Comparing two formulations for the ARM problem

Marcos Goycoolea[1] Alan Murray[2] Juan Pablo Vielma[1]
Andres Weintraub[3]

[1]Department of Industrial and Systems Engineering
Georgia Institute of Technology

[2]Department of Geography
The Ohio State University

[3]Departamento de Ingenieria Industrial
Universidad de Chile

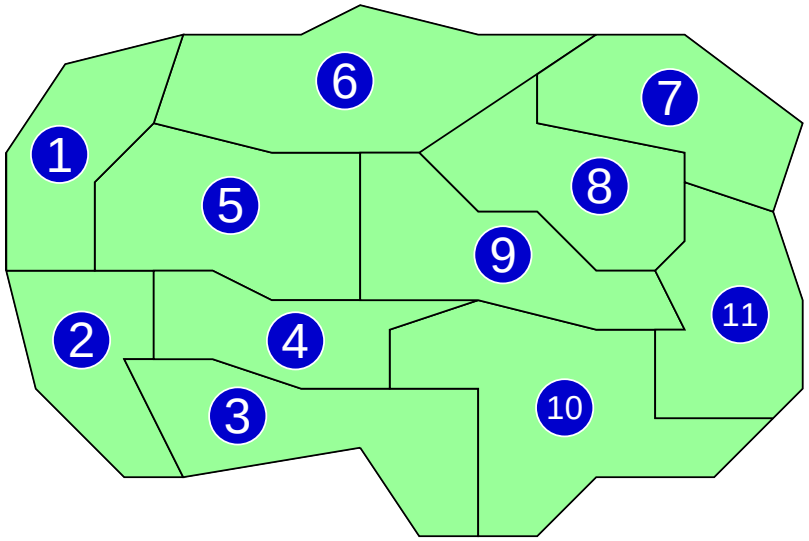
12th Symposium for Systems Analysis in Forest Resources, September 2006

Outline

Obtain Harvest Schedule that Maximizes Profit Subject to Clear Cut Limitations and Side Constraints

- Environmental regulations set Maximum Area Constraints:
  - Reasons include wildlife habitat, scenic beauty, etc.
  - Maximum Clear Cut Area: 40+ to 120+ acres.
  - Thompson et al. 1973, Jones et al. 1991, Barrett et al. 1998, Murray 1999, Boston and Bettinger 2001, Boston and Bettinger 2001, McDill et al. 2002, Bettinger and Sessions 2003. . .
- Side constraints include:
  - Timber Volume Flow Constraints.
  - Average Ending Age.

ARM Includes Aggregation of Cells in the Problem



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- Cluster = Groups of adjacent cells.
- Feasible Cluster = Area-complying clusters.
- Solution is group of non-adjacent feasible clusters.

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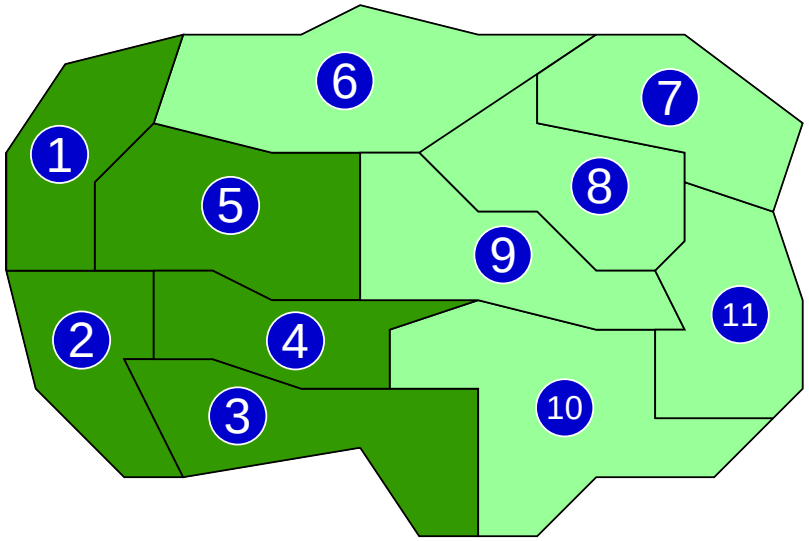

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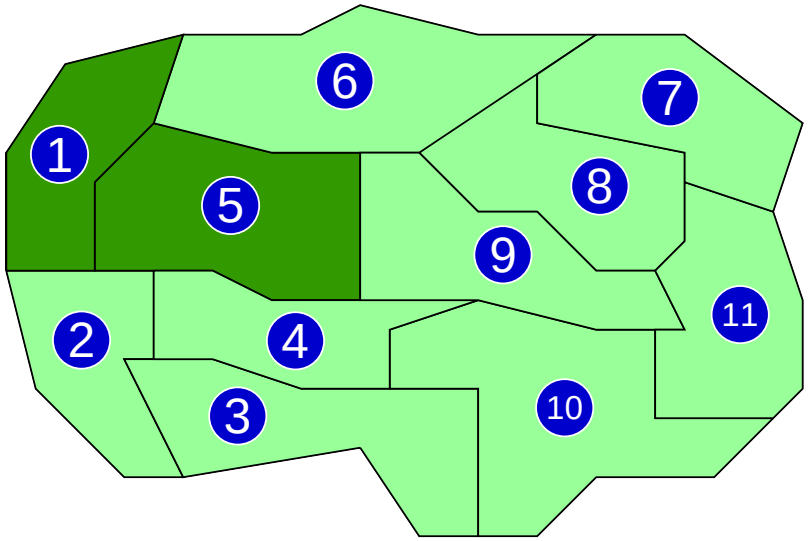

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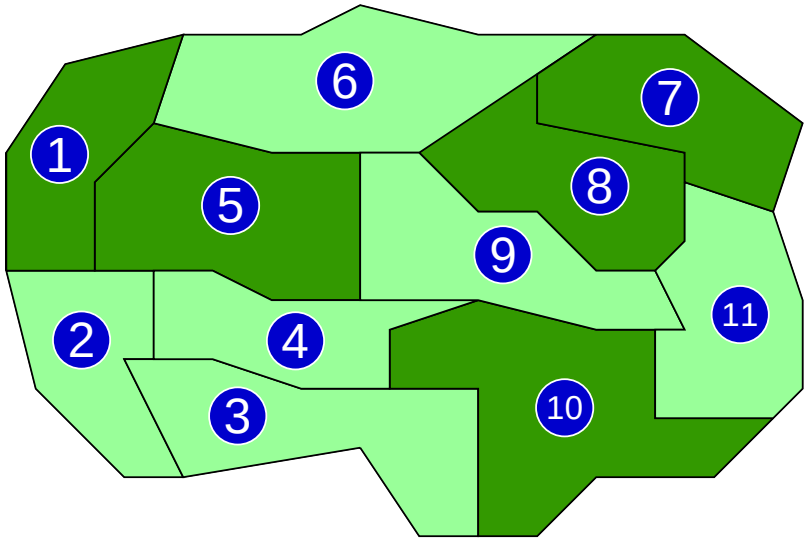

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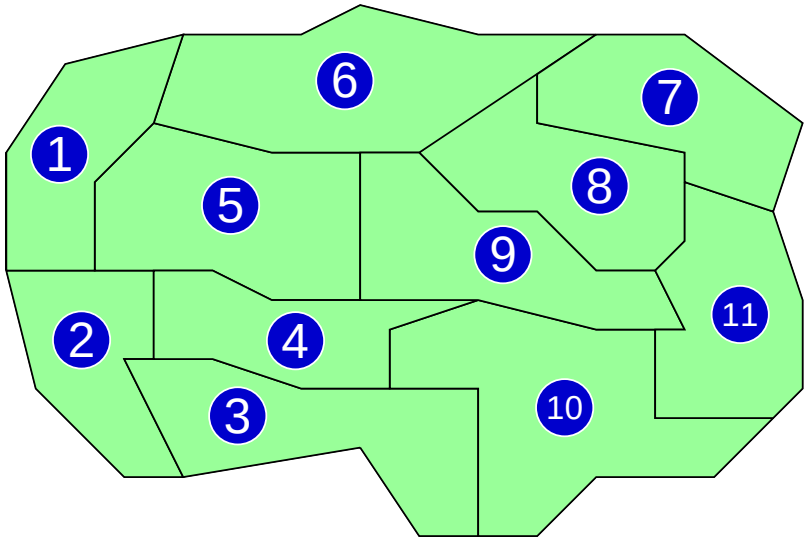
Cell Approach Forbids Infeasible Clusters



- One variable per cell.
- Cover/Path Constraints forbid harvesting (Minimal) Infeasible Clusters. (McDill et al. 2002)
- Strengthening:
  - Crowe et al. 2003 *Clique* Constraints.
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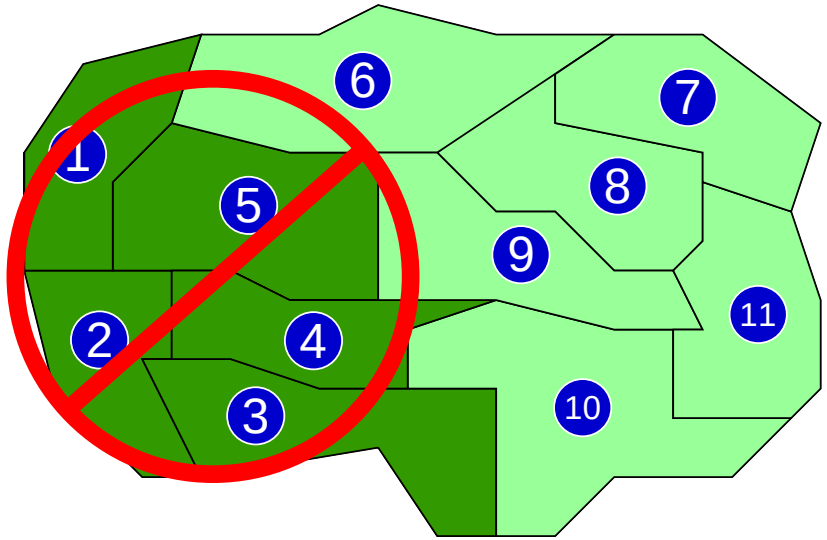

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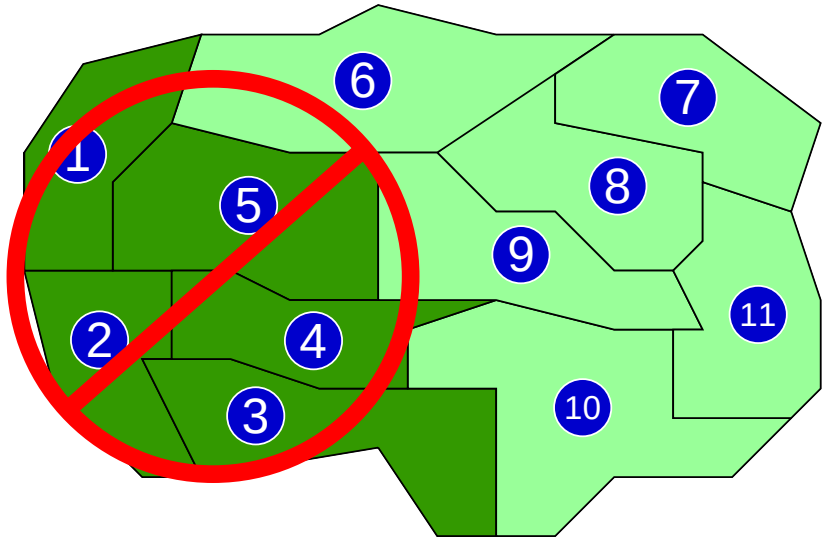

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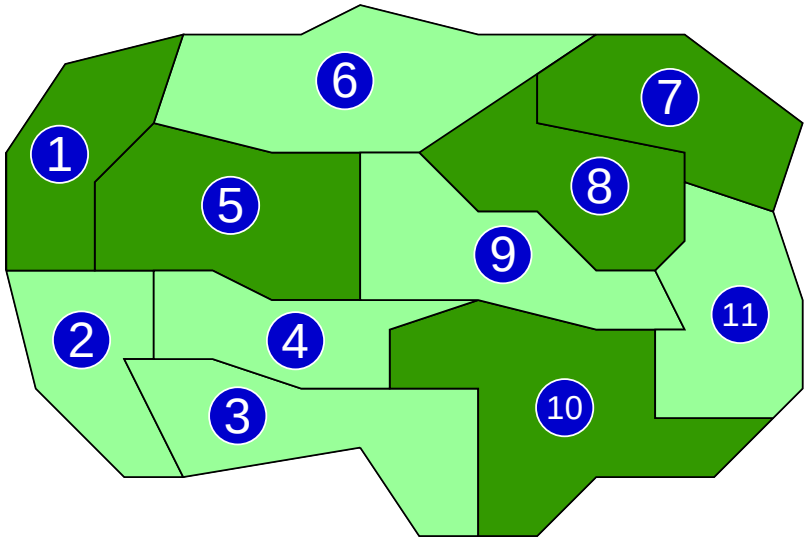

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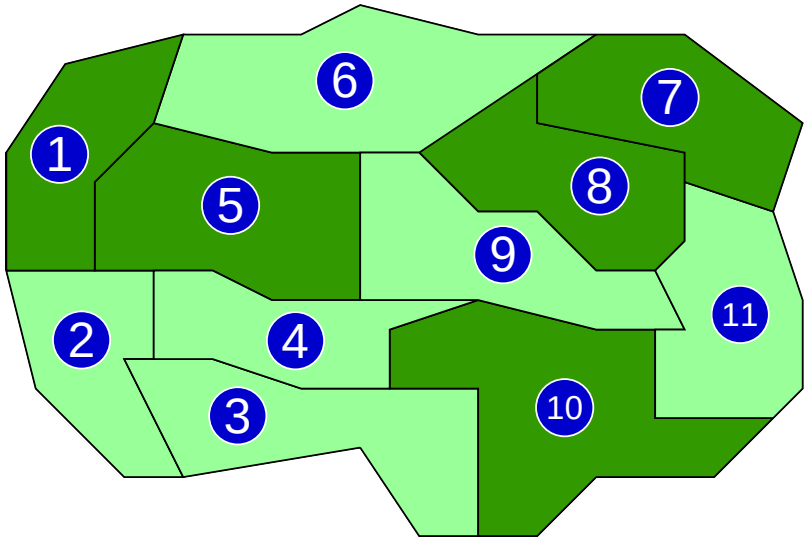

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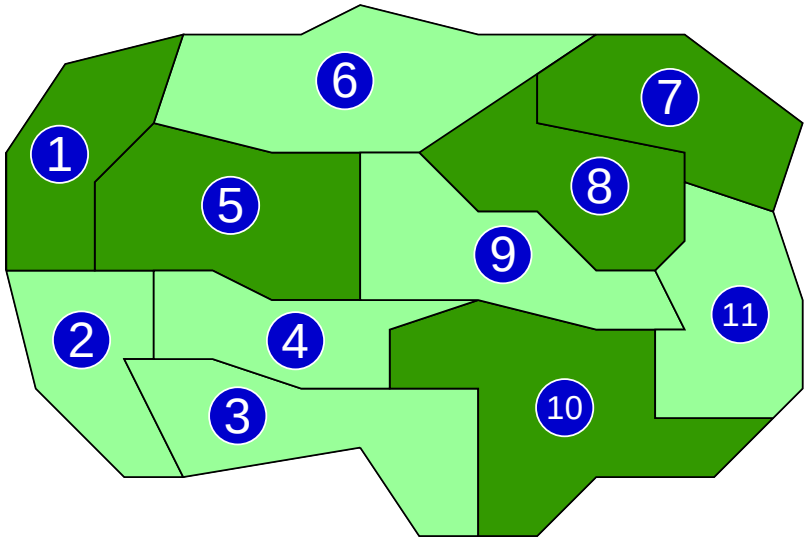

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Cluster Approach Easily Allows for Extra Modeling Requirements

- Fixed Harvesting Costs:
  - Modify objective coefficients in cluster approach.
  - Not clear how to do in cell approach.
- Average area clear-cut constraints:
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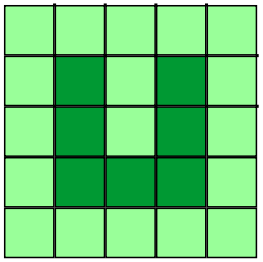
Control Over Clusters Creation Allows to Restrict Clear Cut Shapes

- Easy to forbid inconvenient cluster shapes:
  - U shaped clusters.
  - Long and thin clusters.
  - etc.



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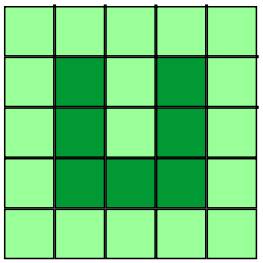

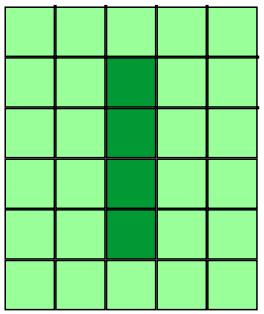

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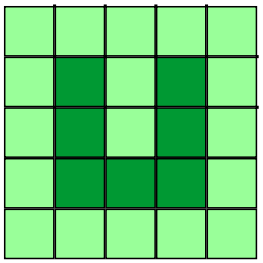

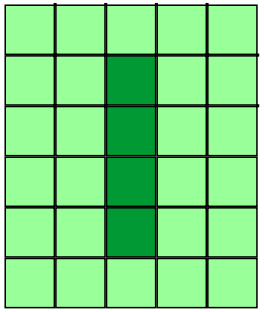

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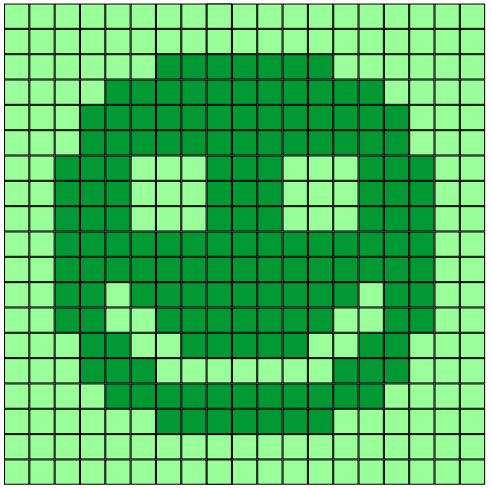

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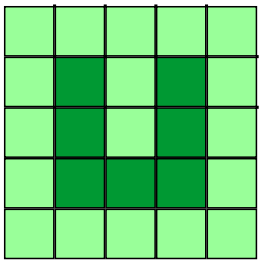

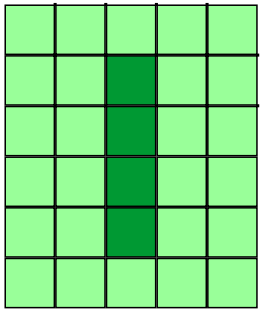

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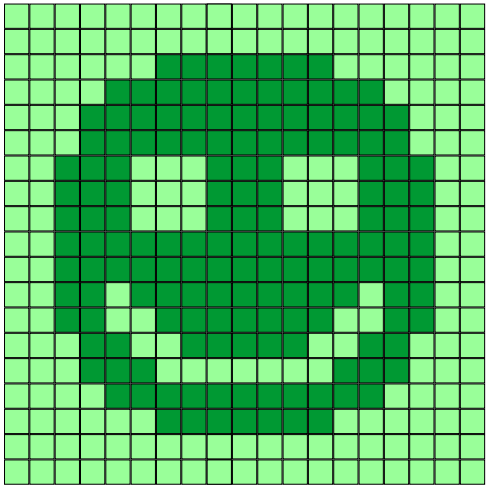
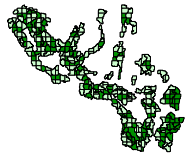


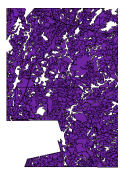
Description of Forest Instances

- Buttercreek
  - 351 nodes and 662 arcs. Max area 120.
  - Feasible clusters ≤ 8 nodes, cliques ≤ 4 nodes.



- El Dorado
  - 1,363 nodes and 3,609 arcs. Max area 120.
  - Feasible clusters ≤ 7 nodes, cliques ≤ 4 nodes.

- Shulkell
  - 1,039 nodes and 2,065 arcs. Max area 40.
  - Feasible clusters ≤ 13 nodes, cliques ≤ 4 nodes.



- Lemon Creek (Partial URM)
  - 6,624 nodes and 18,048 arcs. Max area 40.
  - Feasible clusters ≤ 5 nodes, cliques ≤ 4 nodes.

- 3, 5 and 12 period instances with volume and ending age constraints. Solved with CPLEX 9 for 10,000 seconds. 0.01% GAP considered Optimal

Sizes of Formulations

- Maximum # of cells in a feasible cluster is the key:
  - Can grow if cells become smaller.
  - Can grow if Maximum Area grows.
- For fixed maximum # of cells in a feasible cluster both formulations grow polynomially.
- If maximum # of cells in a feasible cluster is not fixed both formulations can grow exponentially.
- Cell Approach: Size driven by Constraints = Path/Cover.
- Cluster Approach: Size driven by Variables = Feasible Clusters.
- Experiment: Plot Path/Cover and Feasible Clusters v/s Maximum Area.

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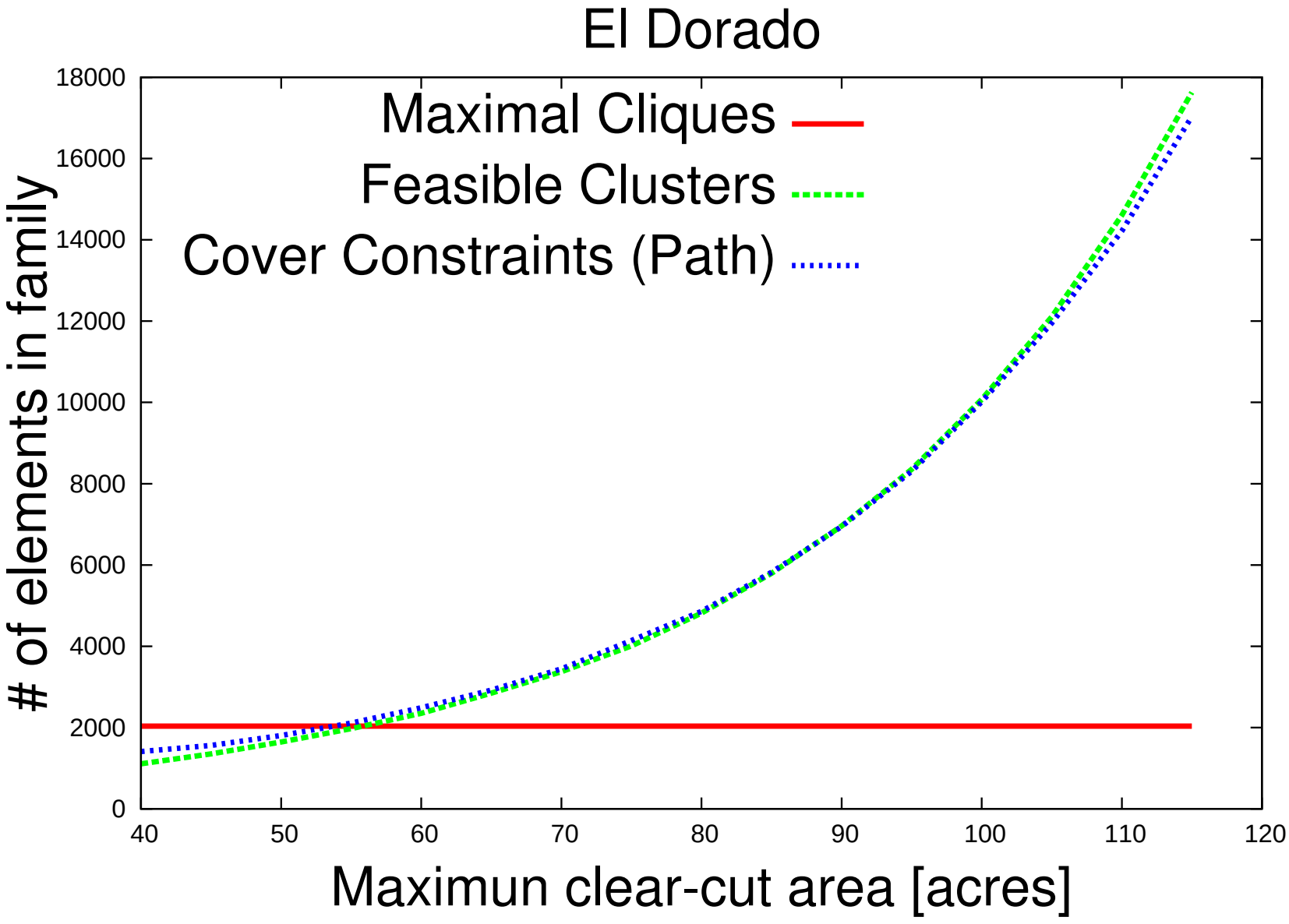

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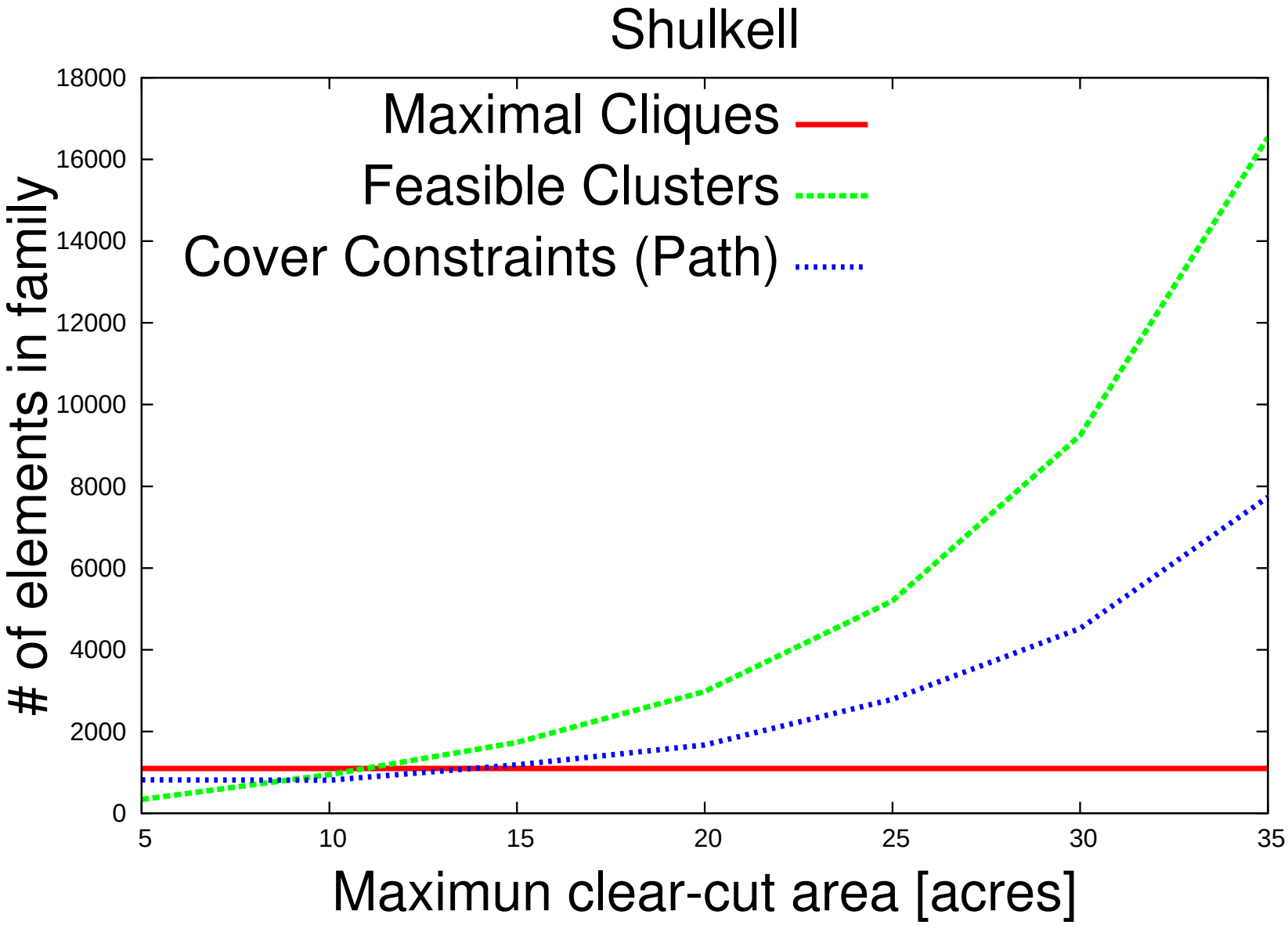

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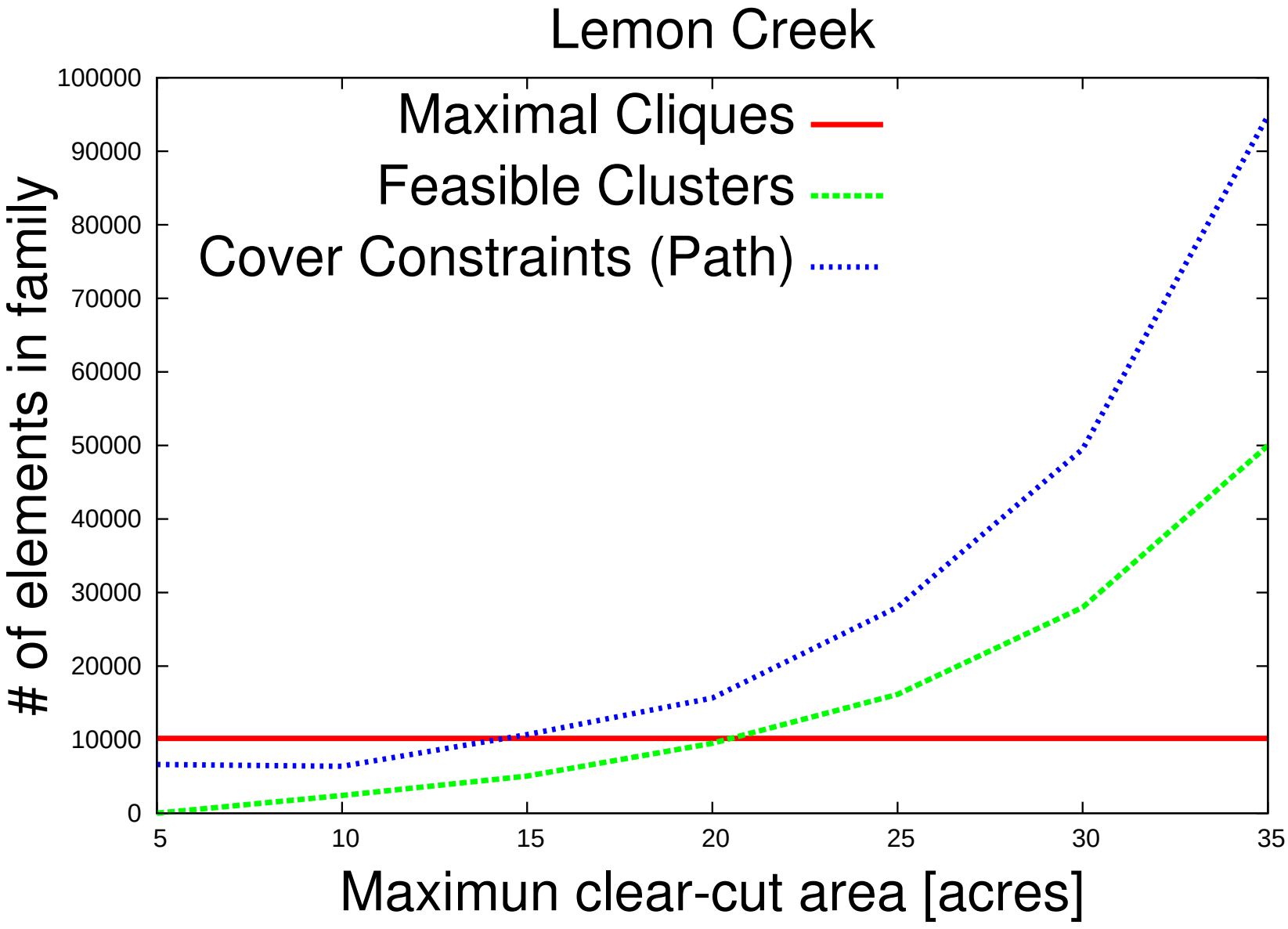


Solving the ARM Model

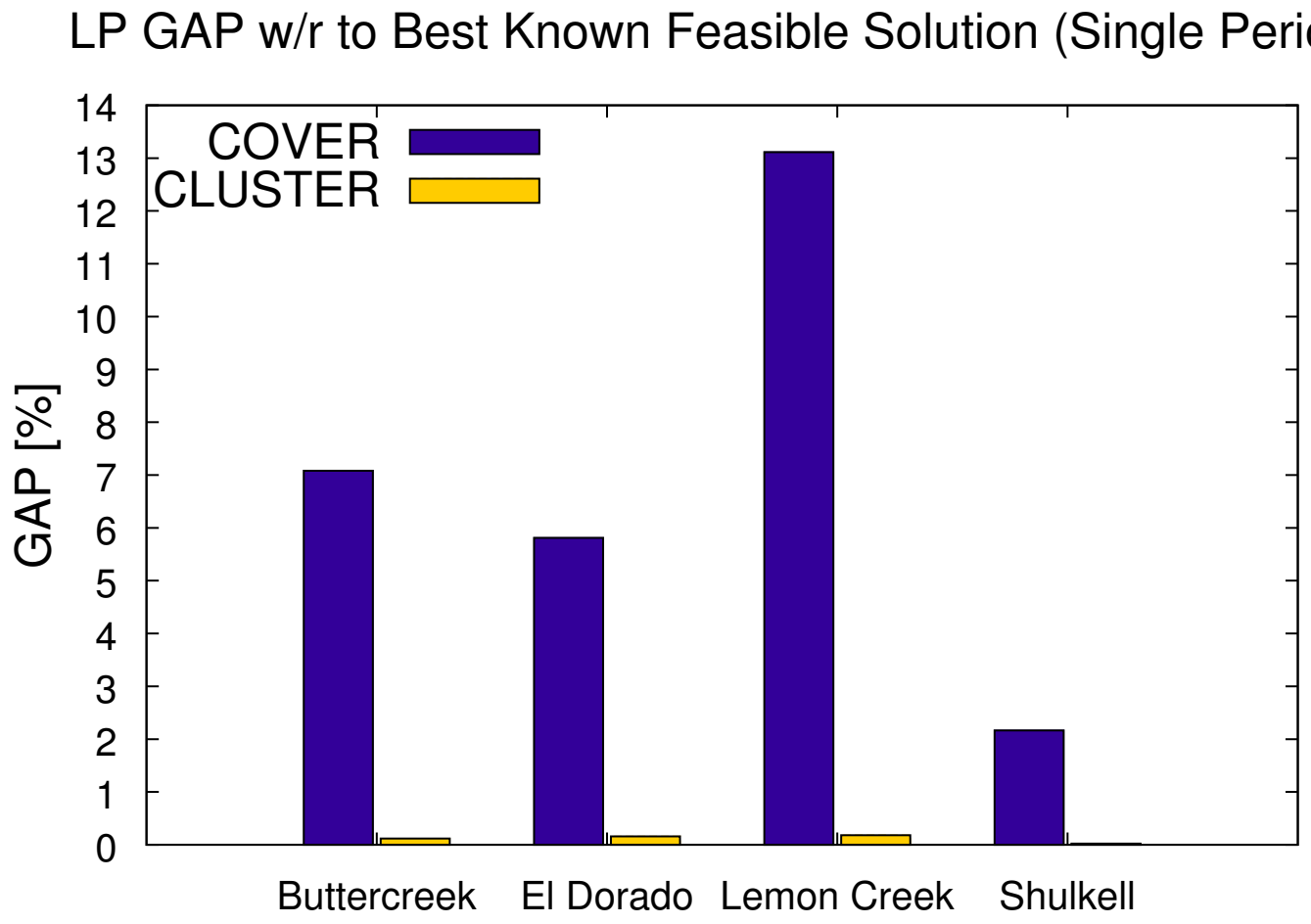
- Feasible solutions are easy to find:
  - CPLEX heuristic usually finds optimum (Some problems with Cluster and Vol. Constraints).
  - Many custom heuristics are available.
- Problem is proving optimality:
  - Tight LP relaxation is very important.

Single and Multi-Period “pure” ARM Model

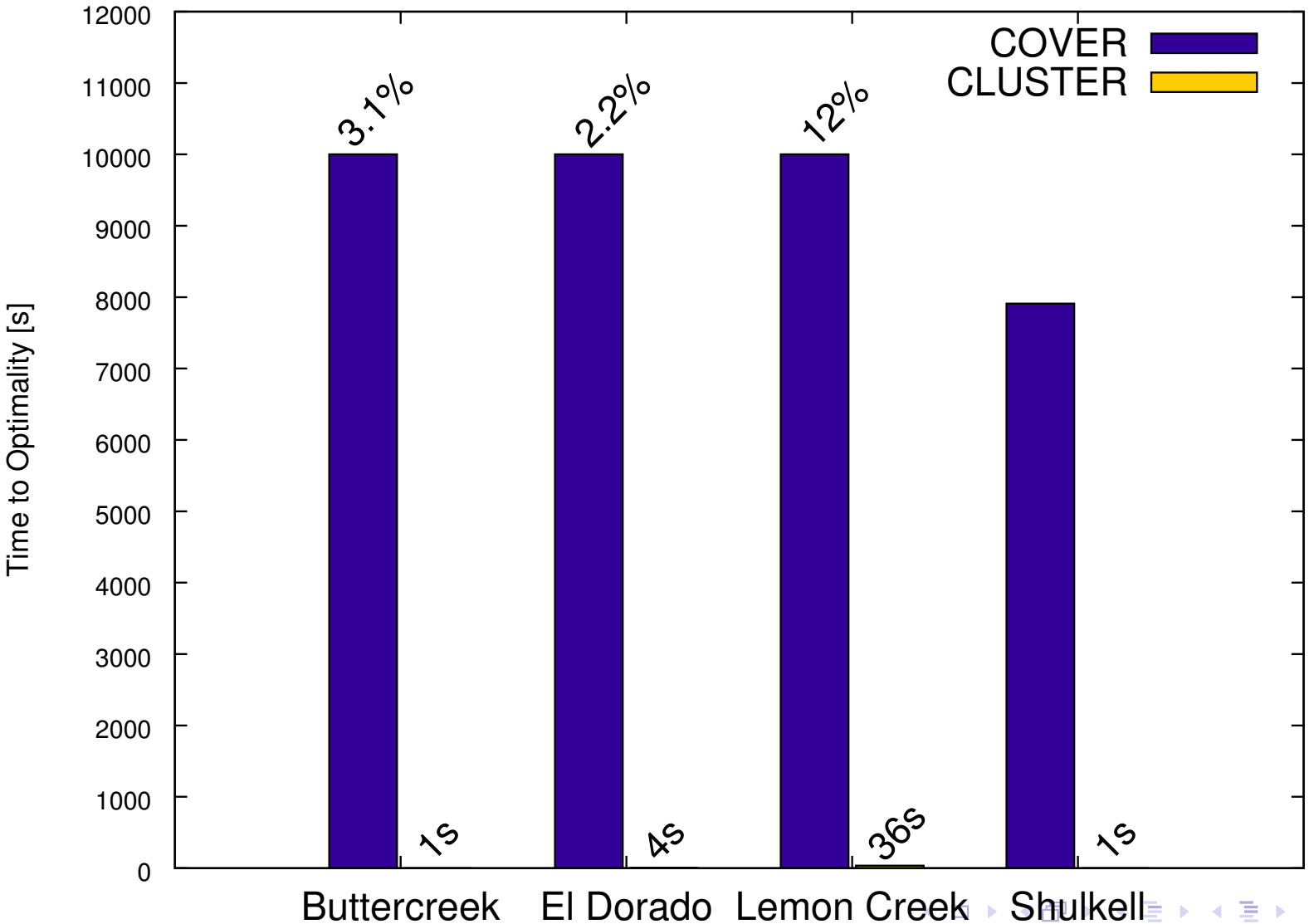
- Problem is pure combinatorial.
- Cluster formulation is far superior:
  - LP relaxation is very tight.
  - Solve times much better that Cell approach.

Cluster Approach LP is Tighter that Cell Approach LP

- Theorem: LP of Cluster Formulation is Stronger than LP of Cell Formulation with Cover Constraints



Tight LP relaxation for Cluster Formulation Translates Into Fast Solve Times

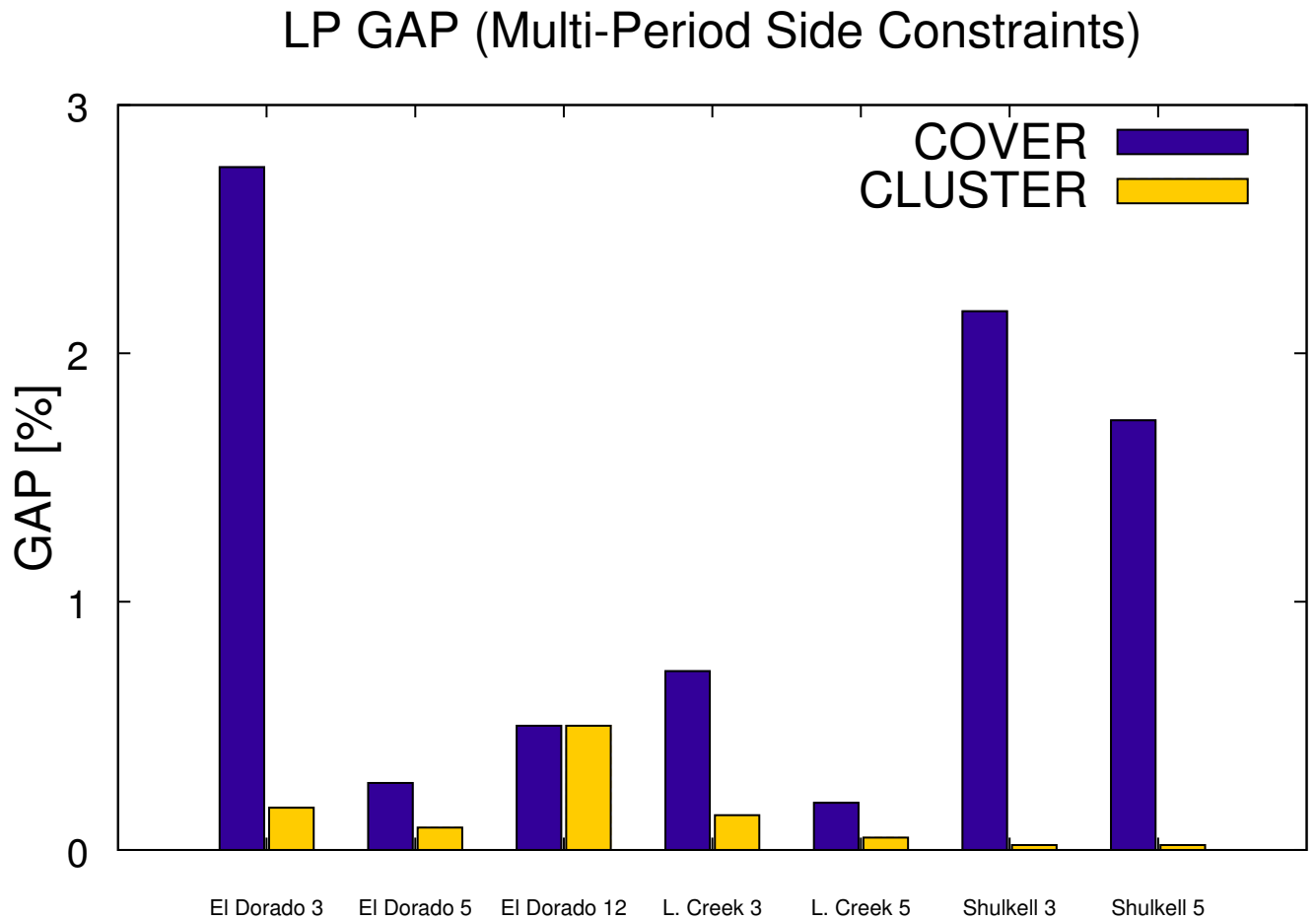


Multi-Period with Side Constraints

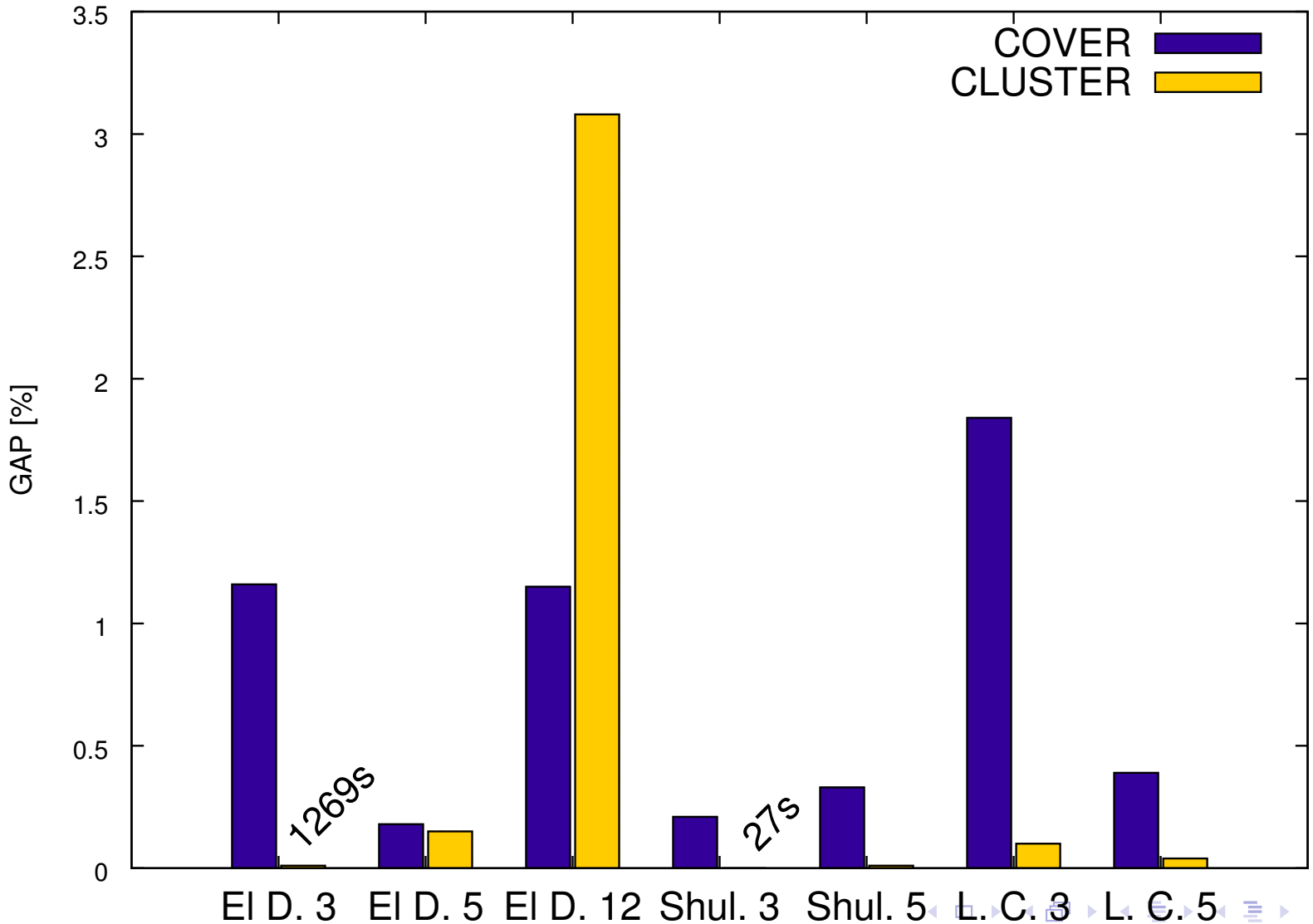
- Side Constraints can be more important that area constraints.
- Both formulations perform simmilarlly:
  - LP relaxations are similar.
  - Solve times are similar.

Multi-Period w. Side Constraints: Cluster LP Relaxation Still Tighter, but Difference is Smaller

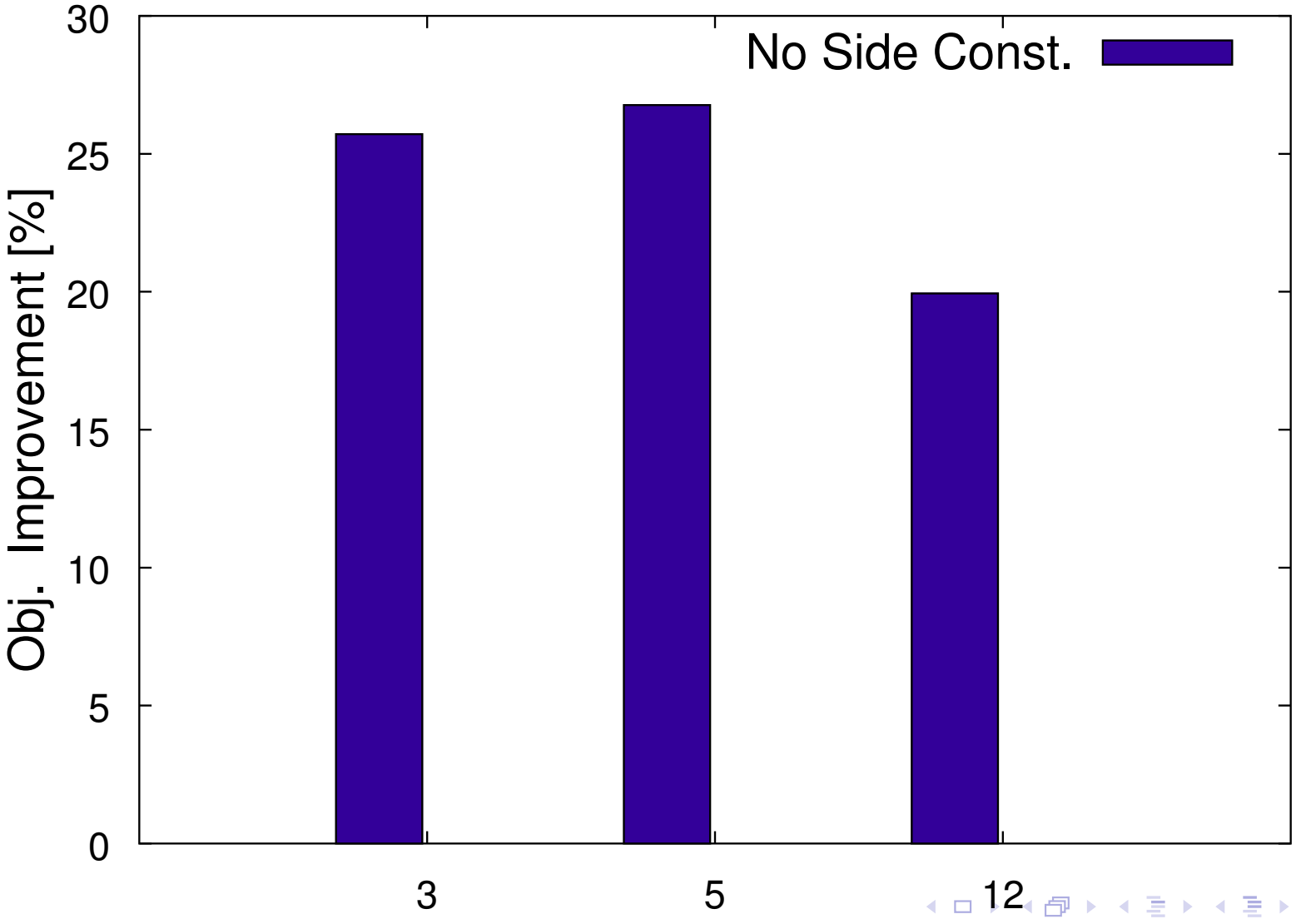
- LP relaxation theorem still holds.



Multi-Period w. Side Constraints: Similar LP Gaps Translates into Similar Solve Times.



Improvement in Objective When Removing Area Constraints (El Dorado)



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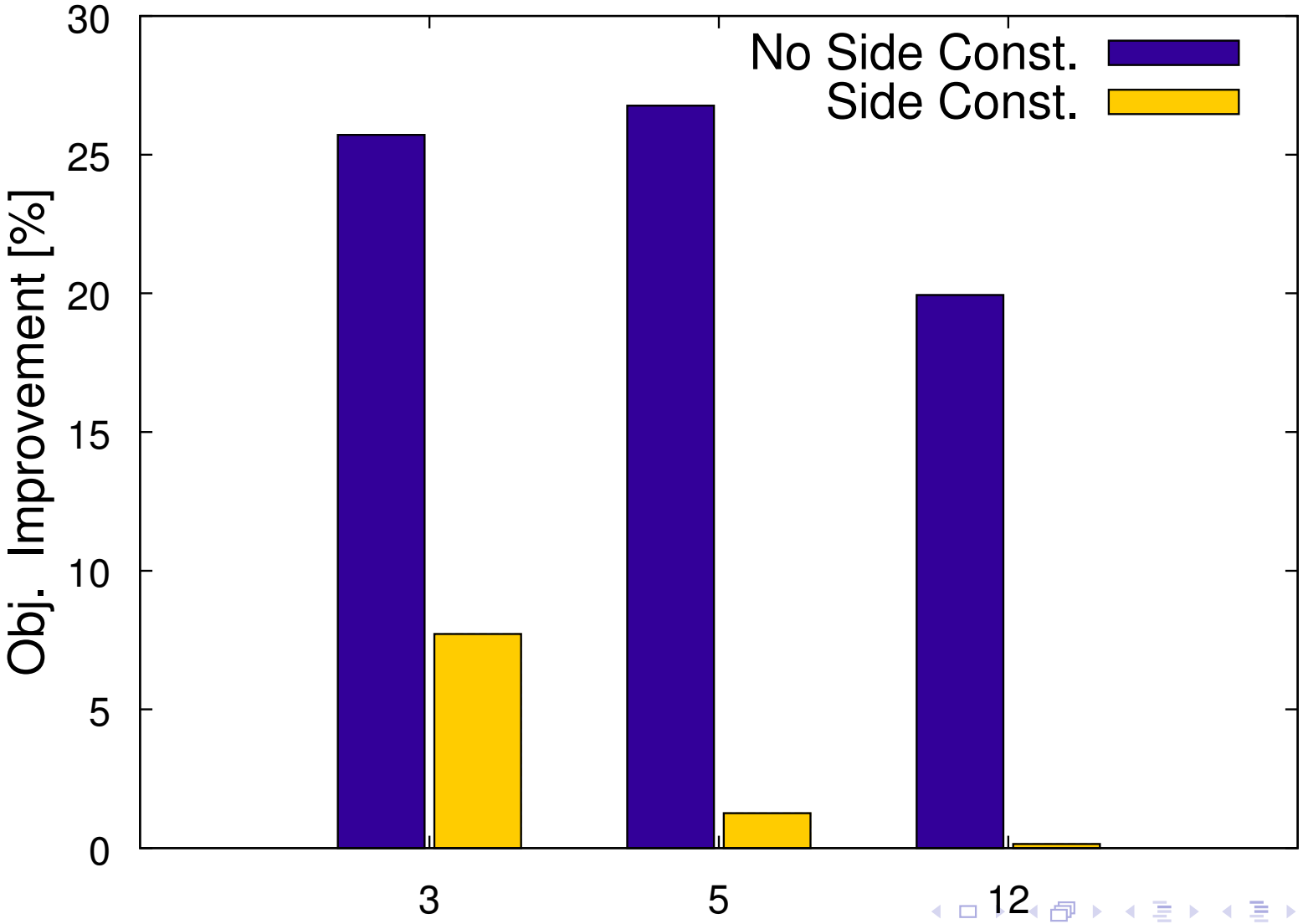

Area Constraints Might not Affect the Objective

- Side constraints can be more important that area constraints:
  - Effect usually stronger for many periods.
  - Area constraints are still needed.
  - Cluster approach particularly sensitive to hard side constraints.
- Green-up>1 can make Area Constraints crucial again.
  - Particular important for many periods.
  - INFORMS 2006.

Conclusions

- Advantages of the Cluster Approach:
  - Models problems which cell approach can not.
  - Better at area constraints aspect of the problem.
- Advantages of the Cell Approach.
  - Much less sensitive to hard side constraints.
- Other aspects of Cell Approach:
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