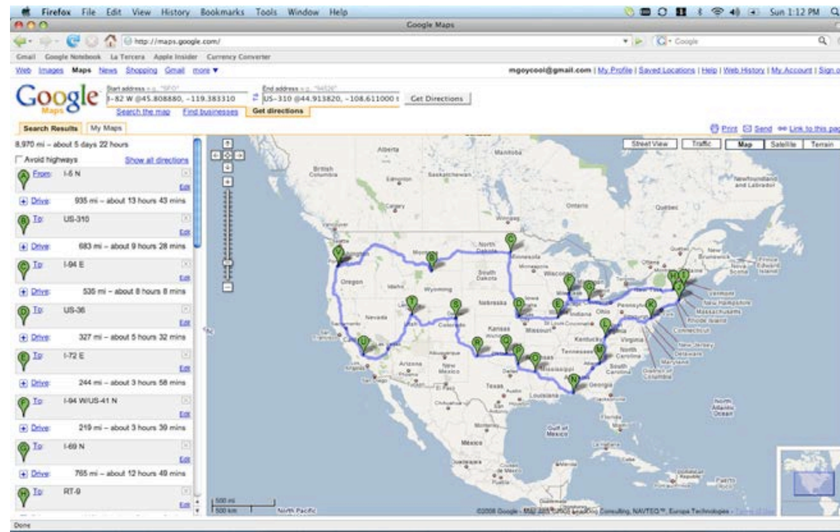
Modelamiento Avanzado con Programación Entera Mixta
Parte 1/3

Juan Pablo Vielma

University of Pittsburgh

Universidad de Antofagasta, 2011 – Antofagasta, Chile

Recorrer Ciudades lo Mas Rápido



Podemos Enumerar las Rutas?

Nro. de Ciudades	Nro. de Soluciones Factibles
10	$10^{5.5}$
100	10^{156}
1,000	$10^{2,565}$
33,810	$10^{138,441}$
85,900	$10^{456,000}$

Edad del universo (en segundos)	10^{18}
Número de átomos en el universo	$< 10^{100}$

Modelo Programación Entera (PE)

$$\min \sum_{e\in E} d_e x_e$$

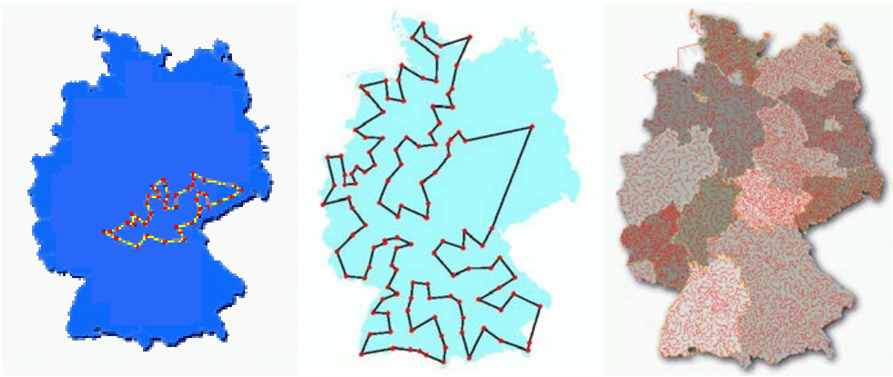
$$st$$

$$\sum_{e\in\delta(i)} x_e = 2 \qquad \forall i \in V$$

$$\sum_{e\in\delta(S)} x_e \geq 2 \qquad \forall S \subsetneq V$$

$$x_e \in \{0,1\} \qquad \forall e \in E$$

Recorrer Alemania con PE



45 ciudades (1832) | 120 ciudades (1977) | 15,112 ciudades (2004)

Problema de Programación Entera

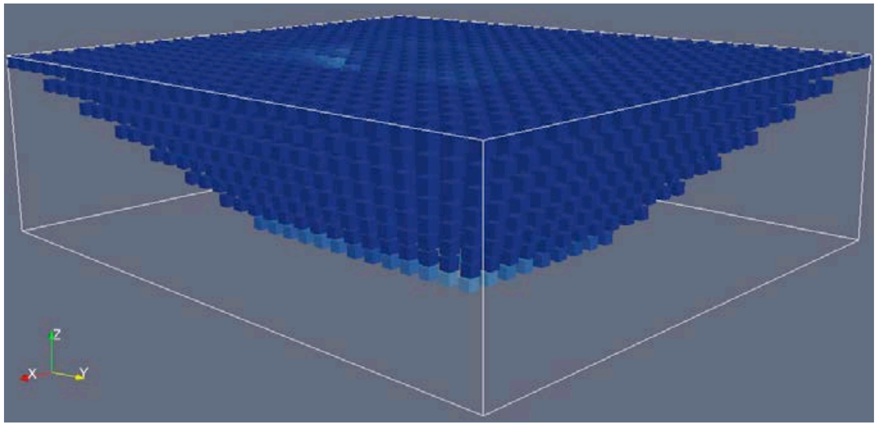
$$\max \sum_{i=1}^{N} a_i x_i$$
$$Ax \leq b$$
$$x_i \in \mathbb{Z} \quad \forall i \in I \subset \{1, \ldots, N\}$$

Ejemplos y Applicaciones

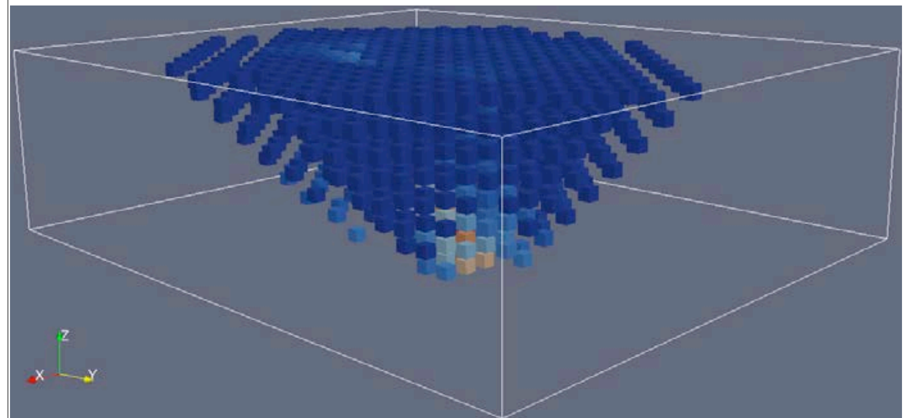
- Combinatorial o 0-1
  - Minería
  - Forestal
  - Computación Biológica
- Entera Mixta
  - Alternativas, Funciones No-lineales, Restricciones Probabilísticas

Mina de Tajo/Rajo Abierto

Modelo de Bloque



Que bloques extraer?



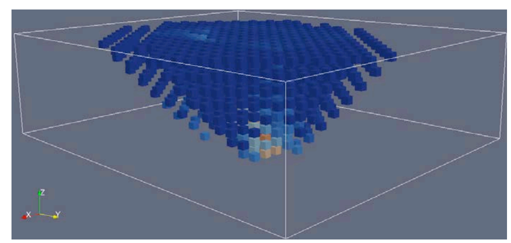
Variables: Extraer o No

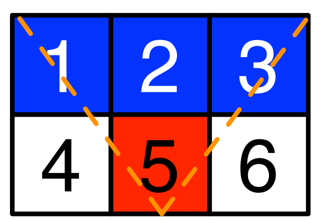
1	2	3
4	5	6

$$x_i = \begin{cases} 1 & \text{si el bloque } i \\ & \text{es extraido} \\ 0 & \text{si no} \end{cases}$$

- Restricciones y Objetivos Lineales:
  - VPN, capacidad de extracción/procesamiento, etc.
- Restricciones Combinatoriales:
  - Reglas de precedencia

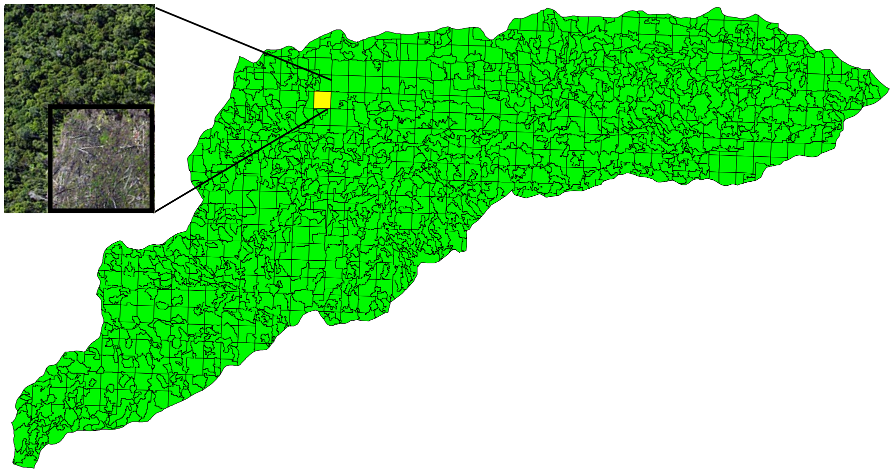
Extraer = Reglas de Precedencia



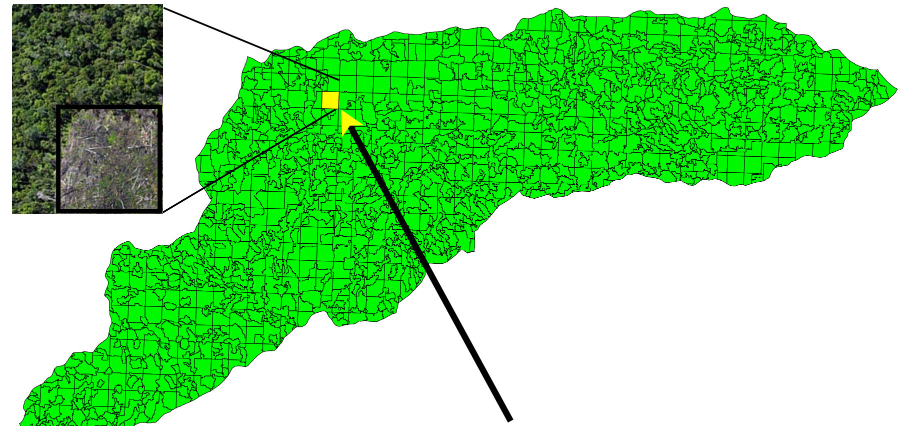


$$x_5 \leq x_1$$
$$x_5 \leq x_2$$
$$x_5 \leq x_3$$

Problema de Planificación Forestal

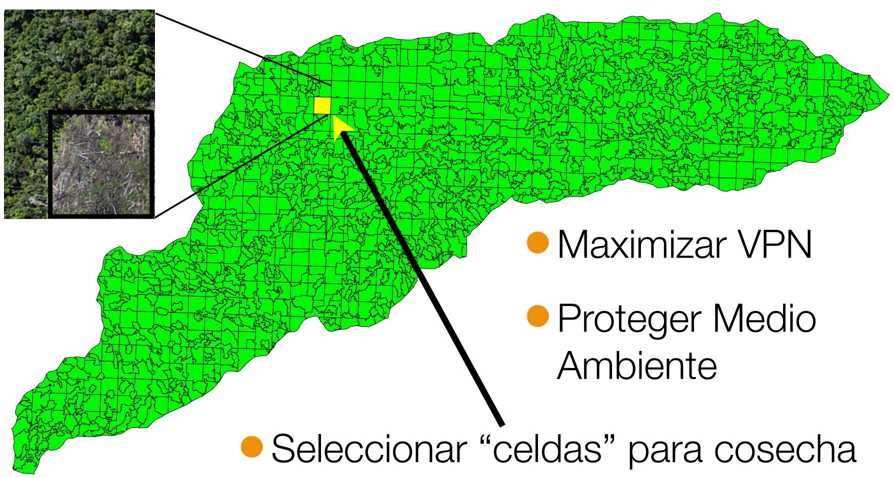


Problema de Planificación Forestal



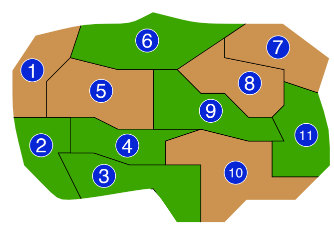
- Seleccionar “celdas” para cosecha

Problema de Planificación Forestal



- Maximizar VPN
- Proteger Medio Ambiente
- Seleccionar “celdas” para cosecha

Variables: Cuando Cosechar



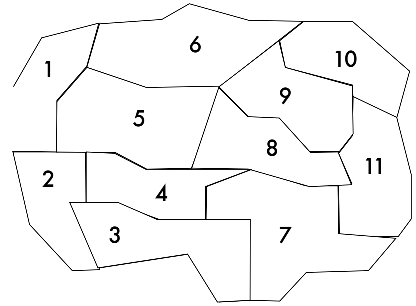
$$x_{v,t} = \begin{cases} 1 & \text{si celda } v \text{ es cosechada} \\ & \text{en el periodo } t. \\ 0 & \text{si no} \end{cases}$$

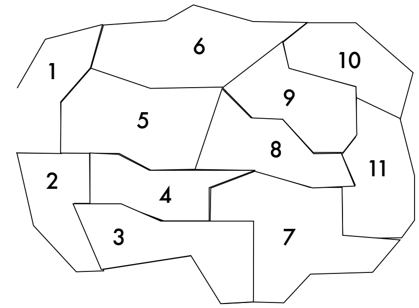
- Restricciones y Objetivos Lineales:
  - VPN, flujo de madera, edad final del bosque, etc.
- Restricciones Combinatoriales:
  - Proteger el Medio Ambiente

Formas de Proteger el Ambiente

- No cosechar grandes áreas contiguas
  - erosión
  - animales no cruzan
  - belleza escénica
- Reservas contiguas

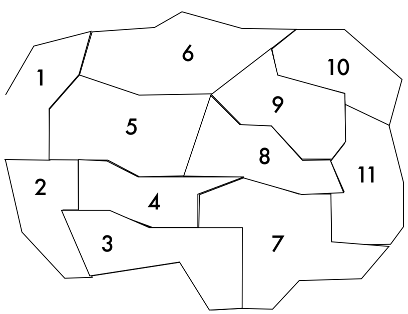
No cosechar grandes áreas contiguas



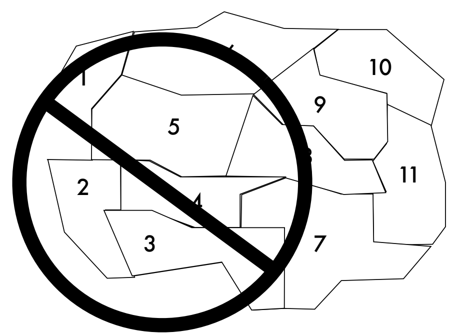


- Aceptable
- No Aceptable

Prohibir Cosechas No Aceptables

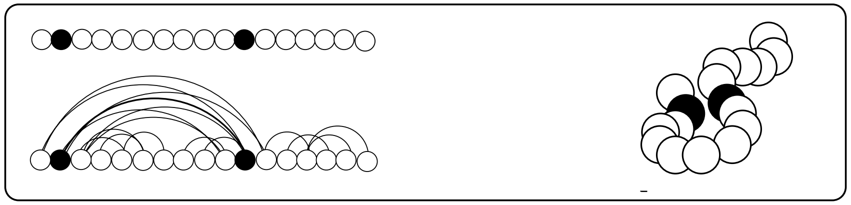


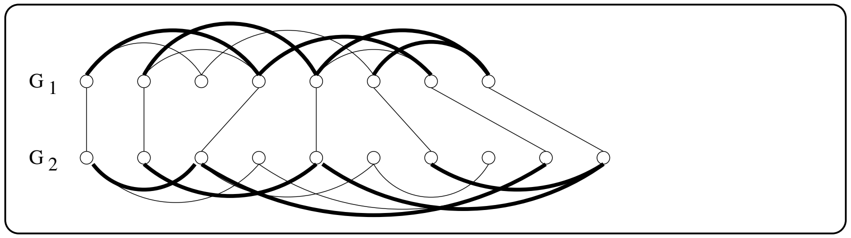
Prohibir Cosechas No Aceptables



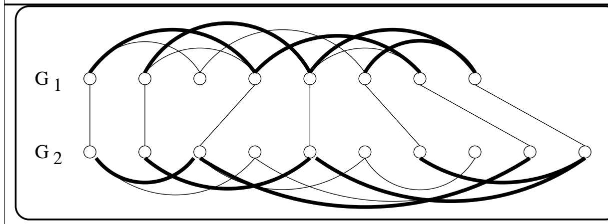
$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 4$$

Alineamiento de Proteinas





- Carr y Lancia 2004



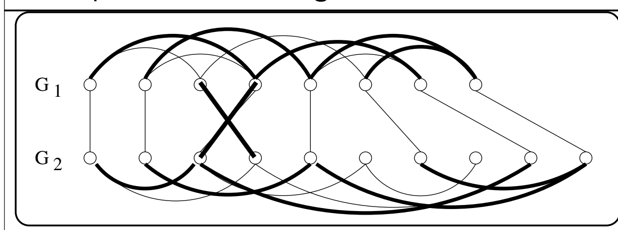
$$\max \sum_{(i,j)\in E_1,\,(u,v)\in E_2} y_{(i,j),(u,v)}$$

$$y_{(i,j),(u,v)} \leq x_{i,u}$$

$$y_{(i,j),(u,v)} \leq x_{j,v}$$

$$\sum_{(u,v)\in F} x_{u,v} \leq 1 \qquad \forall F \in \mathcal{C}$$

$$x_{u,v},\; y_{(i,j),(u,v)} \in \{0,1\} \qquad \forall u,v,i,j$$



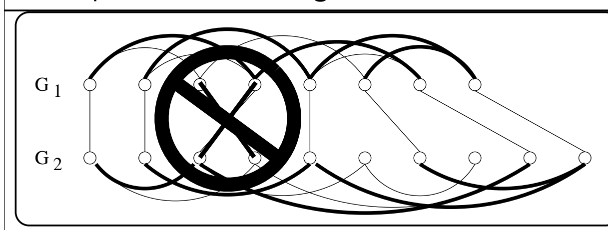
$$\max \sum_{(i,j)\in E_1,\,(u,v)\in E_2} y_{(i,j),(u,v)}$$

$$y_{(i,j),(u,v)} \leq x_{i,u}$$

$$y_{(i,j),(u,v)} \leq x_{j,v}$$

$$\sum_{(u,v)\in F} x_{u,v} \leq 1 \qquad \forall F \in \mathcal{C}$$

$$x_{u,v},\; y_{(i,j),(u,v)} \in \{0,1\} \qquad \forall u,v,i,j$$



$$\max \sum_{(i,j)\in E_1,\,(u,v)\in E_2} y_{(i,j),(u,v)}$$

$$y_{(i,j),(u,v)} \leq x_{i,u}$$

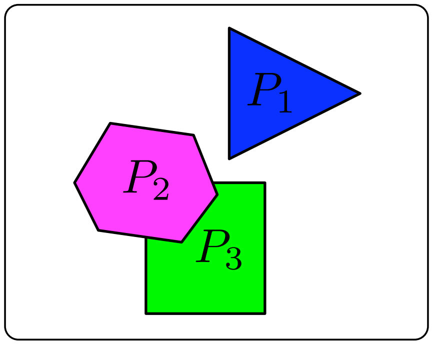
$$y_{(i,j),(u,v)} \leq x_{j,v}$$

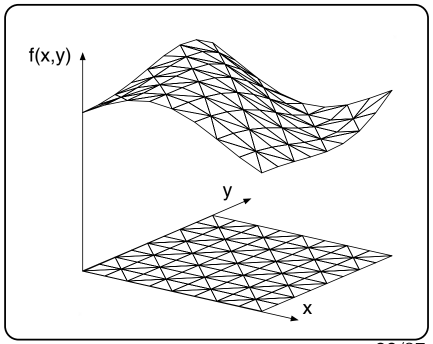
$$\sum_{(u,v)\in F} x_{u,v} \leq 1 \qquad \forall F \in \mathcal{C}$$

$$x_{u,v},\; y_{(i,j),(u,v)} \in \{0,1\} \qquad \forall u,v,i,j$$

Disjunciones o Alternativas

$$x = 0 \quad \vee \quad 1 \leq x \leq 2$$





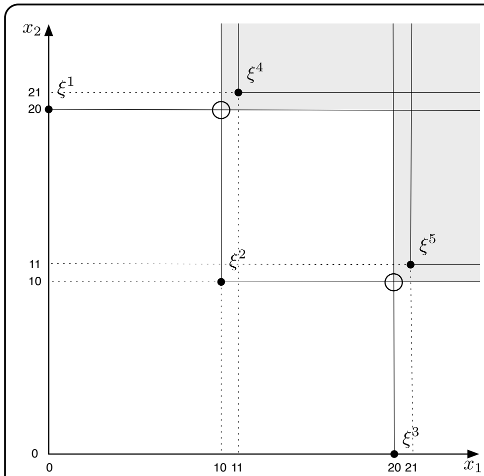
Restricciones Probabilísticas

$$Q := \{x \in \mathbb{R}^d \,:\, \mathbb{P}(x \geq \xi) \geq 1 - \delta\} \qquad \xi \sim U\left(\{\xi^s\}_{s=1}^S\right)$$

$$Q = \{x \in \mathbb{R}^d \,:\, |v(x)| \leq \lfloor \delta D \rfloor =: k\}$$

$$v(x) := \{s \in \{1, \ldots, S\} \,:\, x \not\geq \xi^s\}$$

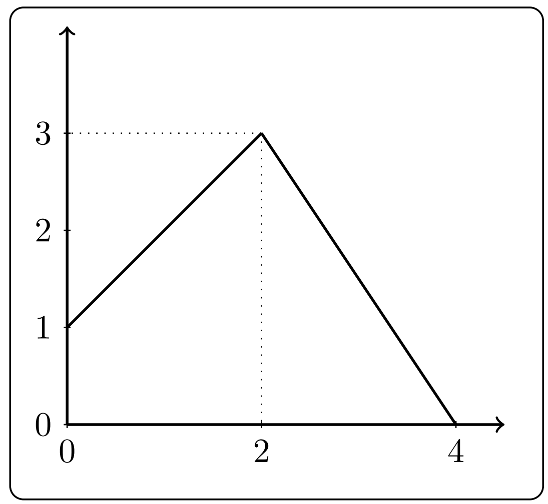
Ejemplo: Violar 3 Restricciones



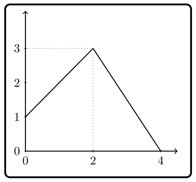
$$k = 3$$

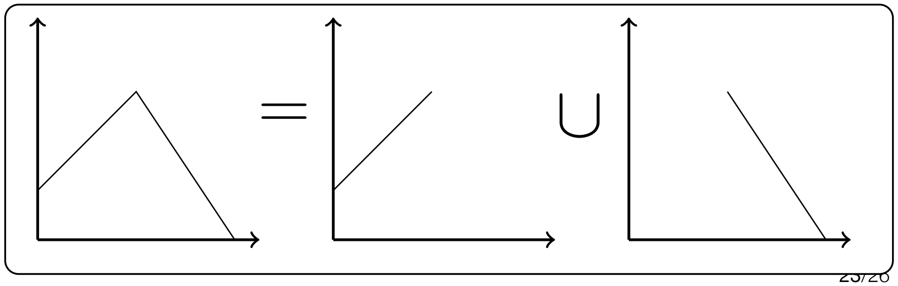
$$Q = \left\{x \in \mathbb{R}^2 \,:\, x \geq \begin{pmatrix} 10 \\ 20 \end{pmatrix}\right\} \cup \left\{x \in \mathbb{R}^2 \,:\, x \geq \begin{pmatrix} 20 \\ 10 \end{pmatrix}\right\}$$

Lineales por trazo = Union de lineas

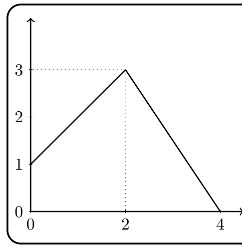


Lineales por trazo = Union de lineas



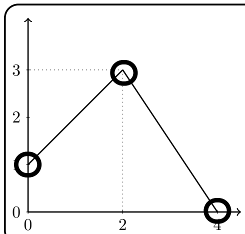


Modelo Combinacion Convexa



$$f(x) := \begin{cases} x+1 & x \in [0,2] \\ 6 - 3/2x & x \in [2,4] \end{cases}$$

Modelo Combinacion Convexa

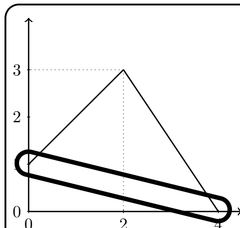


$$f(x) := \begin{cases} x+1 & x \in [0,2] \\ 6 - 3/2x & x \in [2,4] \end{cases}$$

$$\begin{aligned} x &= 0\lambda_1 + 2\lambda_2 + 4\lambda_3 \\ f(x) &= 1\lambda_1 + 3\lambda_2 + 0\lambda_3 \\ 1 &= \lambda_1 + \lambda_2 + \lambda_3, \quad \lambda_1, \lambda_2\lambda_3 \geq 0 \end{aligned}$$

idea: escribir $(x, f(x))$ como combinacion covexa $(0, f(0))$, $(2, f(2))$, $(4, f(4))$

Modelo Combinacion Convexa

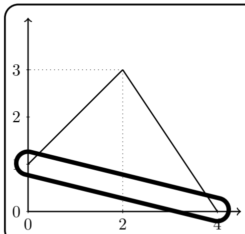


$$f(x) := \begin{cases} x+1 & x \in [0,2] \\ 6 - 3/2x & x \in [2,4] \end{cases}$$

λ_1 y λ_3 no pueden ser > 0 al mismo tiempo.

$$\begin{aligned} x &= 0\lambda_1 + 2\lambda_2 + 4\lambda_3 \\ f(x) &= 1\lambda_1 + 3\lambda_2 + 0\lambda_3 \\ 1 &= \lambda_1 + \lambda_2 + \lambda_3, \quad \lambda_1, \lambda_2\lambda_3 \geq 0 \end{aligned}$$

Modelo Combinacion Convexa

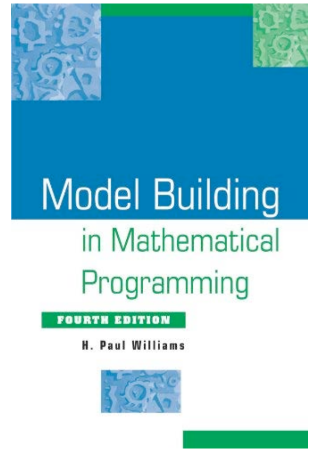


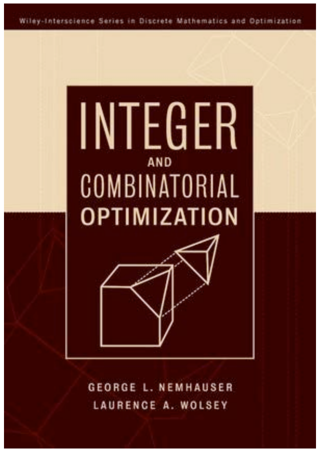
$$f(x) := \begin{cases} x+1 & x \in [0,2] \\ 6 - 3/2x & x \in [2,4] \end{cases}$$

λ_1 y λ_3 no pueden ser > 0 al mismo tiempo.

$$x = \boxed{0}\lambda_1 + 2\lambda_2 + \boxed{4}\lambda_3$$
$$f(x) = \boxed{1}\lambda_1 + 3\lambda_2 + \boxed{0}\lambda_3$$
$$1 = \lambda_1 + \lambda_2 + \lambda_3, \quad \lambda_1, \lambda_2\lambda_3 \geq 0$$
$$\lambda_1 \leq y_1, \quad \lambda_2 \leq y_1 + y_2, \quad \lambda_3 \leq y_2$$
$$1 = y_1 + y_2, \quad y_1,\, y_2 \in \{0,1\}$$

Libros con Modelamiento Básico





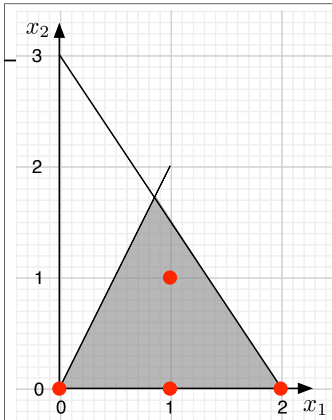
Que significa modelar con PE?

$$x \in S \subset \mathbb{R}^n$$

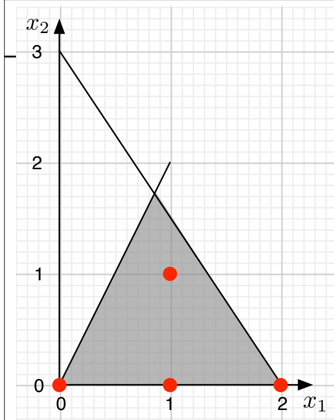
$$\Updownarrow$$

$\exists y \in \mathbb{R}^m$ t.q.

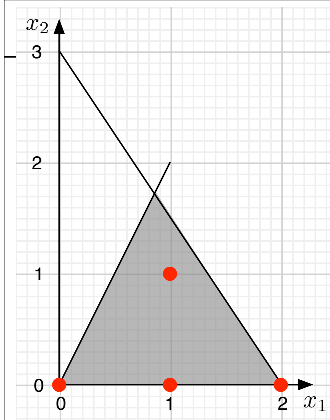
$$Ax + Dy \leq b,$$
$$x_i \in \mathbb{Z} \quad \forall i \in I \subset \{1, \ldots, n\}$$
$$y_j \in \mathbb{Z} \quad \forall j \in J \subset \{1, \ldots, m\}$$

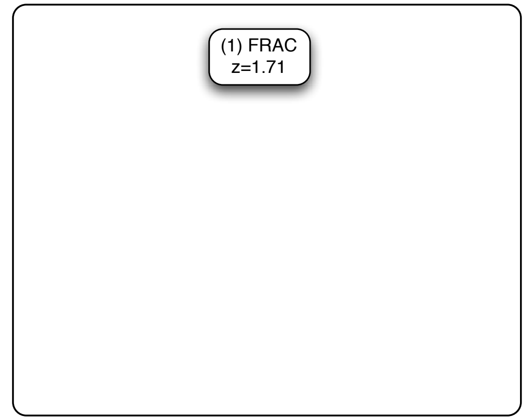


$$\min z := x_2$$
$$3x_1 + 2x_2 \leq 6$$
$$-2x_1 + x_2 \leq 0$$
$$x_1,\, x_2 \geq 0$$
$$x_1,\, x_2 \in \mathbb{Z}$$

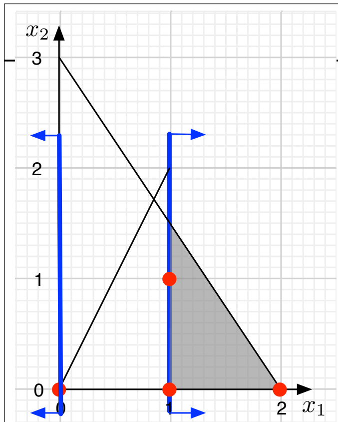


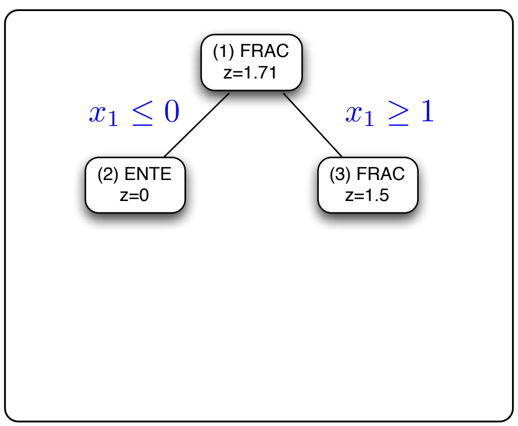
$$\min z := x_2$$
$$3x_1 + 2x_2 \le 6$$
$$-2x_1 + x_2 \le 0$$
$$x_1,\ x_2 \ge 0$$
$$x_1,\ x_2 \in \mathbb{Z}$$



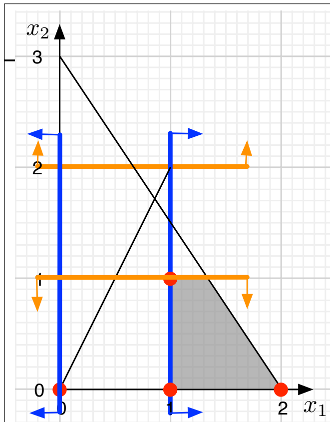


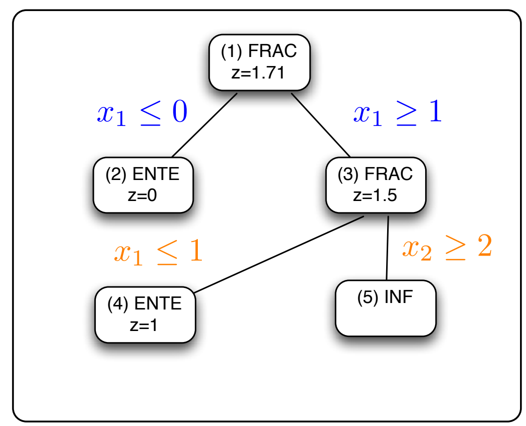
$$\min z := x_2$$
$$3x_1 + 2x_2 \le 6$$
$$-2x_1 + x_2 \le 0$$
$$x_1,\ x_2 \ge 0$$
$$x_1,\ x_2 \in \mathbb{Z}$$





$$\min z := x_2$$
$$3x_1 + 2x_2 \le 6$$
$$-2x_1 + x_2 \le 0$$
$$x_1,\ x_2 \ge 0$$
$$x_1,\ x_2 \in \mathbb{Z}$$





$$\min z := x_2$$
$$3x_1 + 2x_2 \le 6$$
$$-2x_1 + x_2 \le 0$$
$$x_1,\ x_2 \ge 0$$
$$x_1,\ x_2 \in \mathbb{Z}$$