedra

$$P := \left\{ x \in \mathbb{R}^2 \ : \ \begin{array}{r} x_1 + x_2 \le 3, \\ 5x_1 - 3x_2 \le 3 \end{array} \right\}$$

$4x_1 + 3x_2 \le 10.5$ Valid for P



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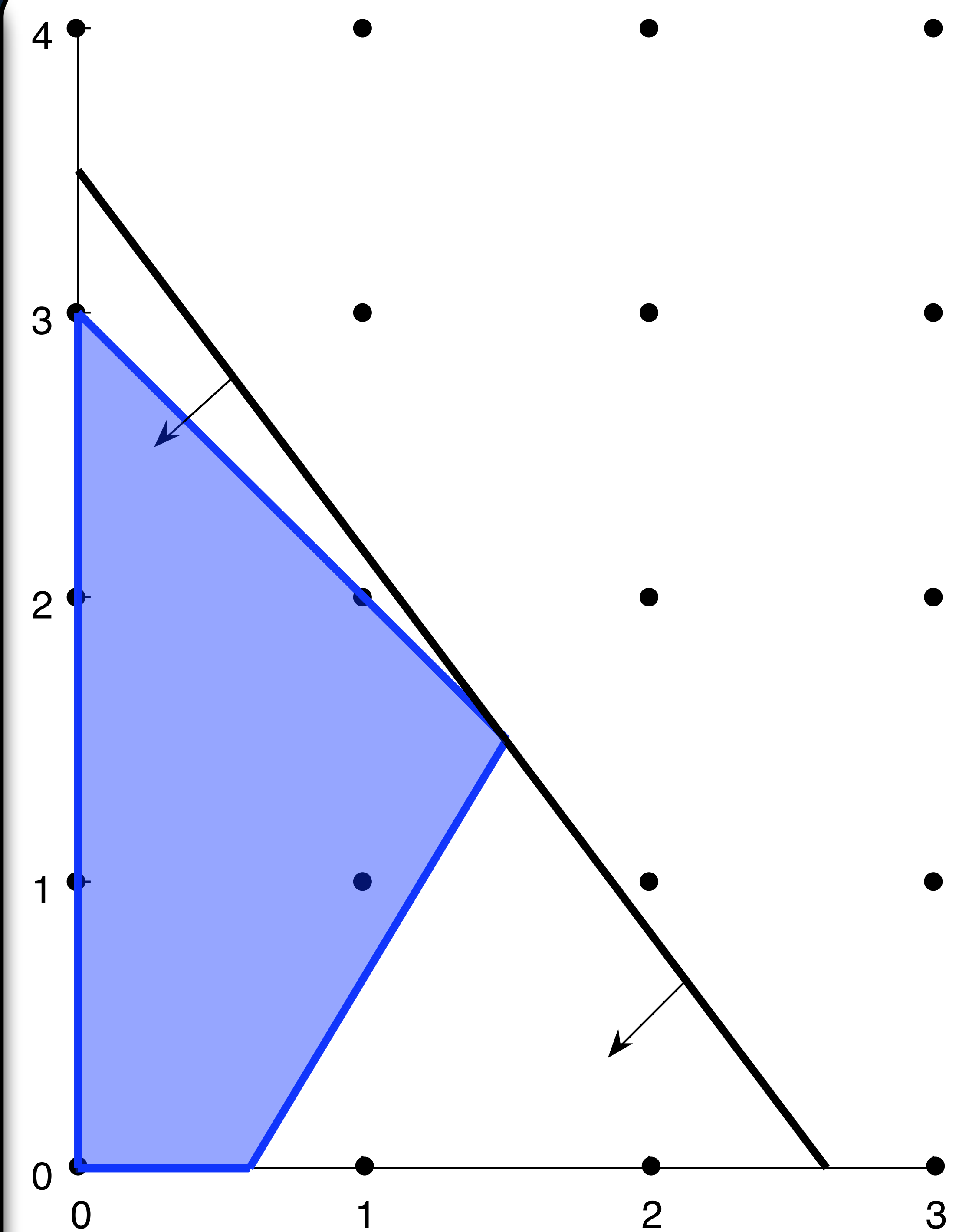

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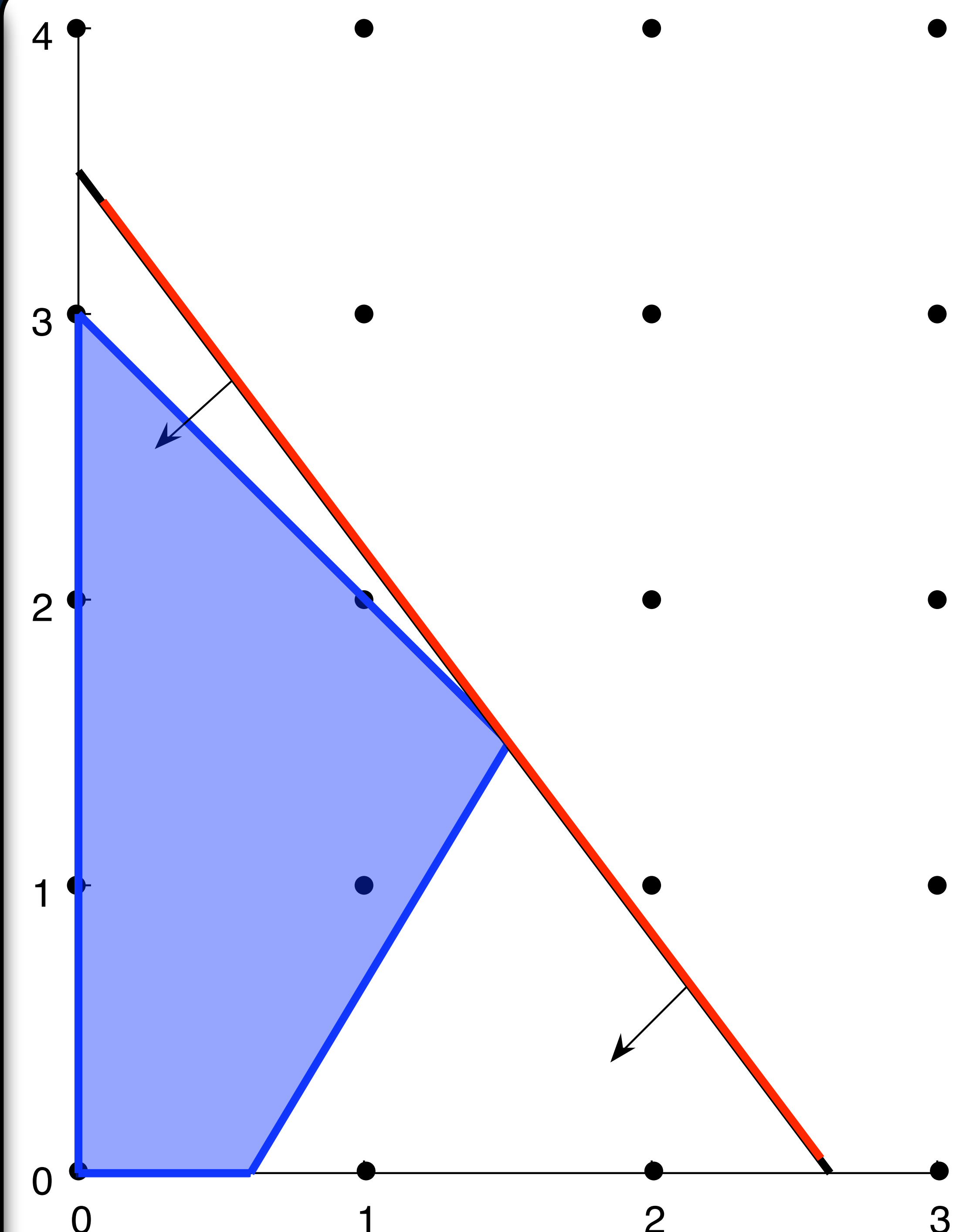

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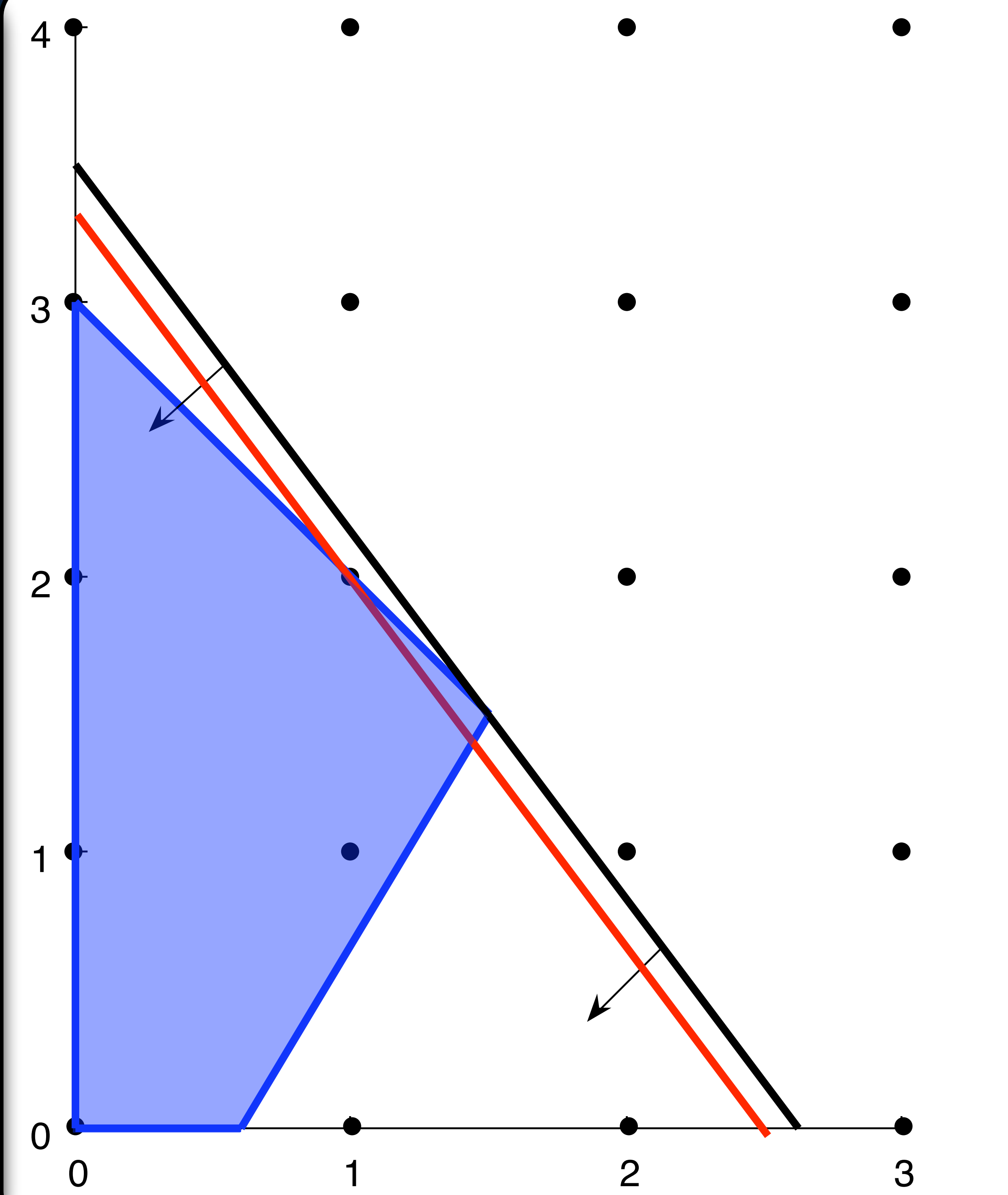
$$\downarrow$$

$$4x_1 + 3x_2 \leq \lfloor 10.5 \rfloor \quad \text{Valid for } P \cap \mathbb{Z}^2$$



CG Cuts for General Convex Sets

$$\sigma_C(a) := \sup\{\langle a, x\rangle \mid x \in C\}$$

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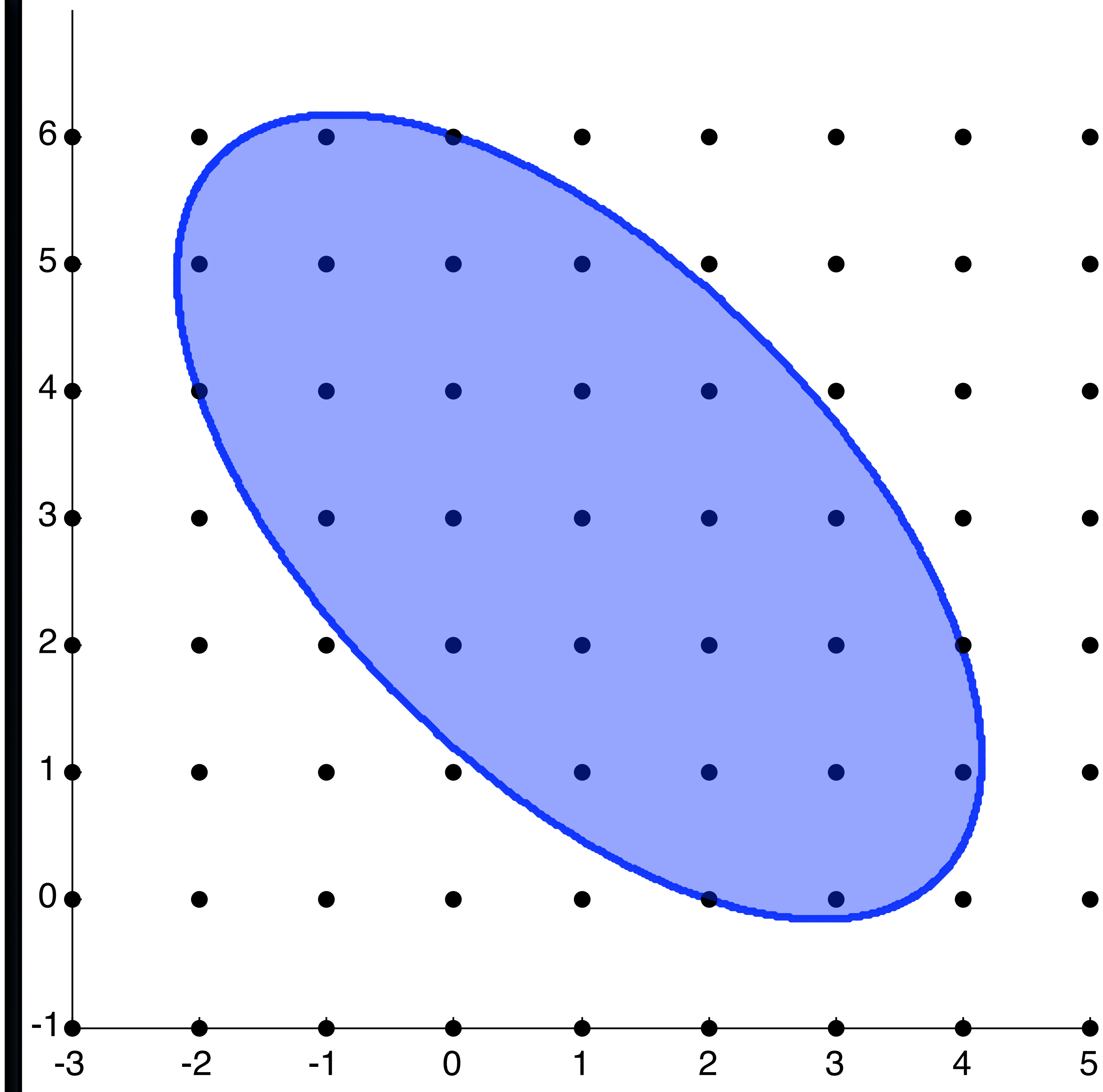

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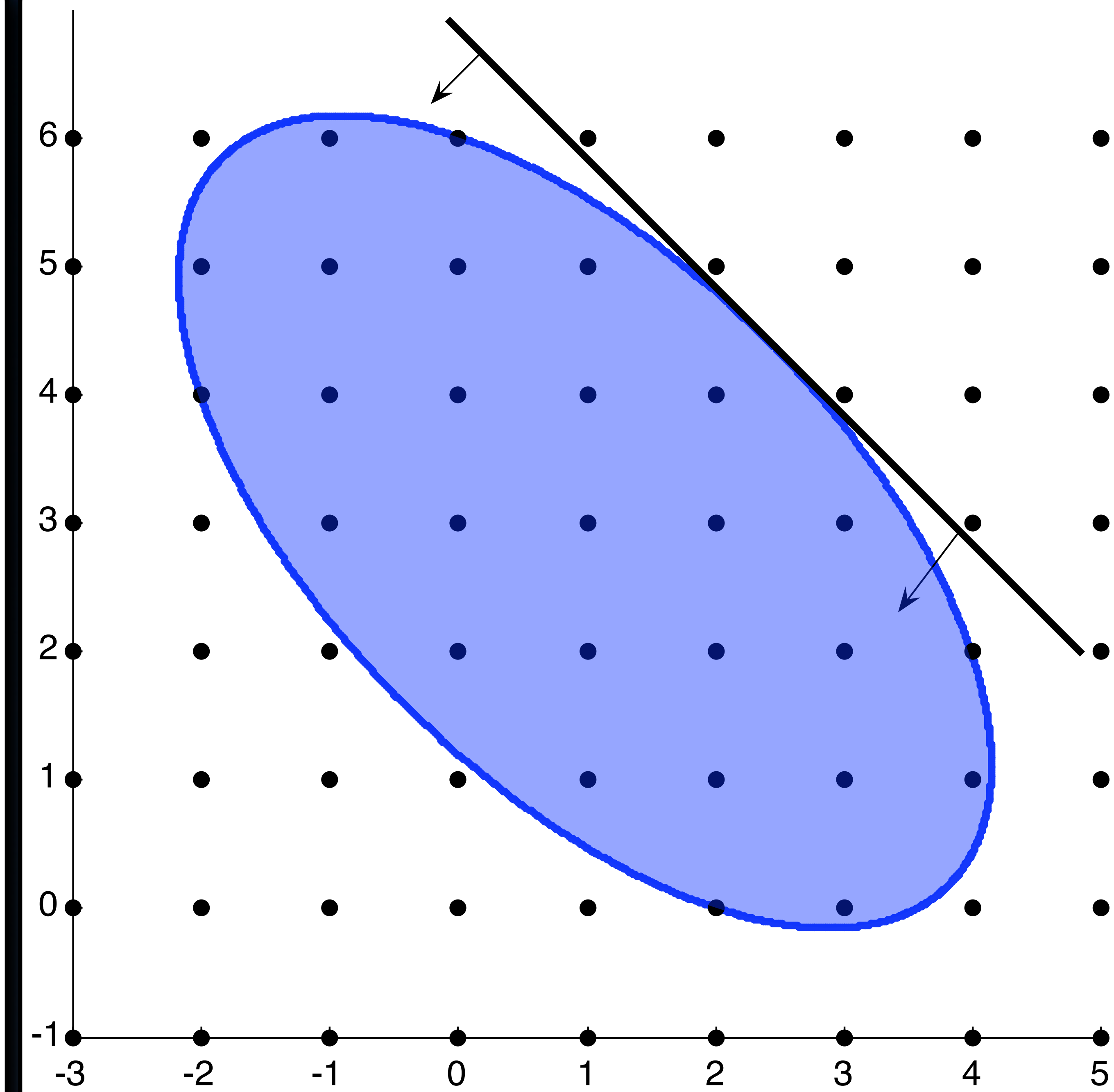

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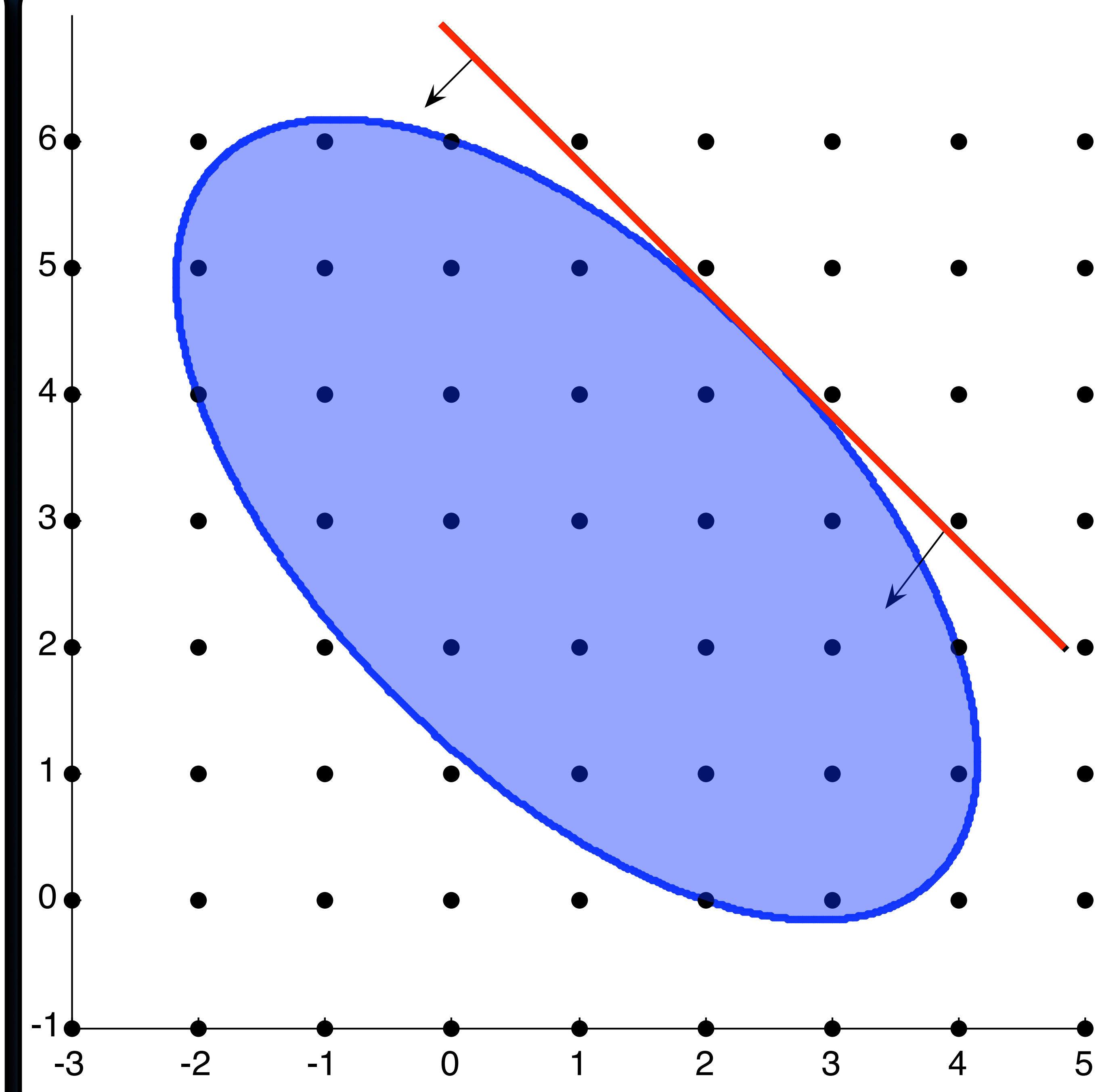

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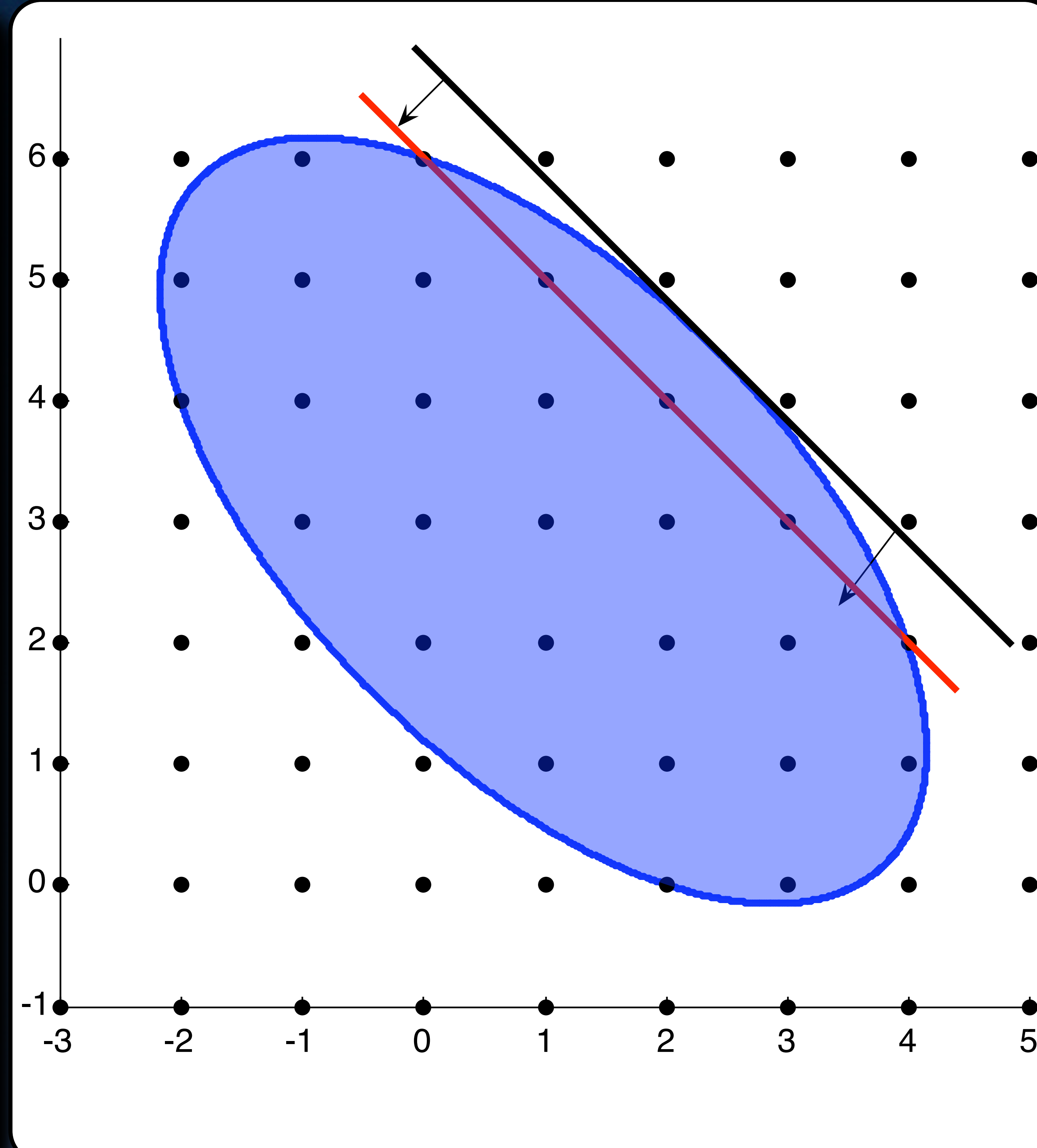
$$\langle a, x\rangle \leq \lfloor \sigma_C(a) \rfloor$$ Valid for $C \cap \mathbb{Z}^n$



CG Closure of a Convex Set

$$\mathtt{CGC}(D,C) := \bigcap_{a \in D} \{x \in \mathbb{R}^n \ : \ \langle a, x \rangle \leq \lfloor \sigma_C(a) \rfloor \}$$

- CG Closure: $\mathtt{CGC}(\mathbb{Z}^n, C)$
- Is CG closure a polyhedron?
  - Finite set $S \subset \mathbb{Z}^n$ s.t. $\mathtt{CGC}(\mathbb{Z}^n, C) = \mathtt{CGC}(S, C)$
  - Yes, for rational polyhedra (Schrijver, 1980)
  - What about other convex sets?

What we know for Convex Bodies

$$C^0 := C, \quad C^k := \texttt{CGC}(\mathbb{Z}^n, C^{k-1})$$

- There exists k s.t. $C^k = \operatorname{conv}(C \cap \mathbb{Z}^n)$ (Chvátal, 1973)
- Also for unbounded rational polyhedra (Schrijver, 1980).
- Result does not imply polyhedrality of C^1

Proof Outline: Generation Procedure

- Step 1: Construct a finite set $S^1 \subset \mathbb{Z}^n$ such that
  - $\texttt{CGC}(S^1, C) \subseteq C$
  - $\texttt{CGC}(S^1, C) \cap \mathrm{bd}(C) \subset \mathbb{Z}^n$

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- $\mathtt{CGC}(\mathbb{Z}^n, C) = \mathtt{CGC}(S^1, C) \cap \mathtt{CGC}(S^2, C)$

Outline of Step 1

- Step 1: Construct a finite set $S^1 \subset \mathbb{Z}^n$ such that
  - $\texttt{CGC}(S^1, C) \subseteq C$ and $\texttt{CGC}(S^1, C) \cap \mathrm{bd}(C) \subset \mathbb{Z}^n$

(a) Separate non-integral points in $\mathrm{bd}(C)$.

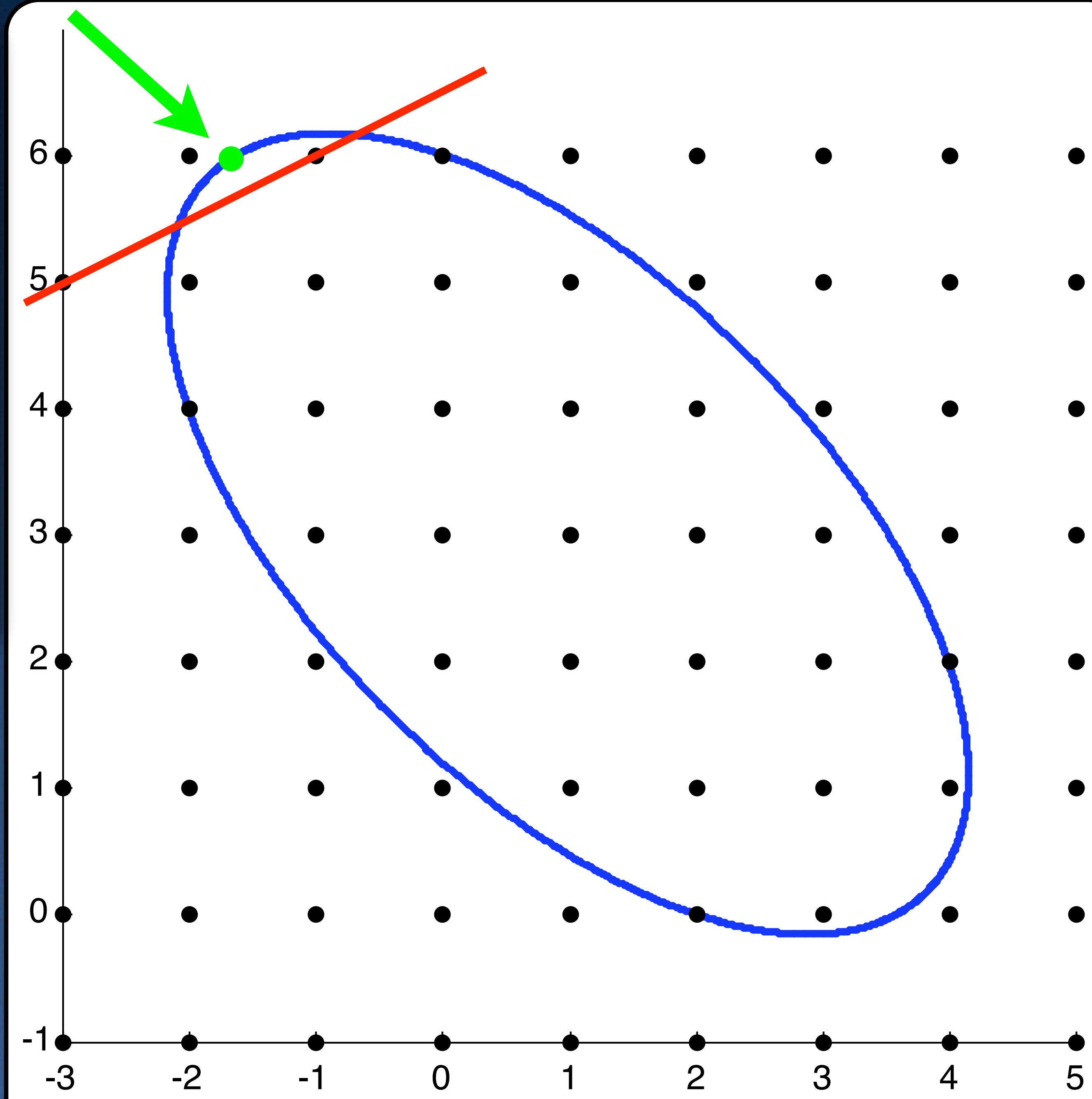
(b) Separate neighborhood of integral points in $\mathrm{bd}(C)$.

(c) Compactness argument to construct finite S^1.

Separate non-integral points in $\mathrm{bd}(C)$

$u \in \mathrm{bd}(C) \setminus \mathbb{Z}^n \qquad \exists\, a^u \in \mathbb{Z}^n$

$$\langle a^u, u \rangle > \lfloor \sigma_C(a^u) \rfloor$$



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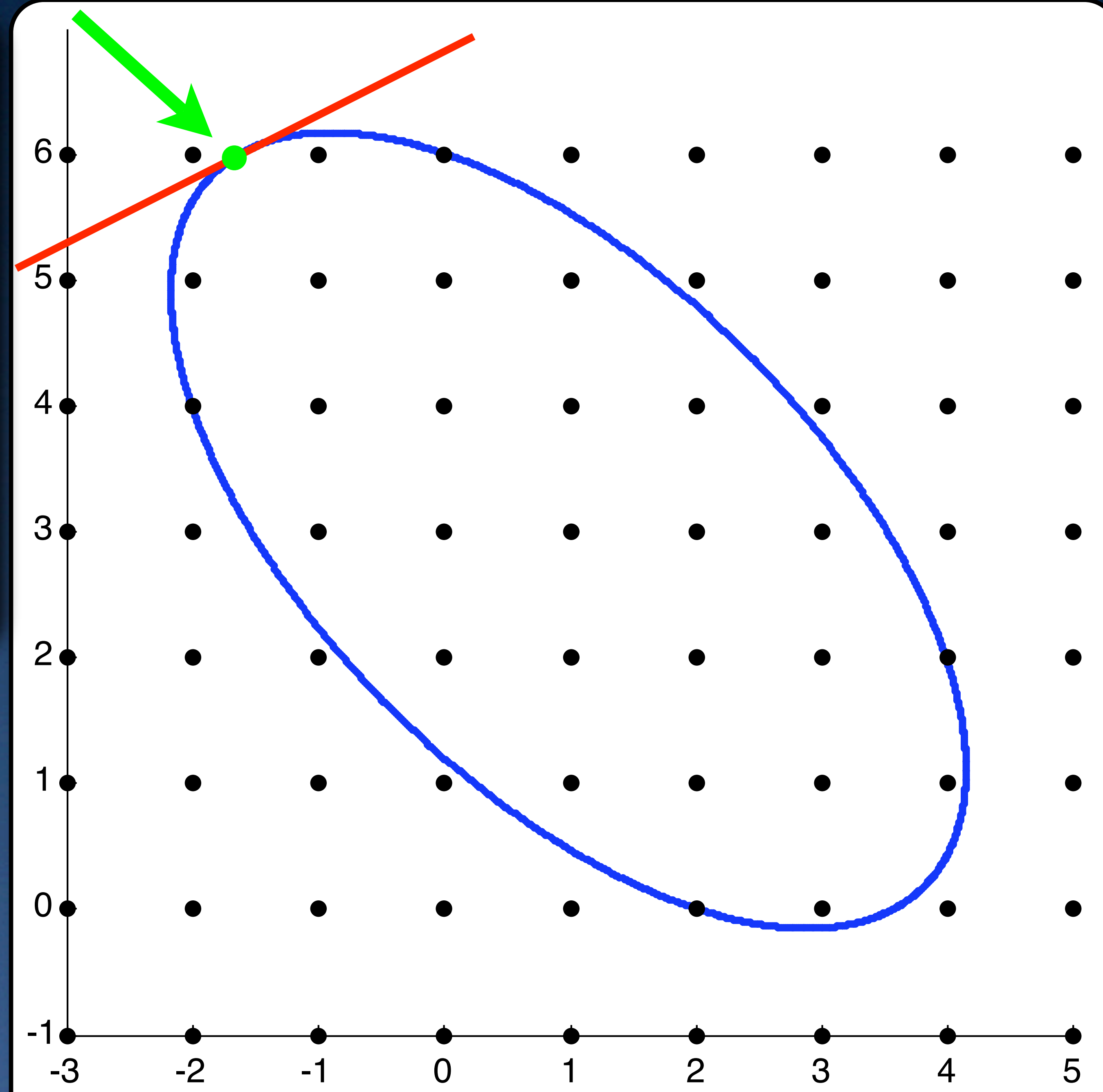

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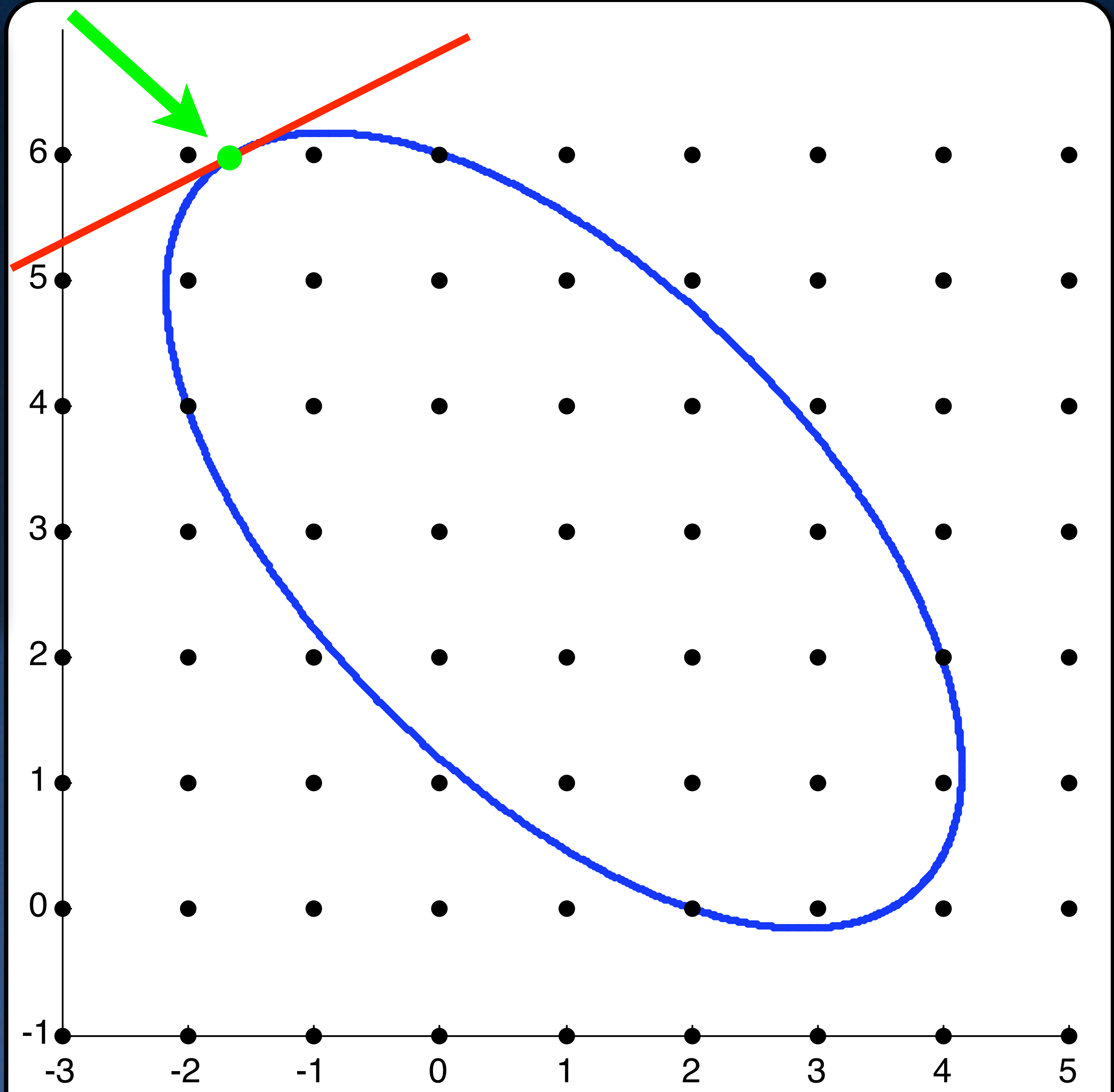

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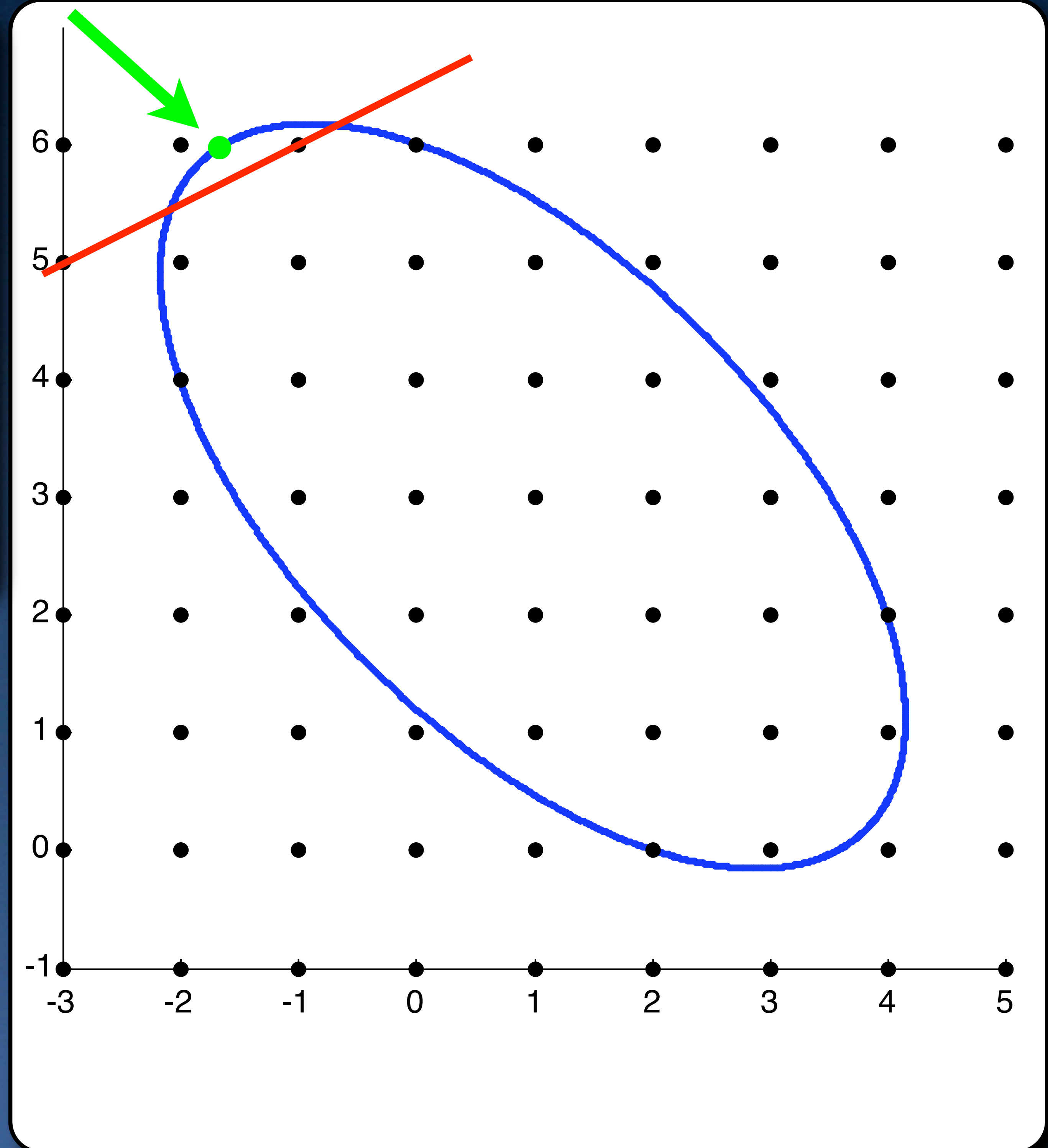

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$$C = \left\{ x \in \mathbb{R}^2 \;:\; \sqrt{x_1^2 + x_2^2} \leq 1 \right\}$$

$$u = (1/2, \sqrt{3}/2)^T \in \mathrm{bd}(C)$$

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$$u = (25/13, 60/13)^T \in \operatorname{bd}(C)$$

$$s(u) = (5, 12)^T,\ \sigma_C(s(u)) = 65$$

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$$\frac{s^i}{\|s^i\|} \xrightarrow{i\to\infty} \frac{s(u)}{\|s(u)\|}$$

$$\lim_{i\to\infty} \langle s^i, u\rangle - \lfloor \sigma_C(s^i)\rfloor > 0$$

Diophantine approx. of $s(u)$

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Separate neighborhood of integers

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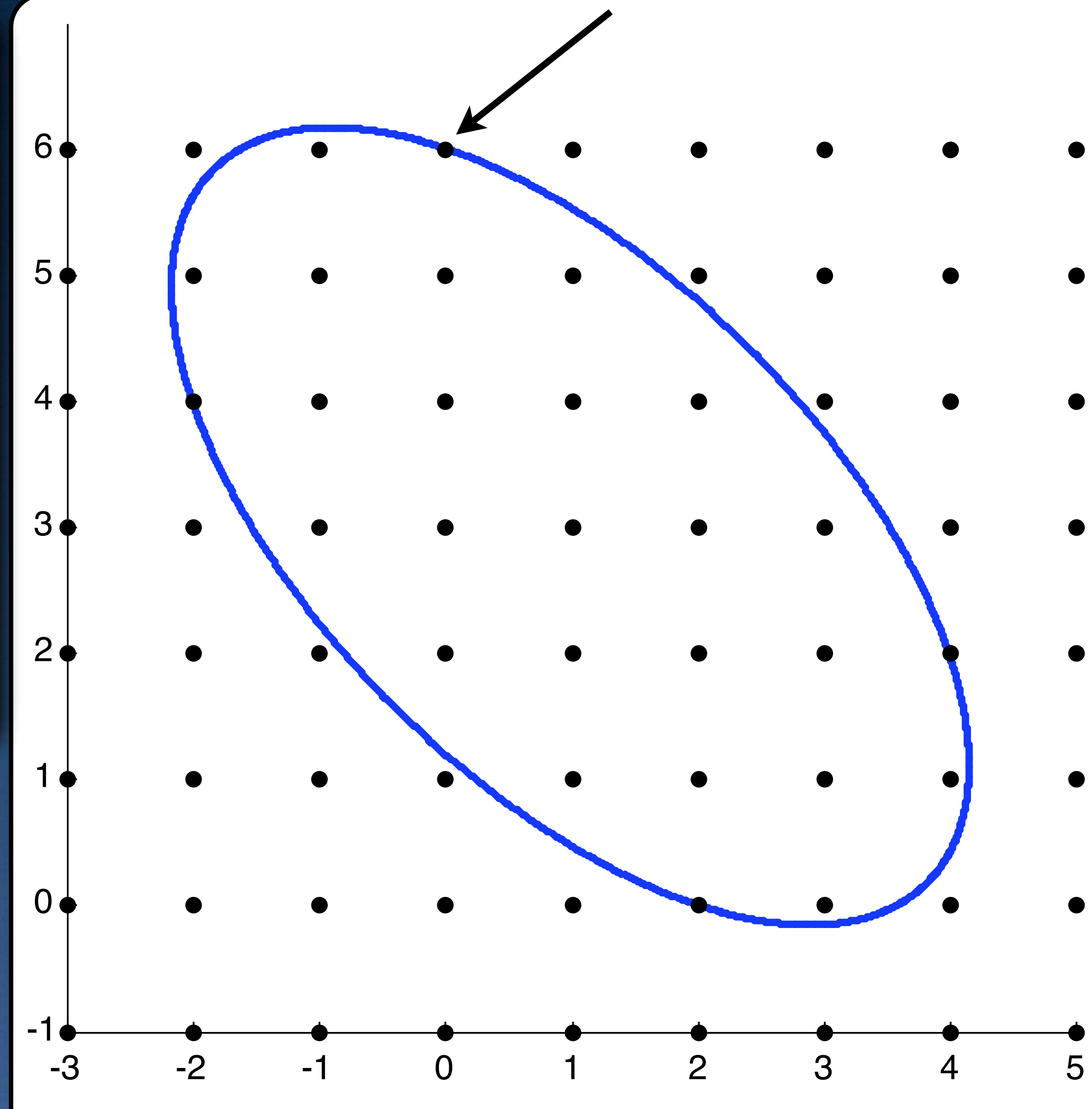

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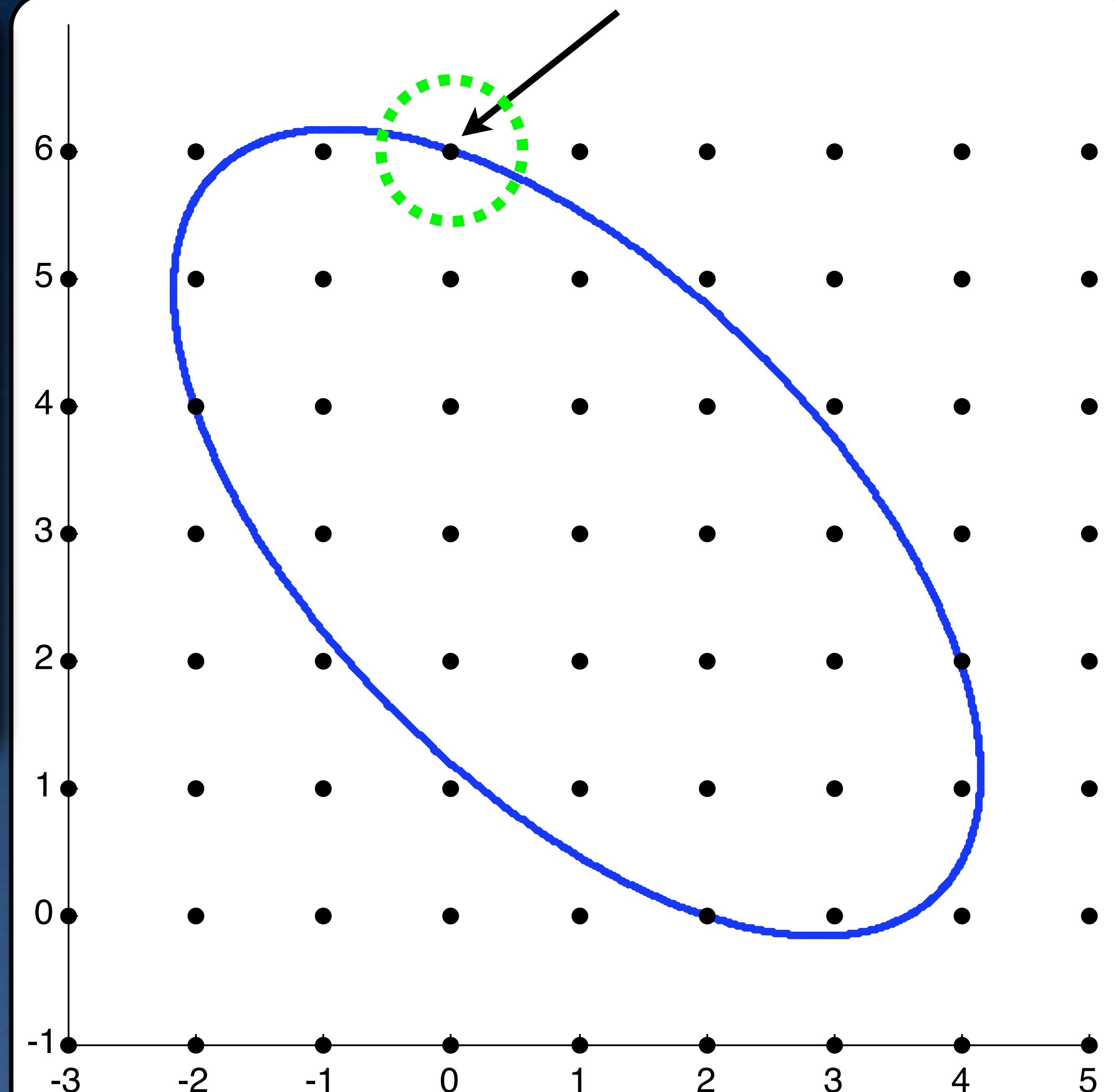

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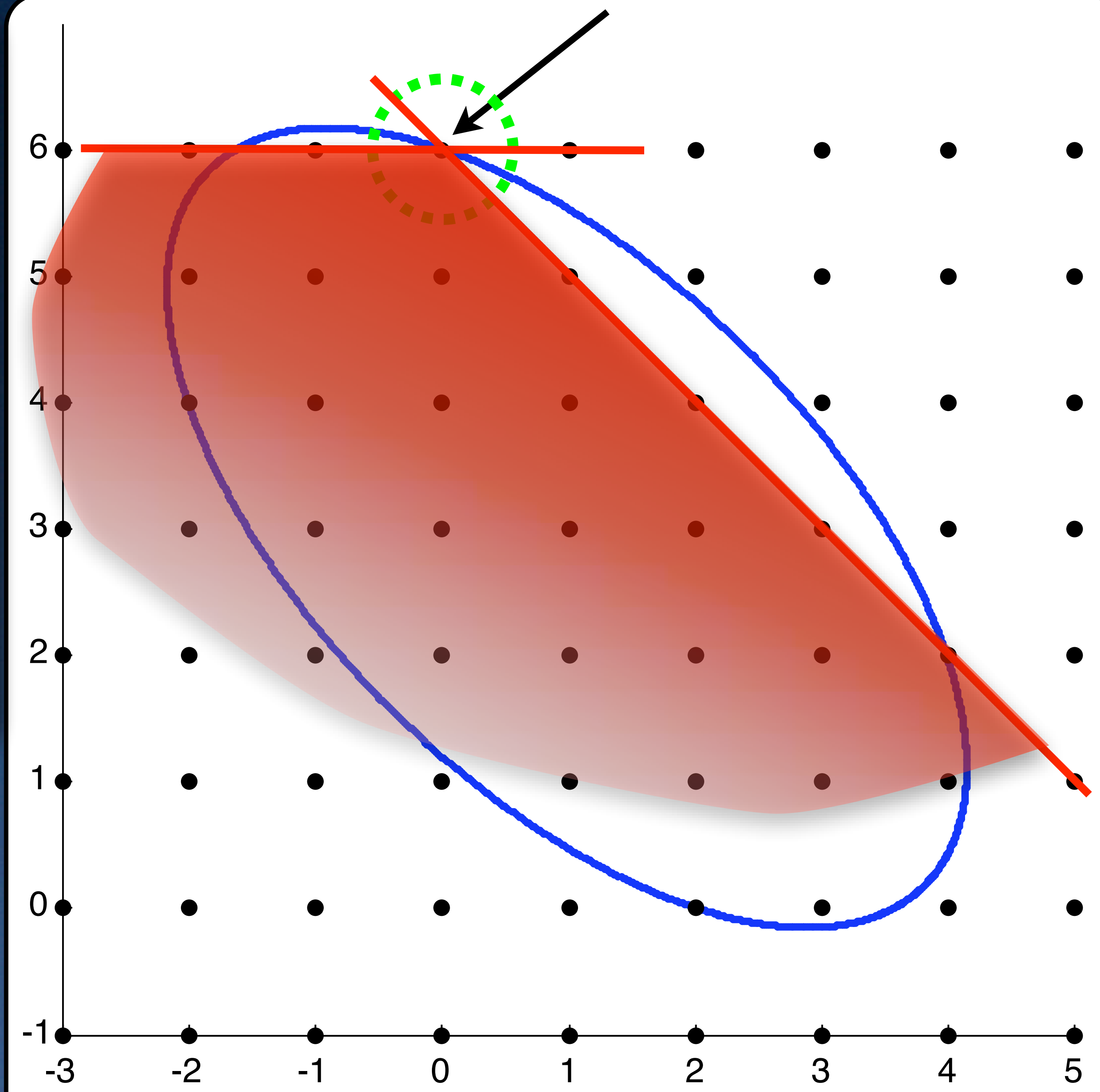

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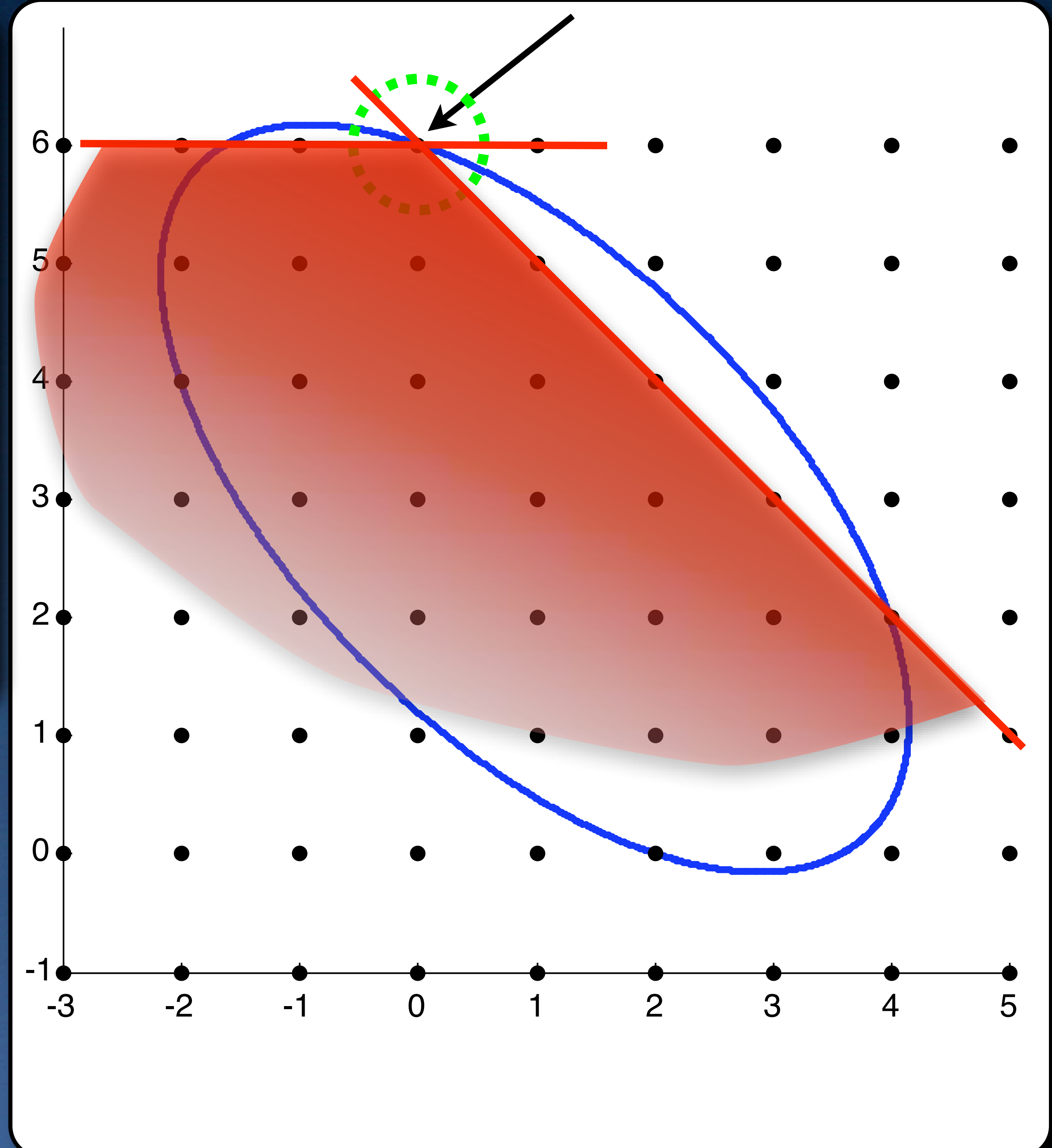
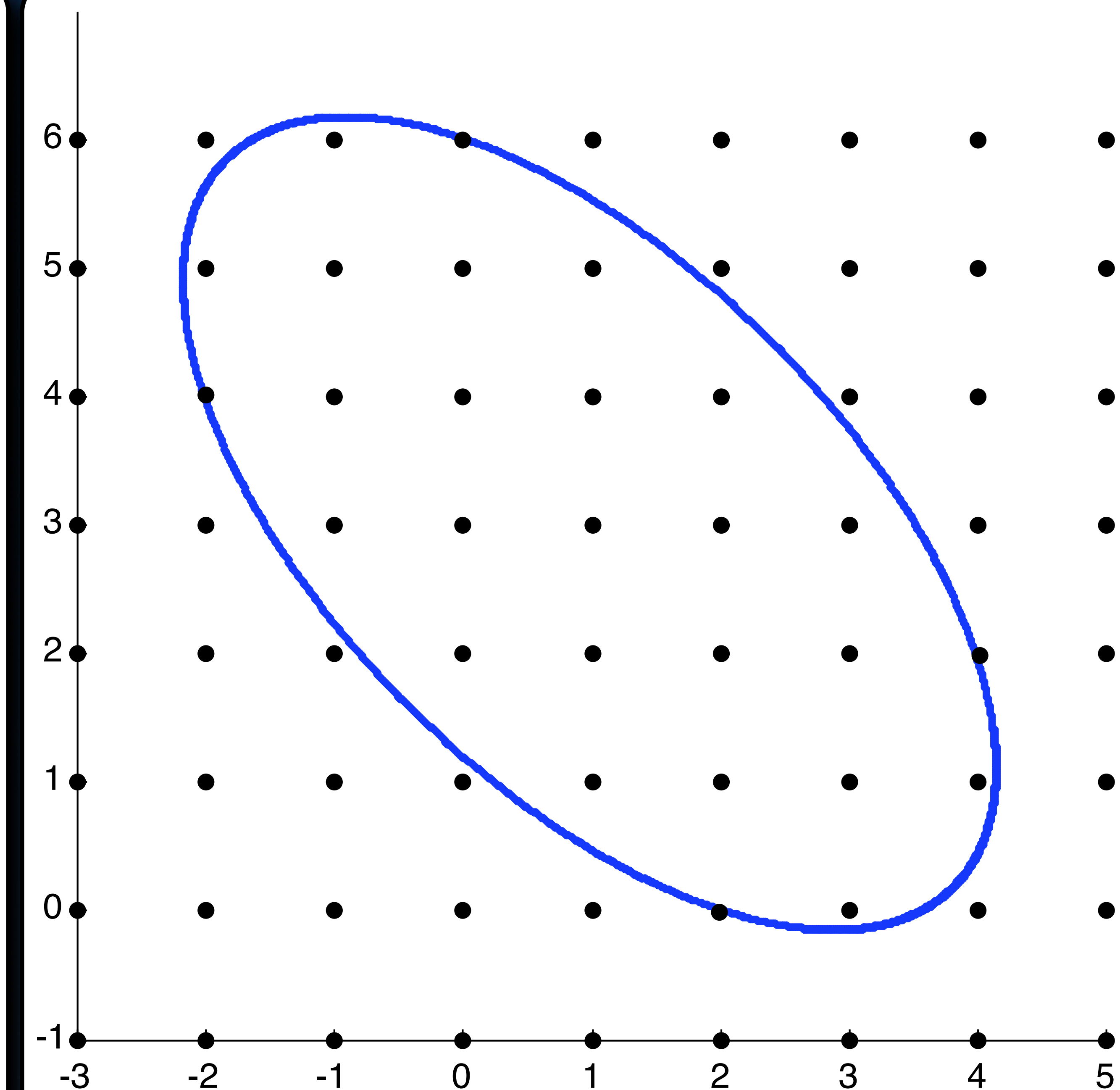
- Similar to non-integer separation + compactness argument



Compactness Argument

$$K := \mathrm{bd}(C) \setminus \bigcup_{v \in \mathrm{bd}(C) \cap \mathbb{Z}^n} \mathcal{N}_v$$



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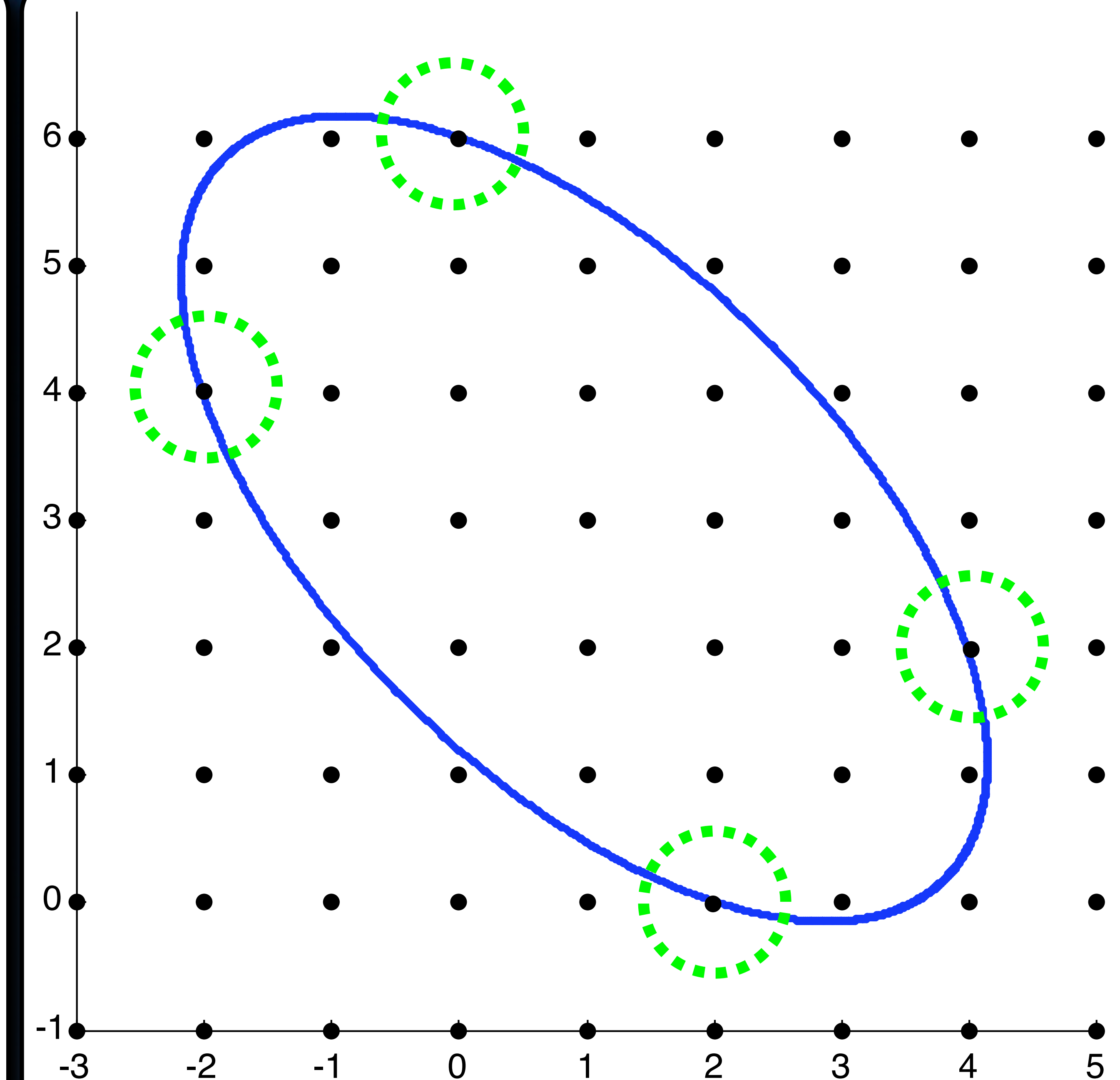

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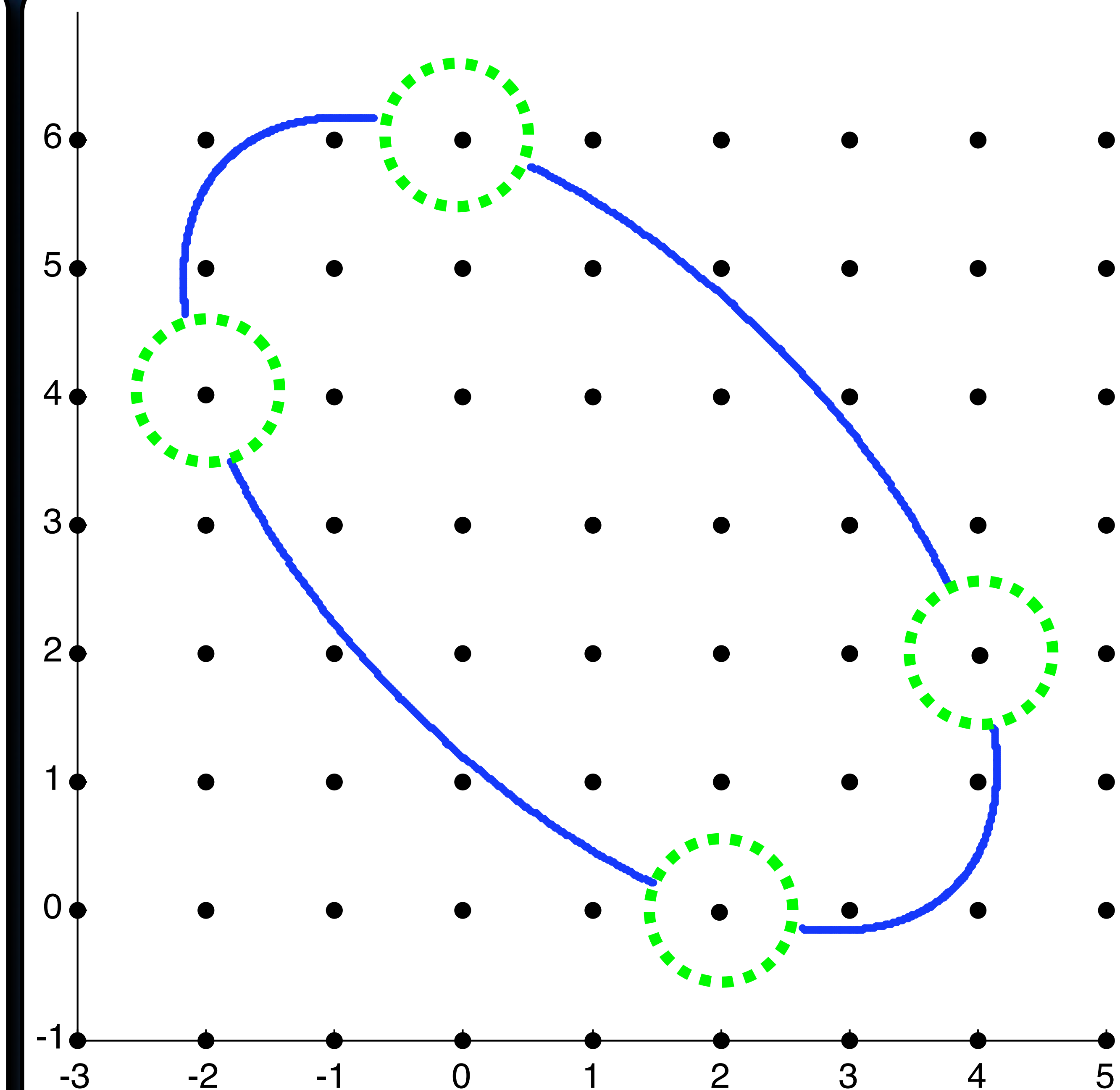

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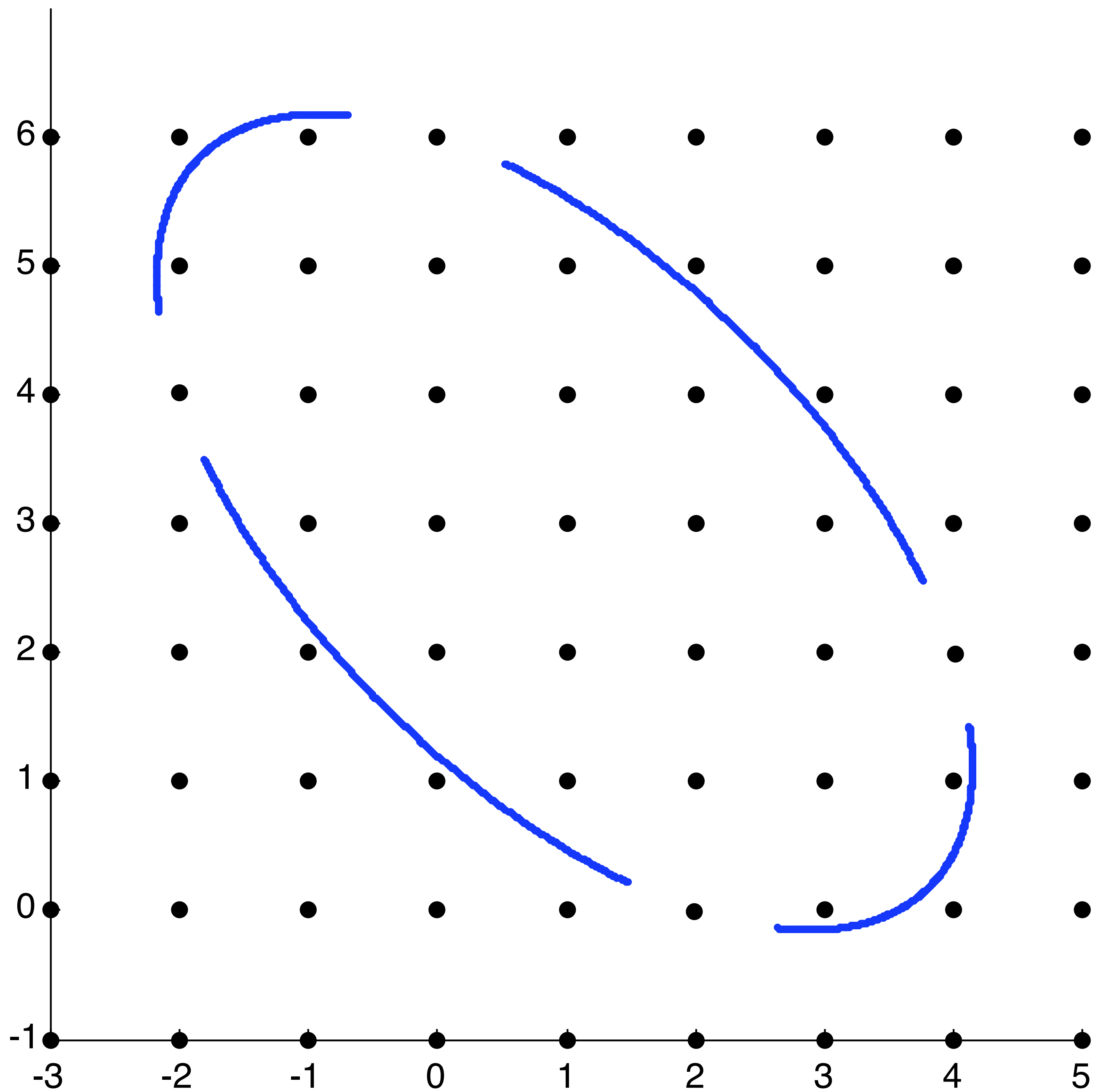

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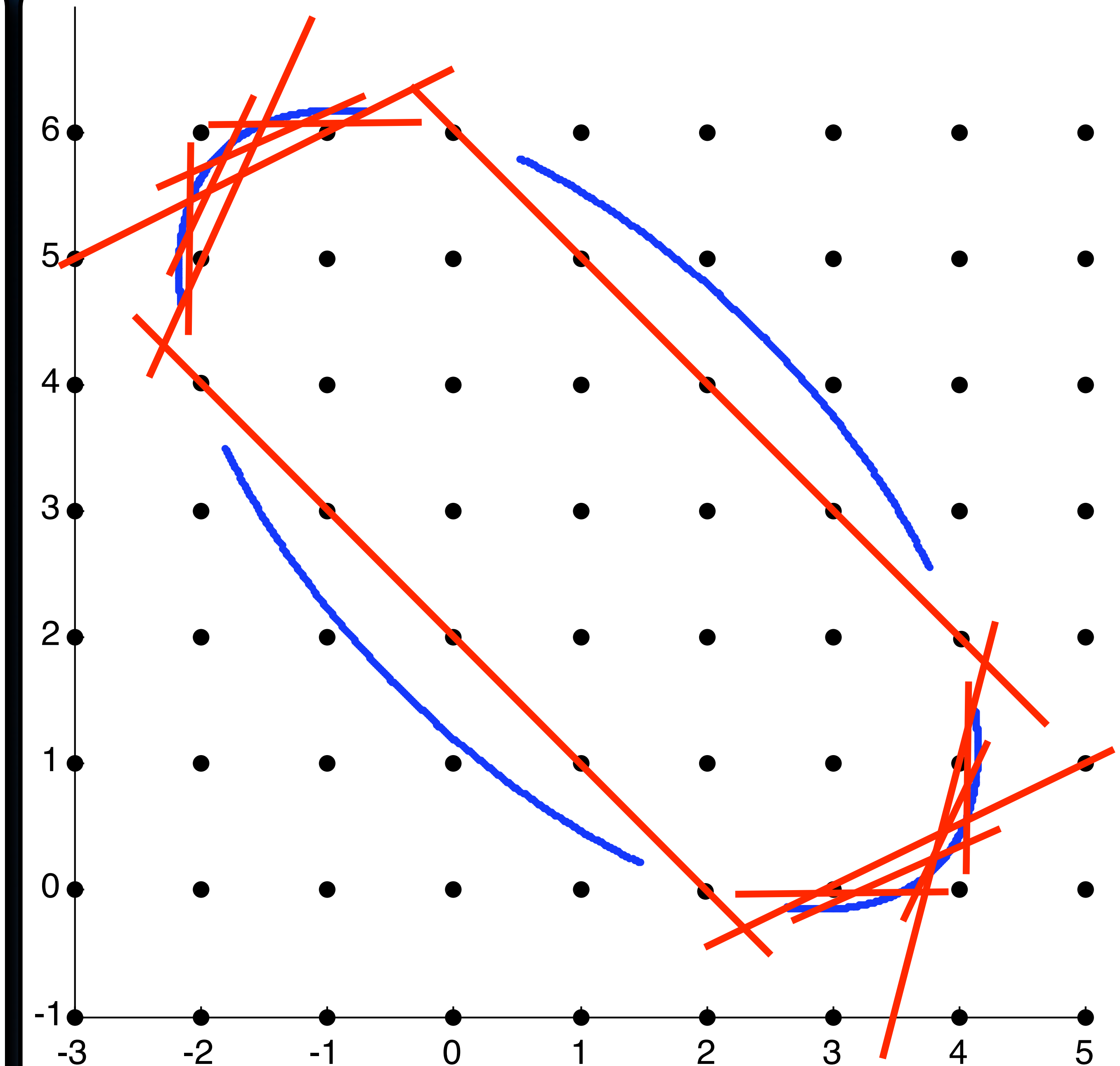

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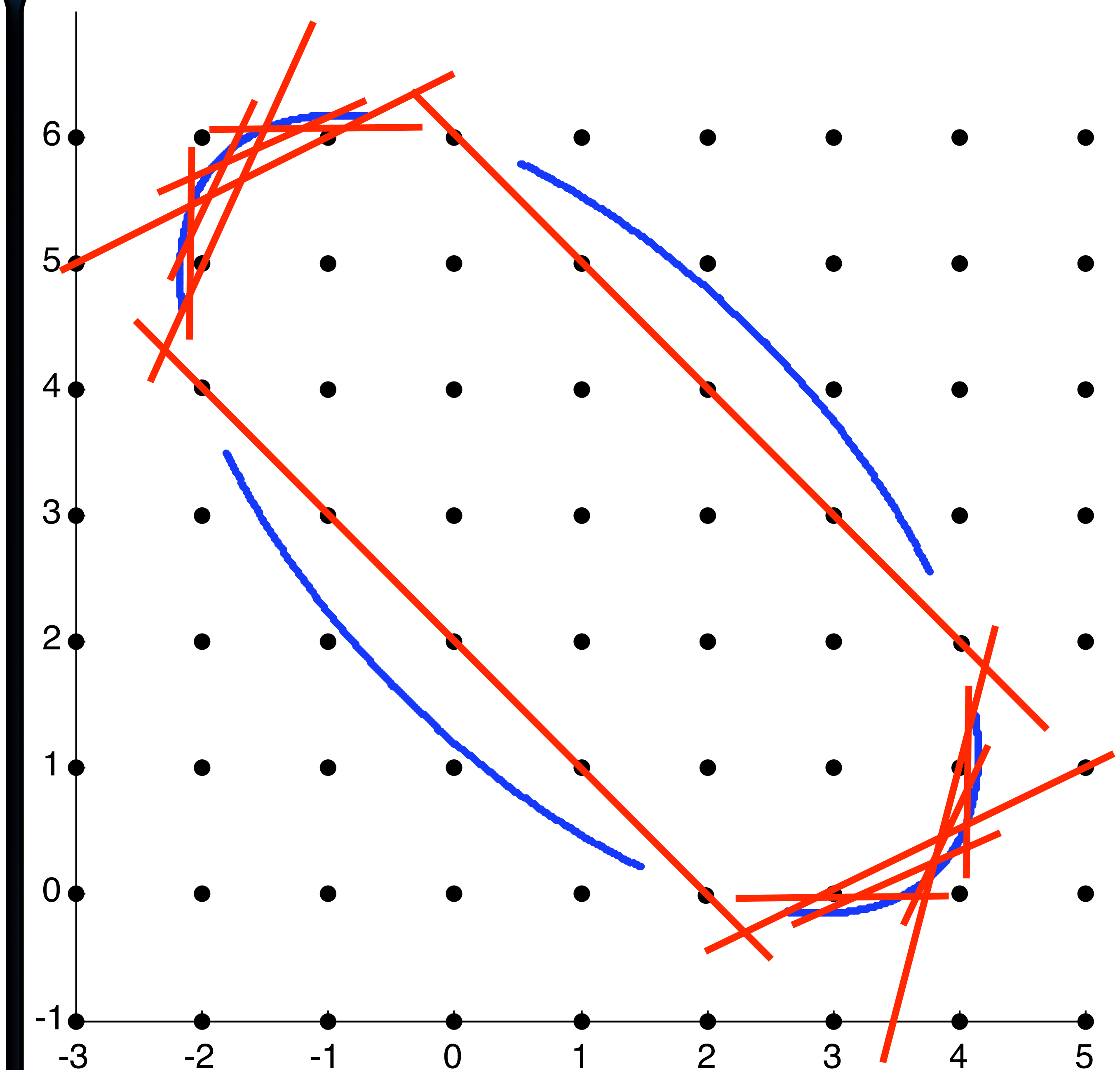
compact → $K \subset \bigcup_{u \in K} \mathcal{S}_u$

$$K \subset \bigcup_{i=1}^{m} \mathcal{S}_{u^i}$$



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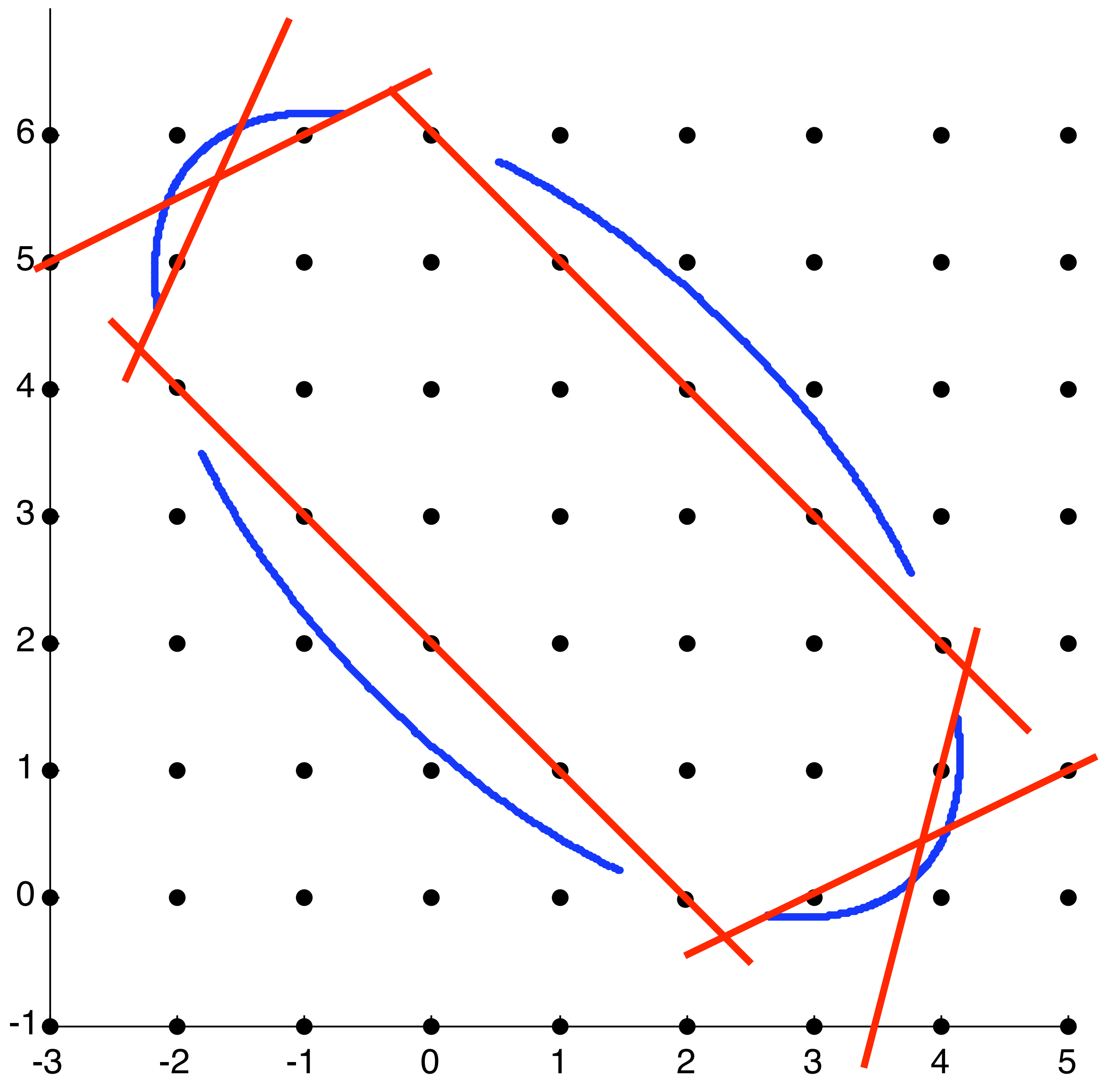

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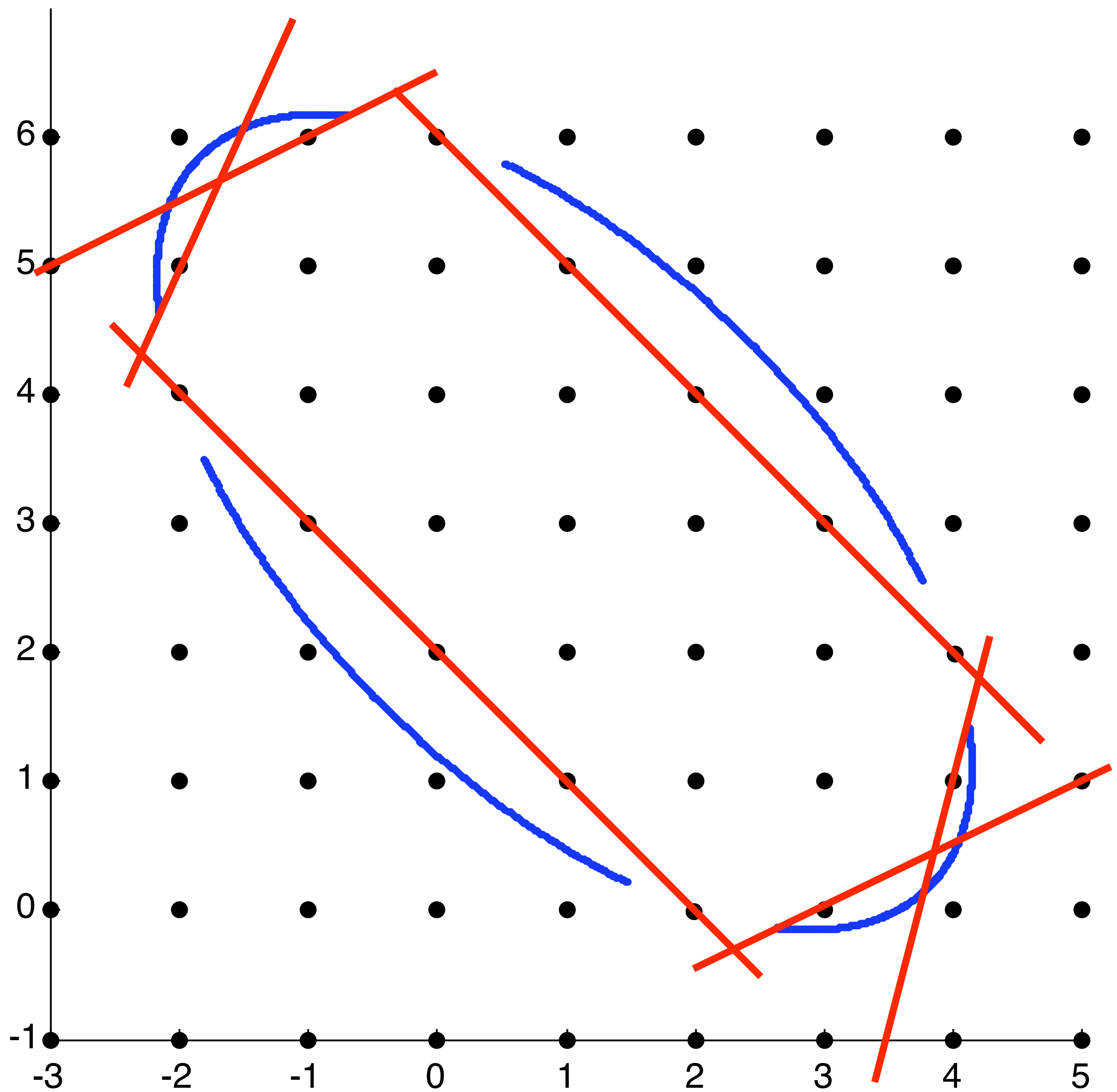

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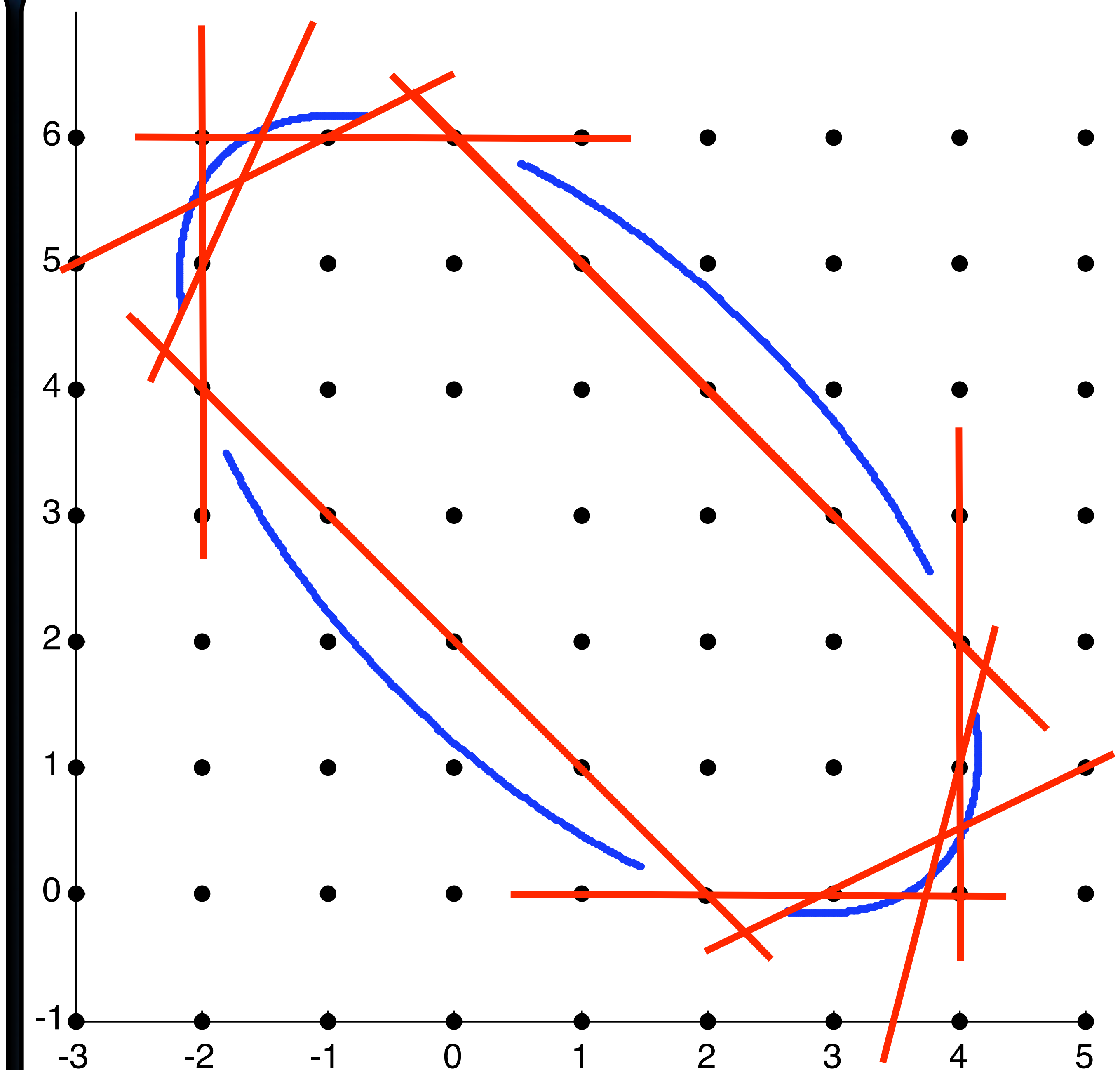

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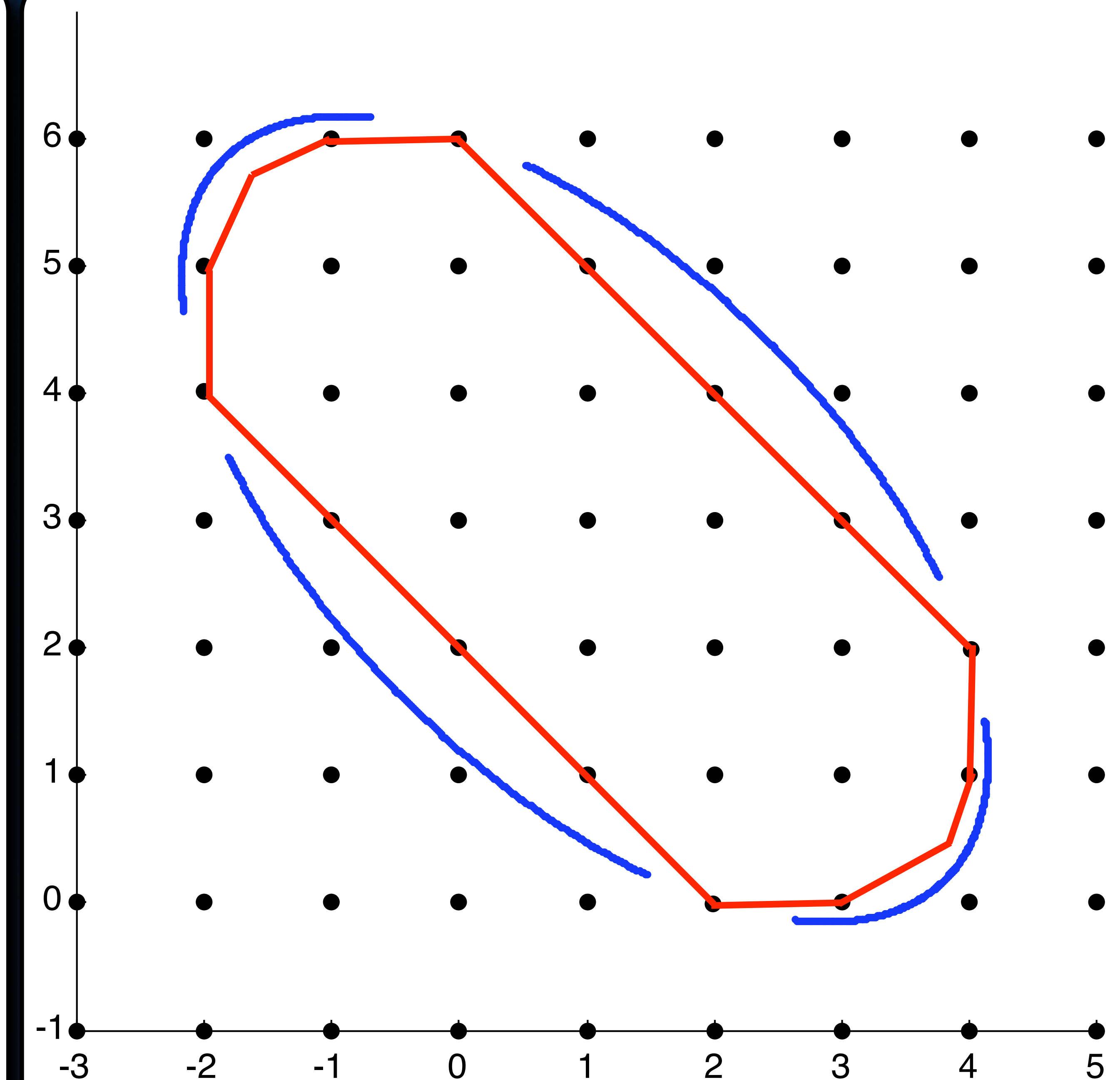

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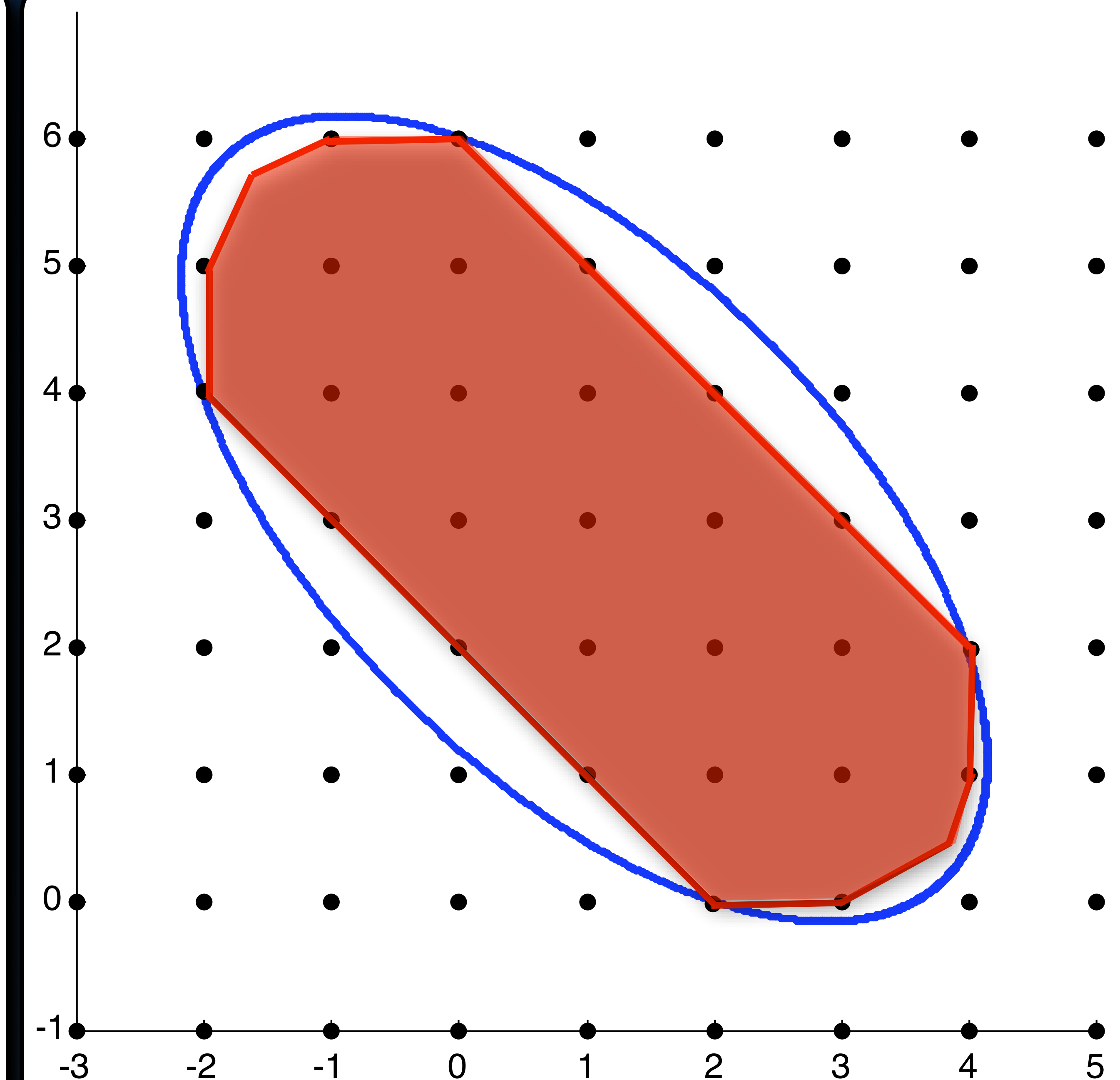

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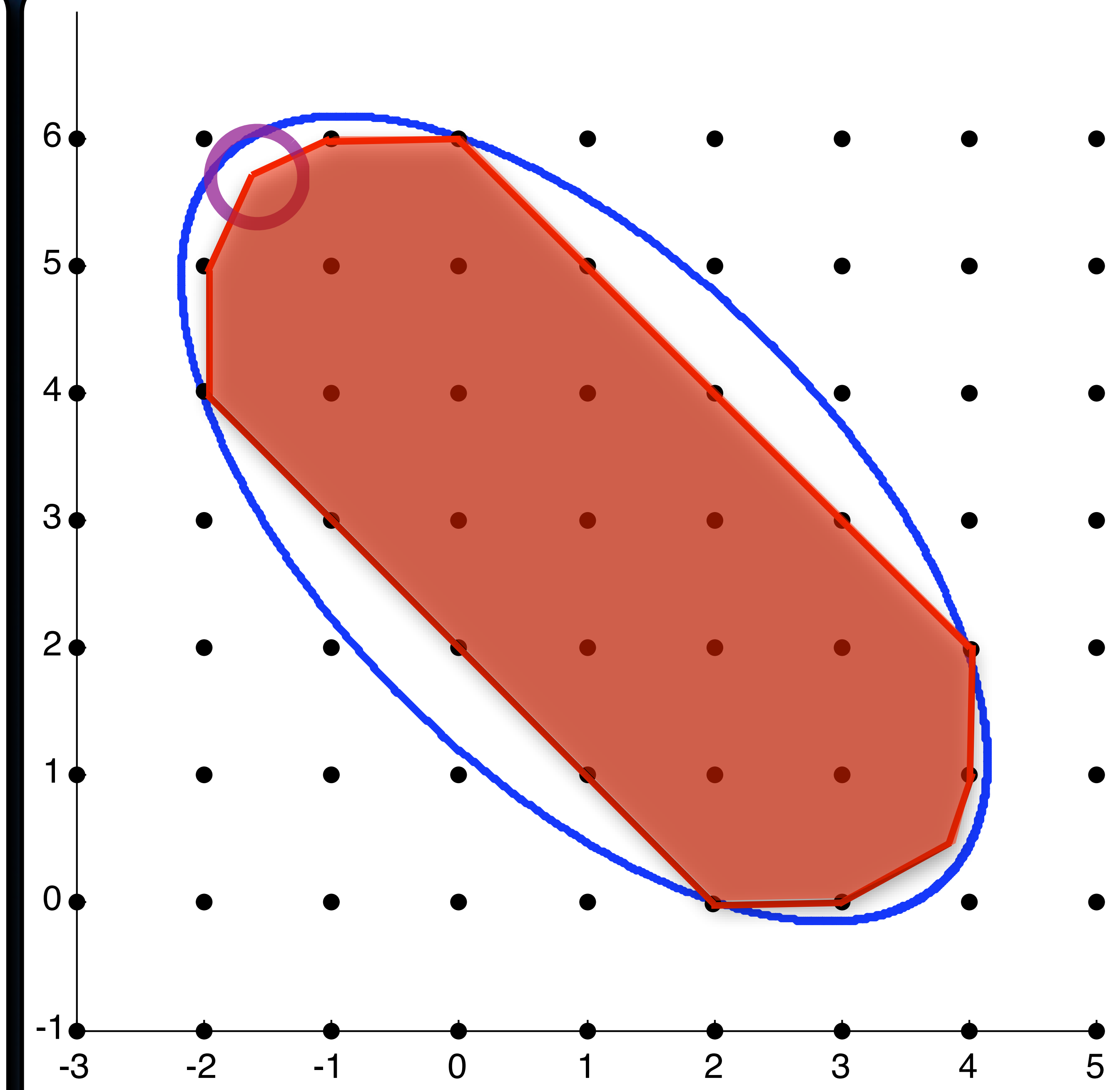


Compactness Argument

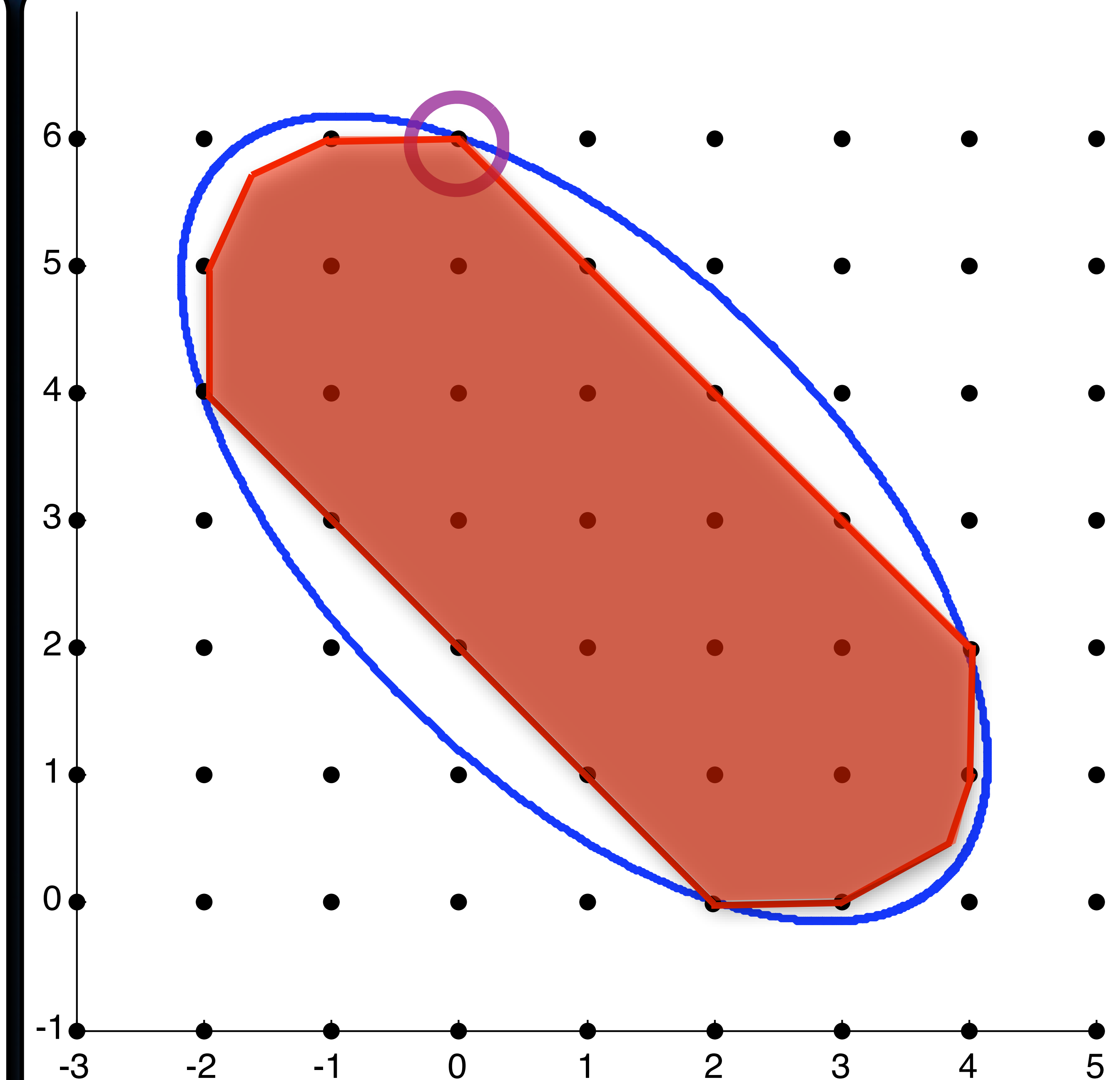
$$K := \mathrm{bd}(C) \setminus \bigcup_{v \in \mathrm{bd}(C) \cap \mathbb{Z}^n} \mathcal{N}_v$$

$$K \subset \bigcup_{i=1}^{m} \mathcal{S}_{u^i}$$

$$v \in \mathrm{bd}(C) \cap \mathbb{Z}^n$$

$$\mathtt{CGC}(I_v, C) \cap \mathcal{N}_v \cap \mathrm{bd}(C) = \{v\}$$

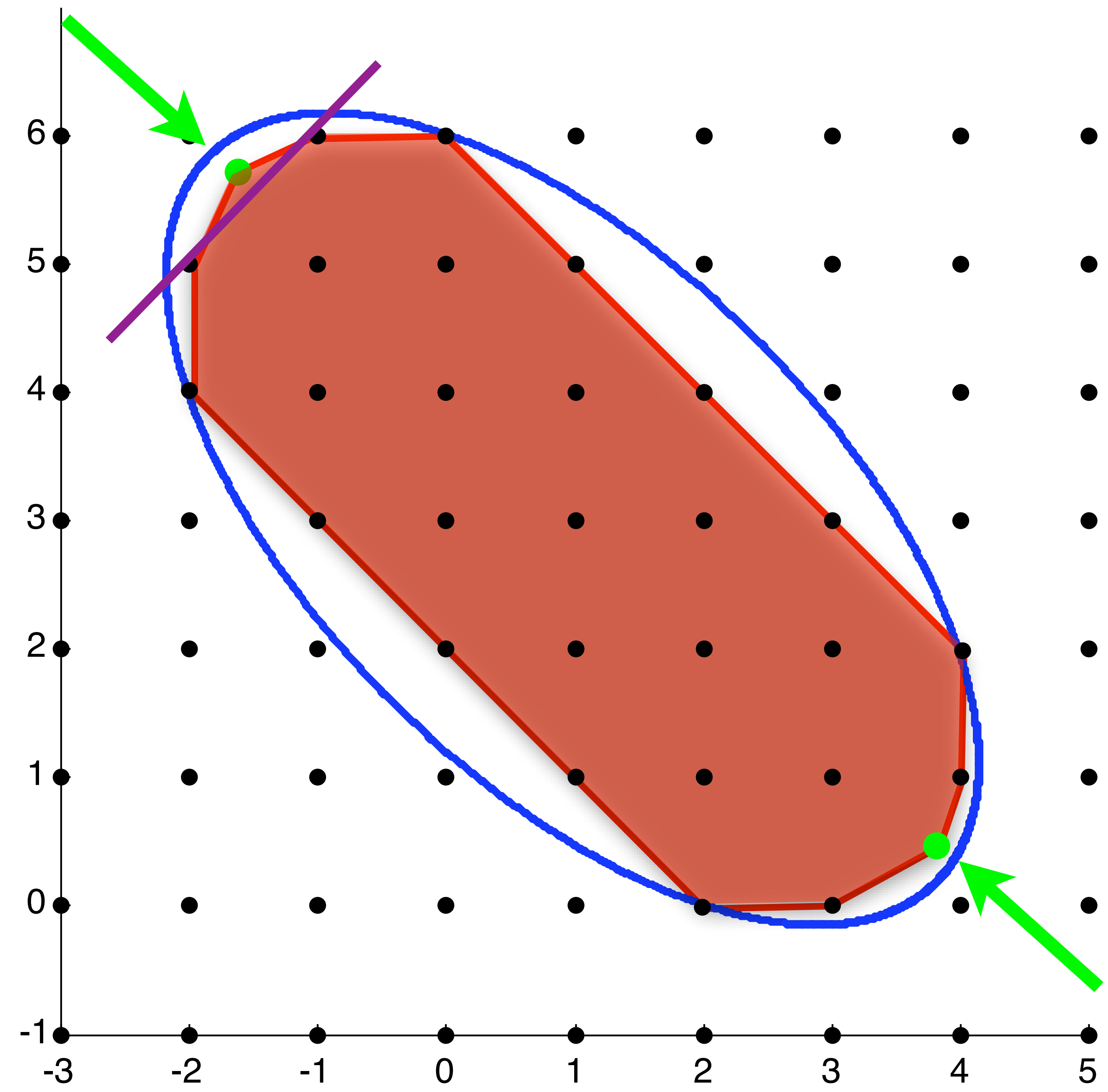
$$S^1 = \bigcup_{i=1}^{m} \left\{a^{u^i}\right\} \cup \bigcup_{v \in \mathrm{bd}(C) \cap \mathbb{Z}^n} I_v$$



Step 2 : Separate $\mathtt{CGC}(S^1, C) \setminus \mathtt{CGC}(\mathbb{Z}^n, C)$

$$V := \operatorname{Ext}\left(\mathtt{CGC}(S^1, C)\right) \setminus \mathbb{Z}^n$$

$$\langle a, v\rangle > \lfloor \sigma_C(a) \rfloor$$



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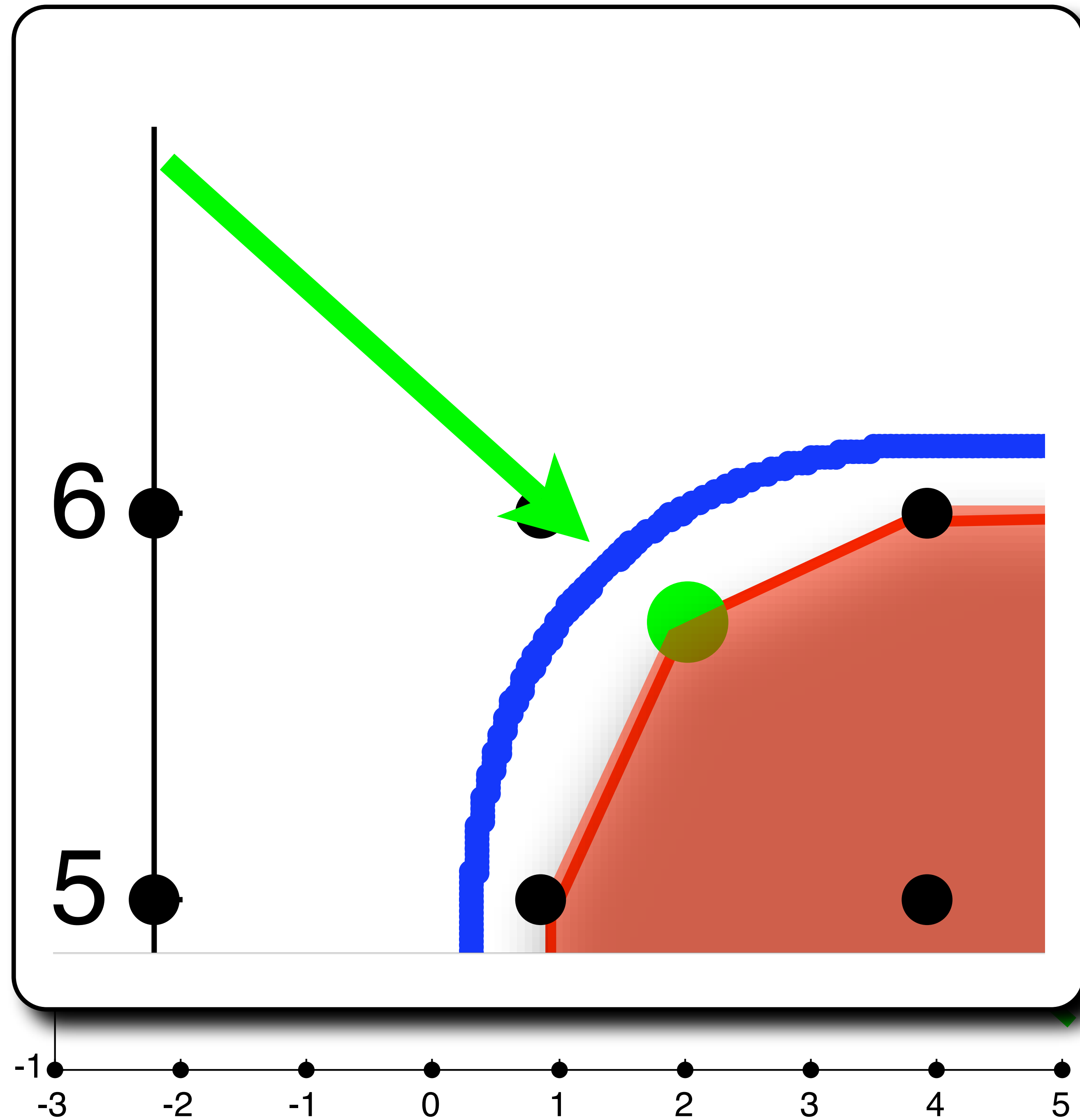

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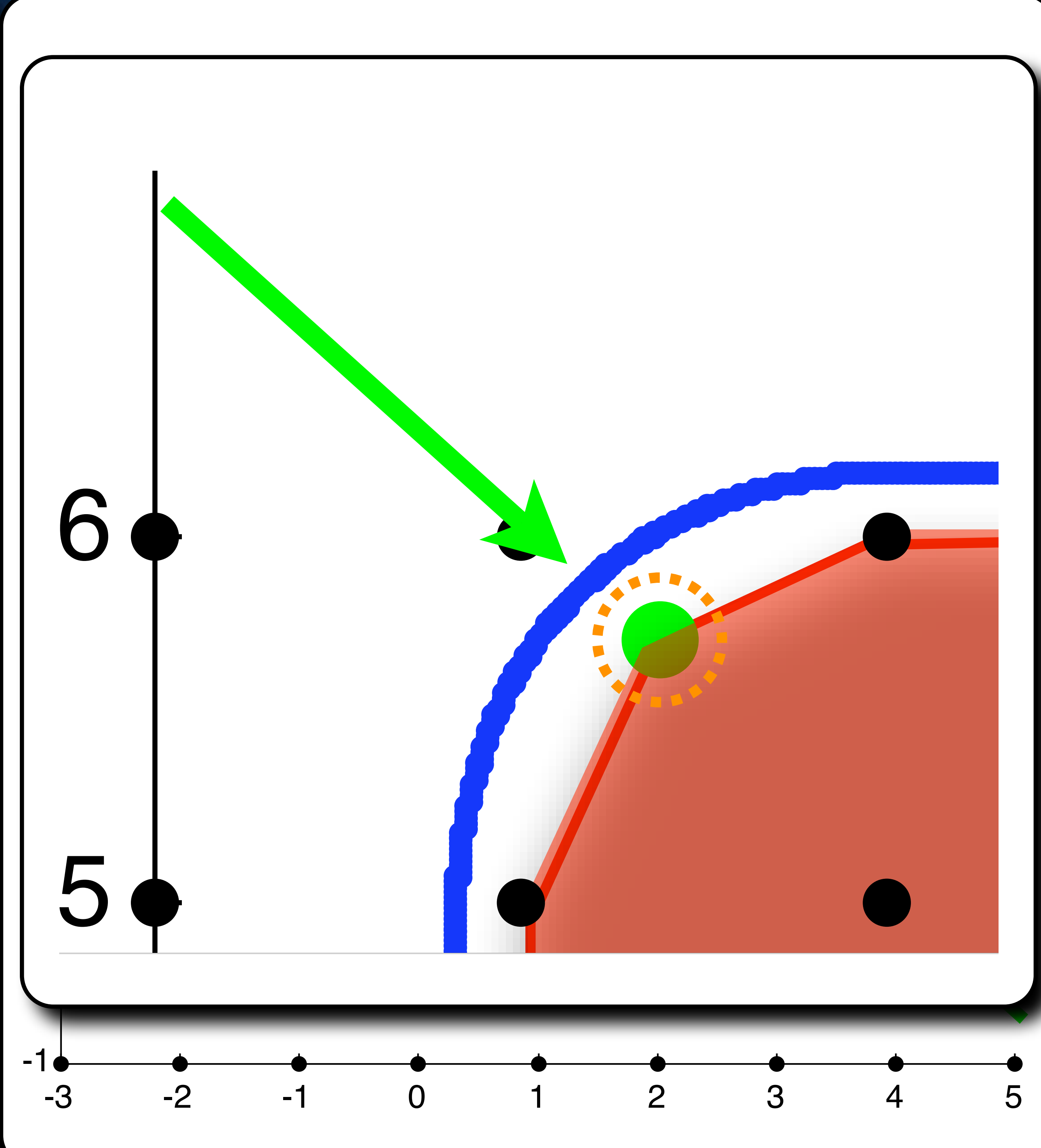

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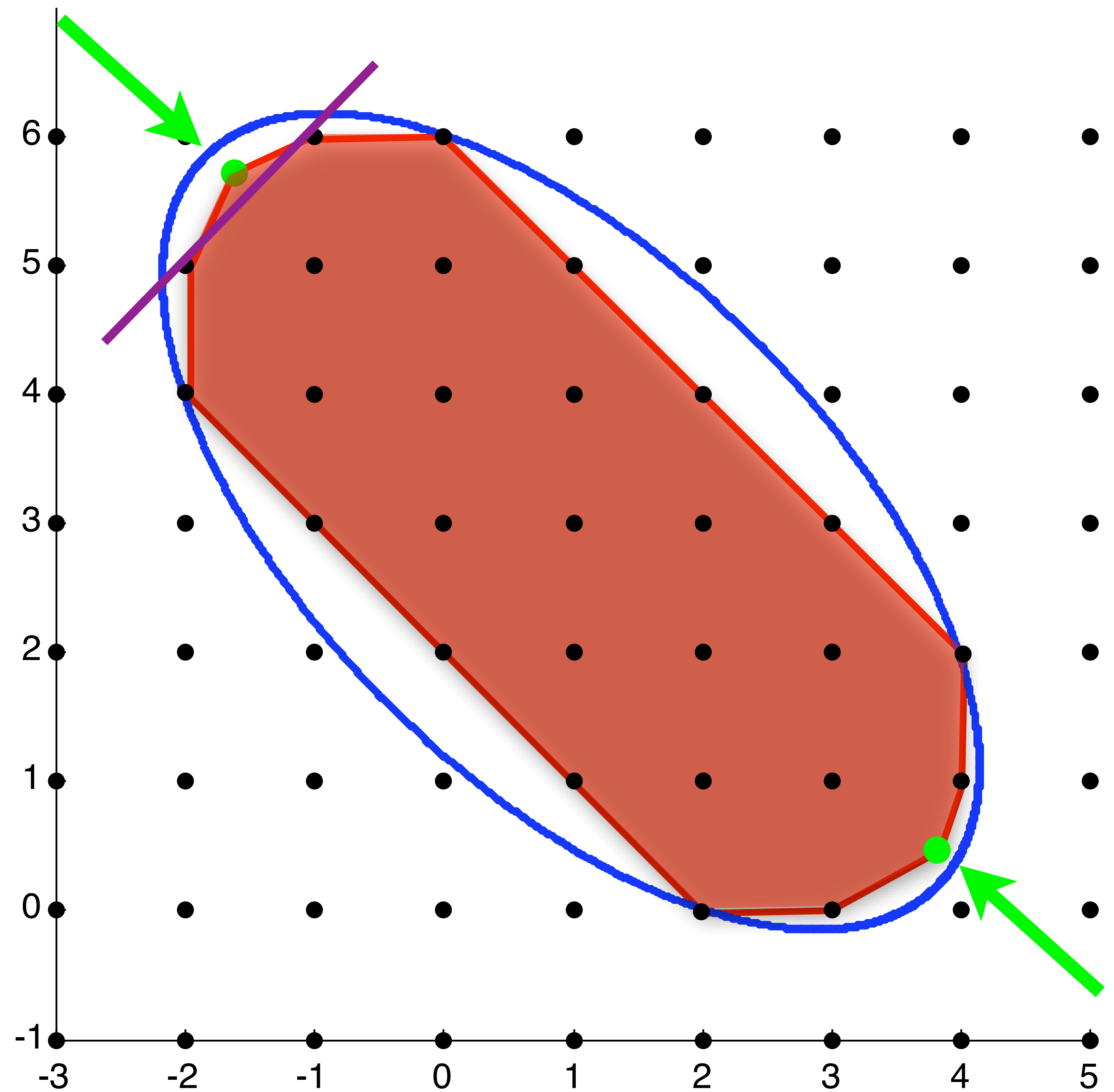

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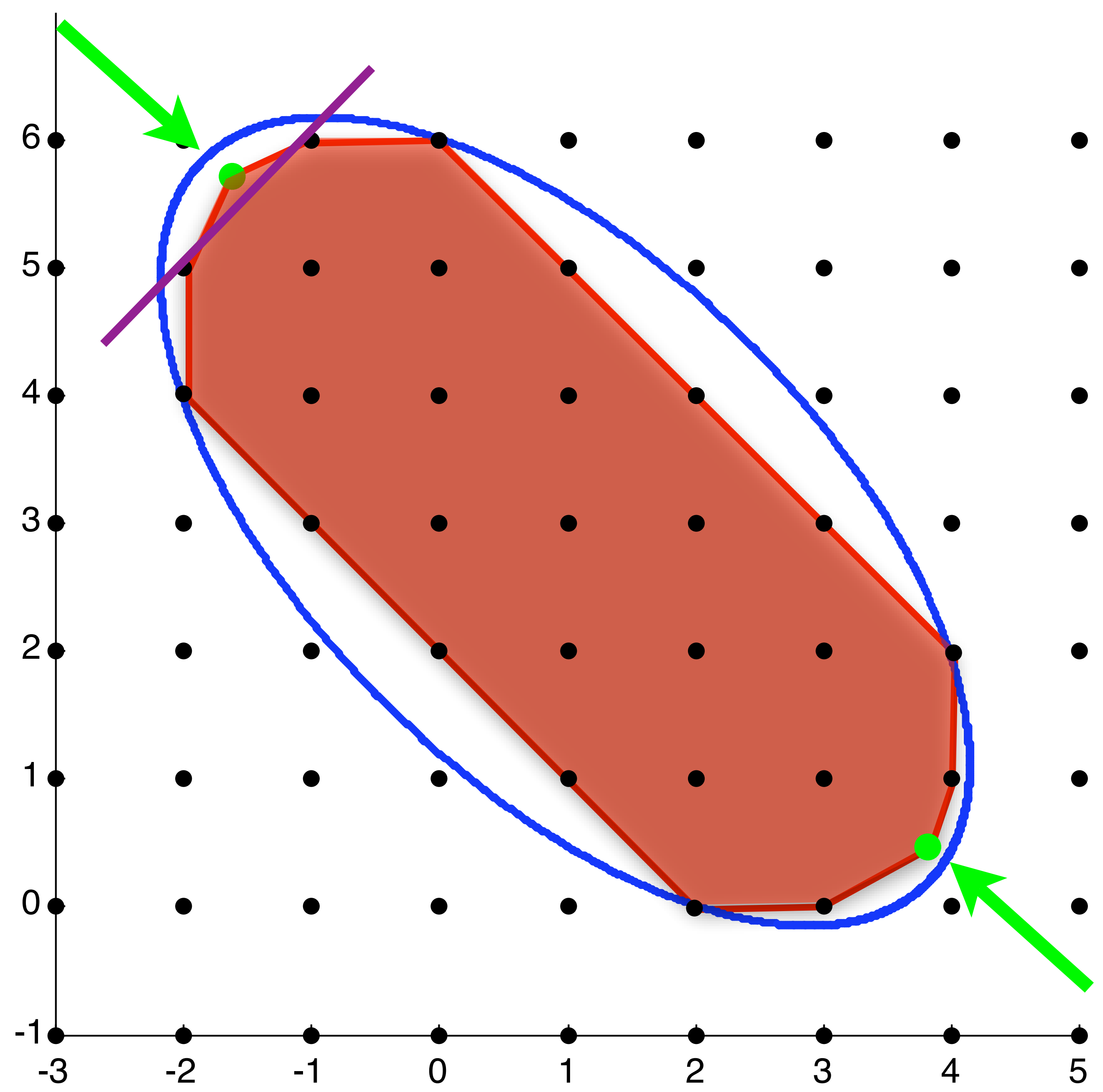
$$\|a\| \geq \frac{1}{\varepsilon} \Rightarrow$$

$$\begin{aligned} \lfloor \sigma_C(a) \rfloor &\geq \sigma_C(a) - 1 \\ &\geq \sigma_{v+\varepsilon B^n}(a) - 1 \\ &= \langle v, a\rangle + \varepsilon\|a\| - 1 \\ &\geq \langle v, a\rangle \end{aligned}$$



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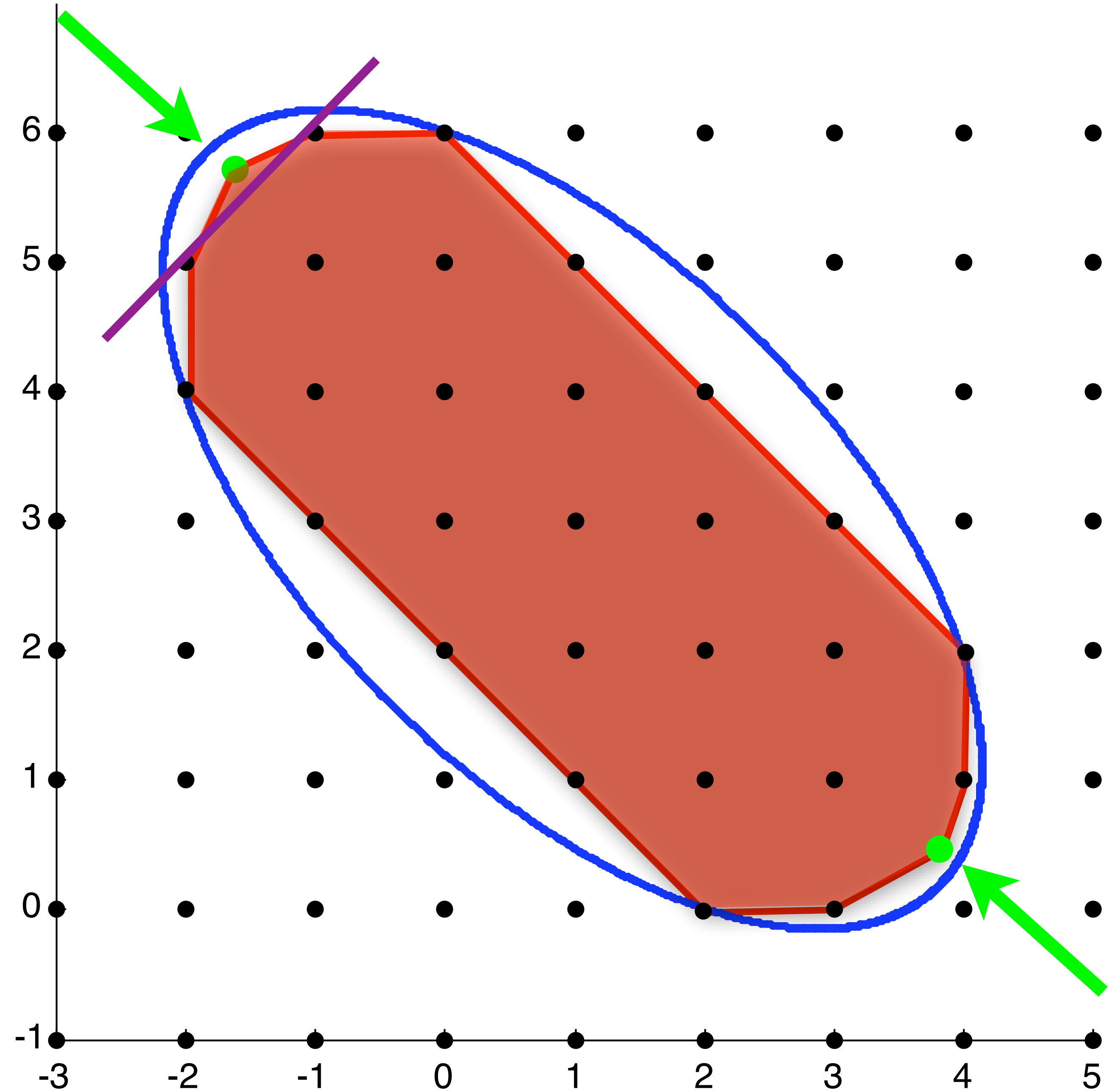
$$S^2 = (1/\varepsilon) B \cap \mathbb{Z}^n$$

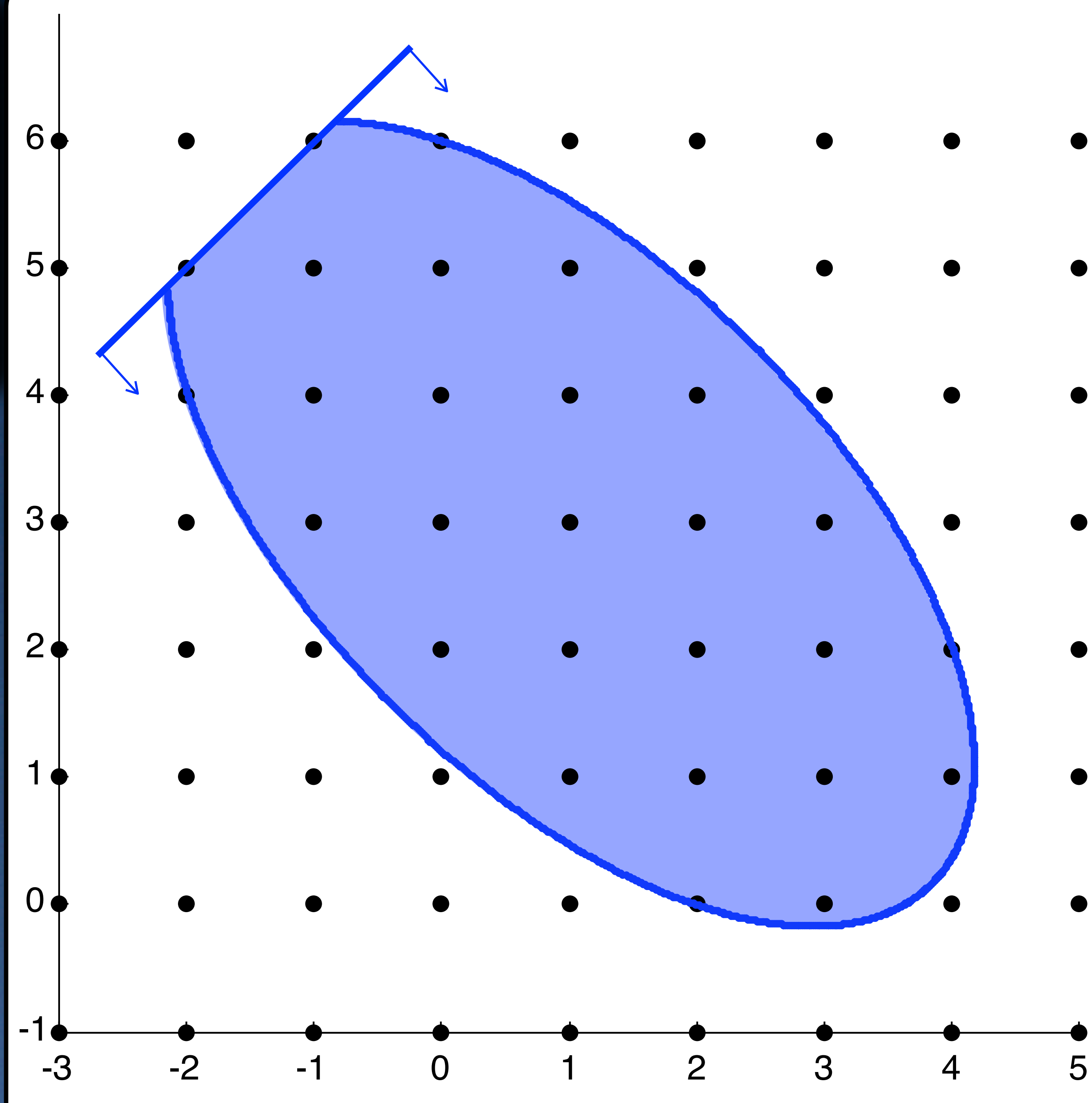


Example: Ellipsoid and Halfspace

P polyhedron, F face of P

$\mathtt{CGC}(F) = \mathtt{CGC}(C) \cap F$

(Schrijver, 1986)



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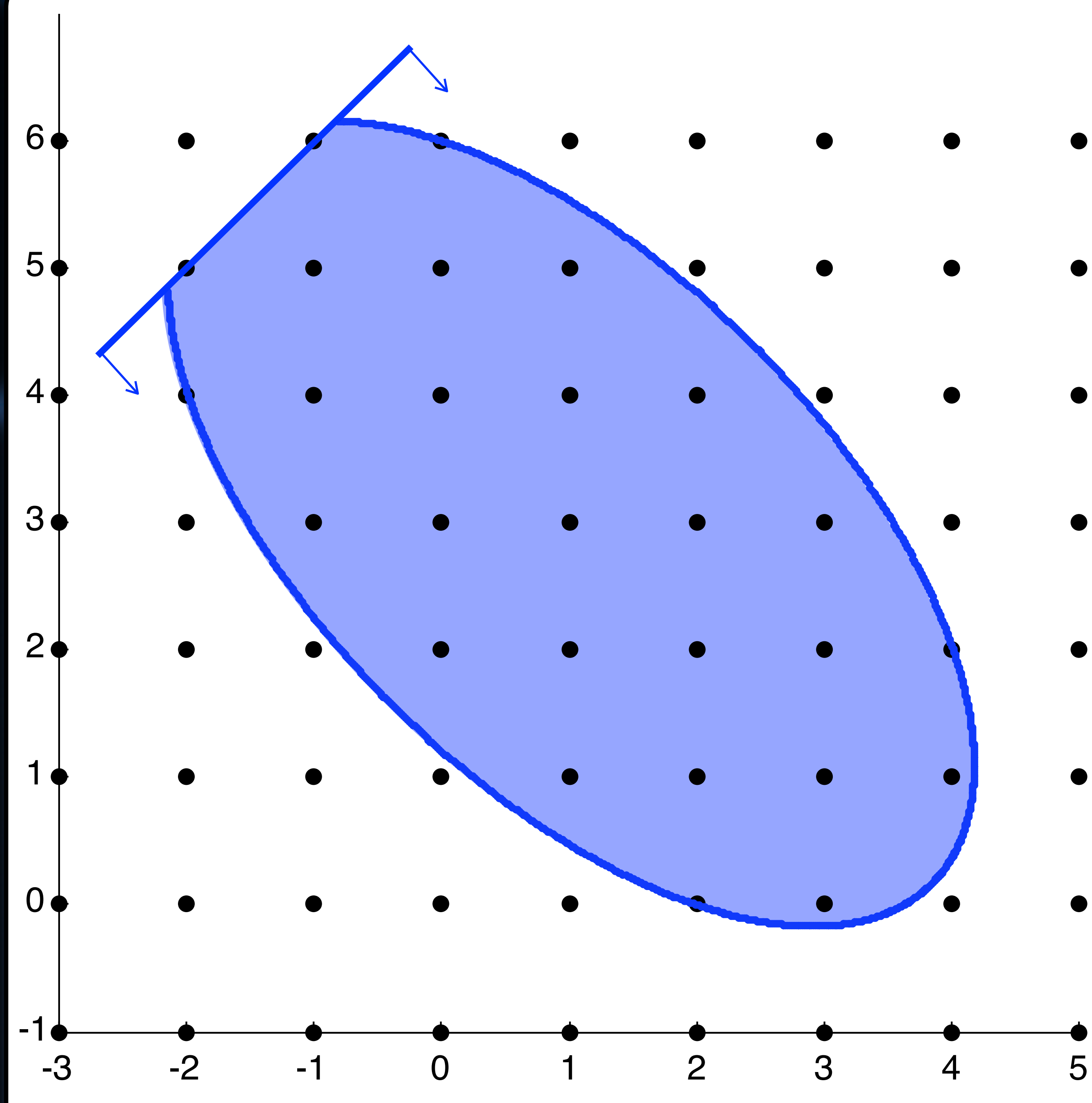

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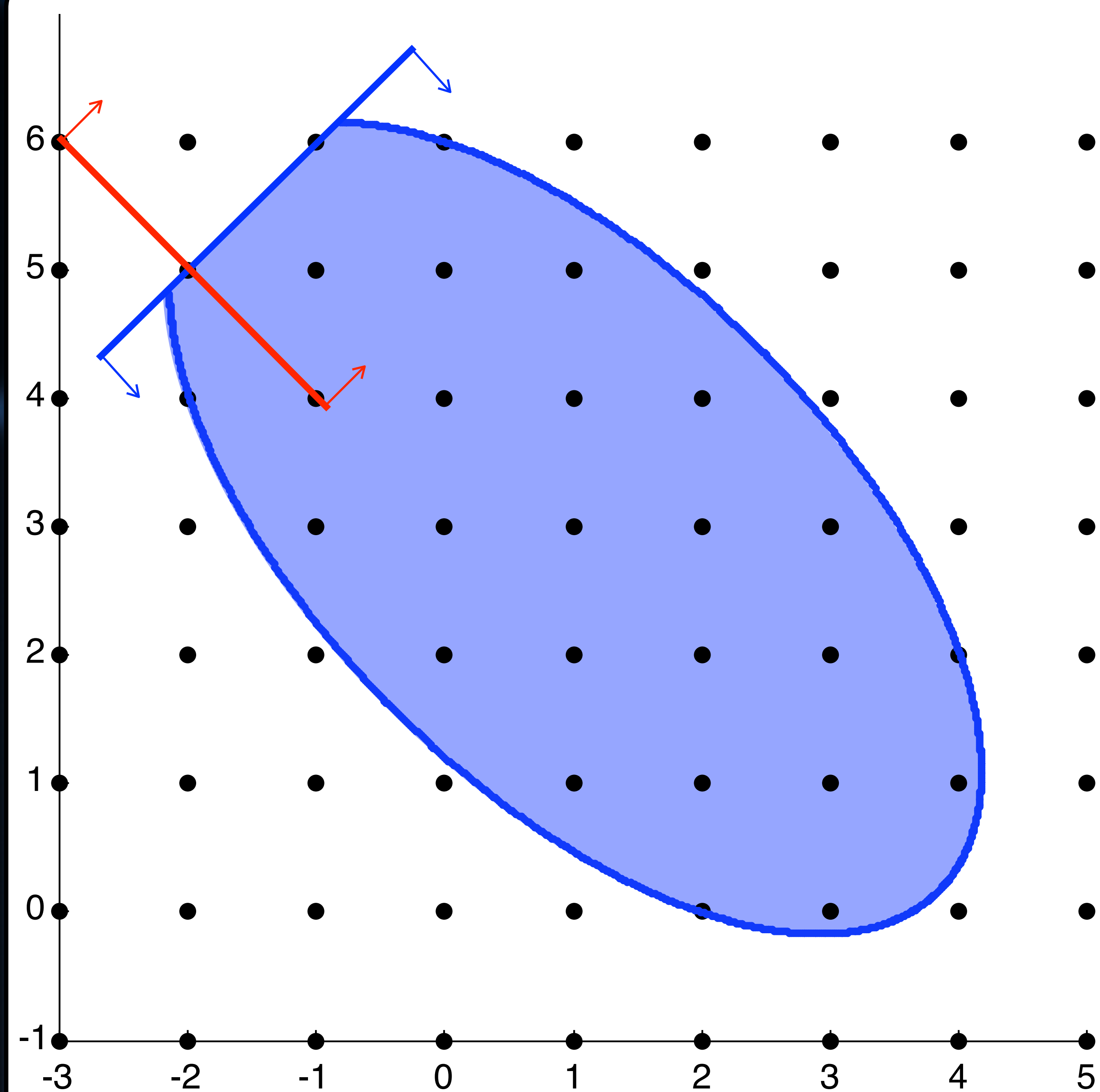

Example: Ellipsoid and Halfspace

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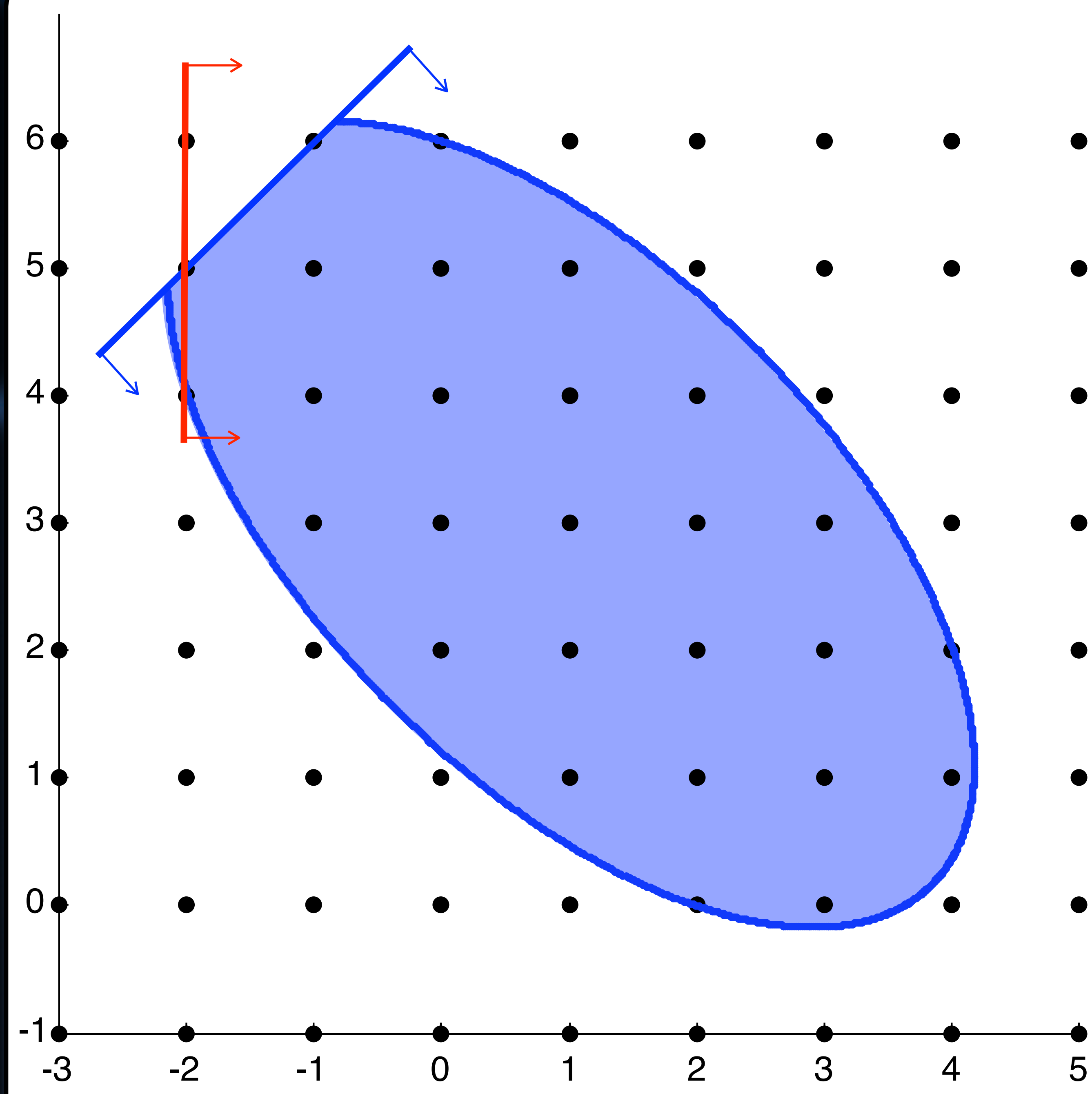
(Schrijver, 1986)

We can generalize it.

$$\langle a, x\rangle \le \lfloor \sigma_F(a) \rfloor$$

$$\downarrow$$

$$\langle a', x\rangle \le \lfloor \sigma_C(a') \rfloor$$



Split Closure of an Ellipsoid

- Pure Integer Case:

$$C = \{x \in \mathbb{R}^3 \ : \ \|A(x-c)\|_2 \leq 1\}$$

$$A = \frac{1}{\sqrt{33/64}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/100 \end{bmatrix}, \quad c = (1/2, 1/2, 1/2)^T$$

- Two split cuts:

$$x_1 \leq 0 \vee x_1 \geq 1 \qquad\qquad x_2 \leq 0 \vee x_2 \geq 1$$

Conclusions and Future Work

- Non-Constructive because of compactness argument in step 1.
- Current work:
  - General compact convex sets including non-rational polytopes ("Almost" done).
  - Split closure is "finitely generated".
- Open Problems:
  - Simpler Proof (Circle in $\mathbb{R}^2$?).
  - Constructive/Algorithmic proof.