Winning at Daily Fantasy Hockey Using Analytics

David Hunter,

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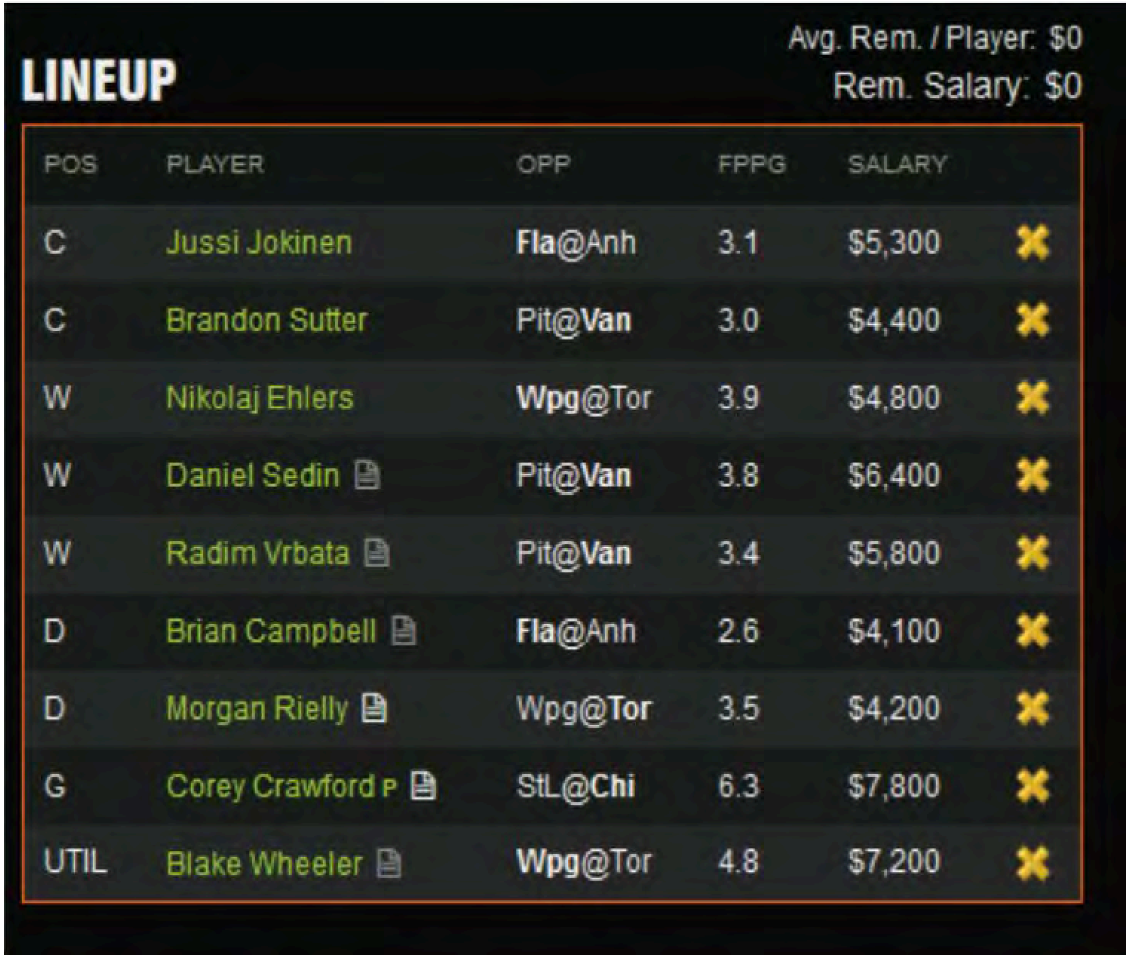
Tauhid Zaman (@zlisto)

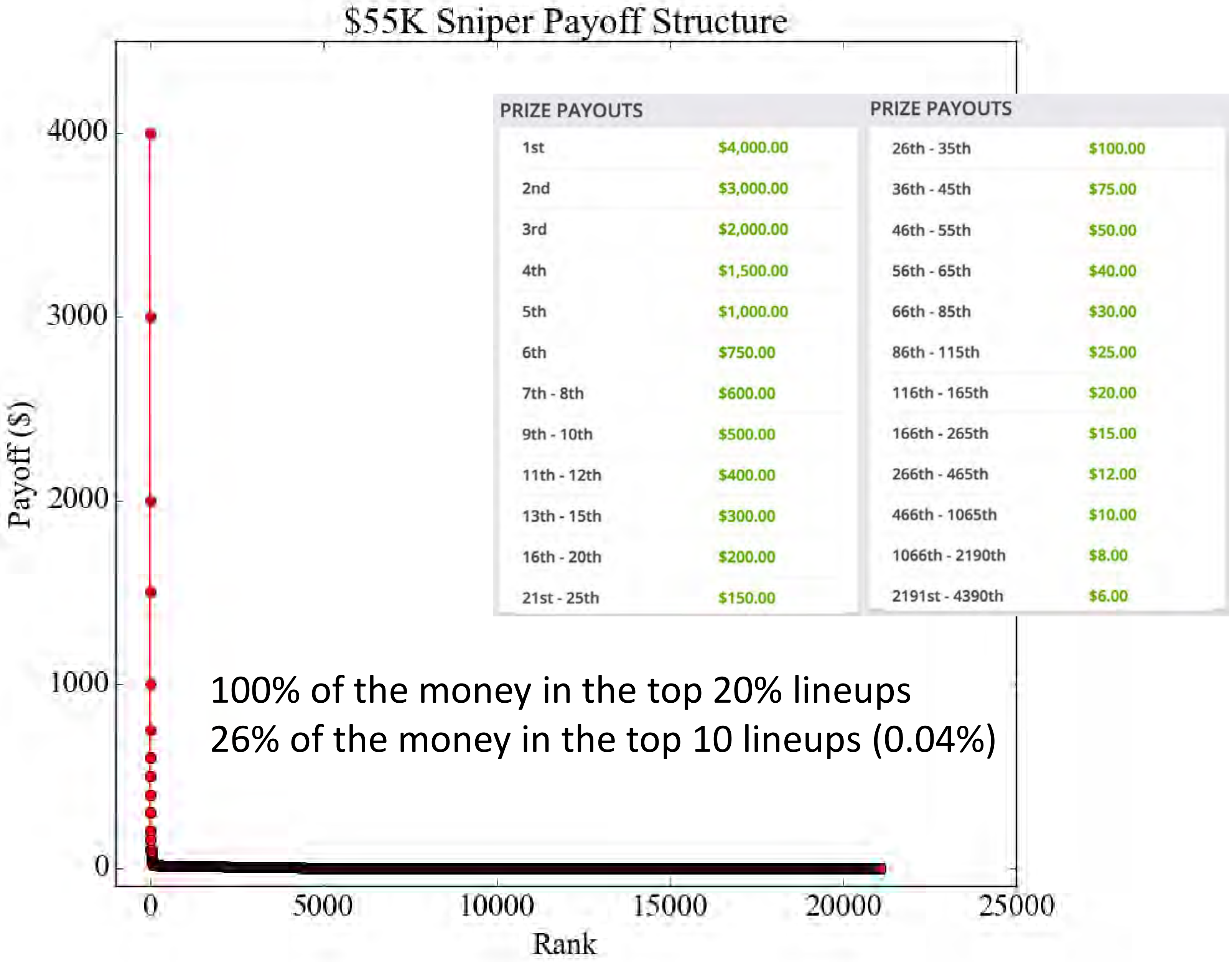
Example Entry



LINEUP

Avg. Rem. / Player: $0
Rem. Salary: $0

POS	PLAYER	OPP	FPPG	SALARY
C	Jussi Jokinen	Fla@Anh	3.1	$5,300
C	Brandon Sutter	Pit@Van	3.0	$4,400
W	Nikolaj Ehlers	Wpg@Tor	3.9	$4,800
W	Daniel Sedin	Pit@Van	3.8	$6,400
W	Radim Vrbata	Pit@Van	3.4	$5,800
D	Brian Campbell	Fla@Anh	2.6	$4,100
D	Morgan Rielly	Wpg@Tor	3.5	$4,200
G	Corey Crawford P	StL@Chi	6.3	$7,800
UTIL	Blake Wheeler	Wpg@Tor	4.8	$7,200

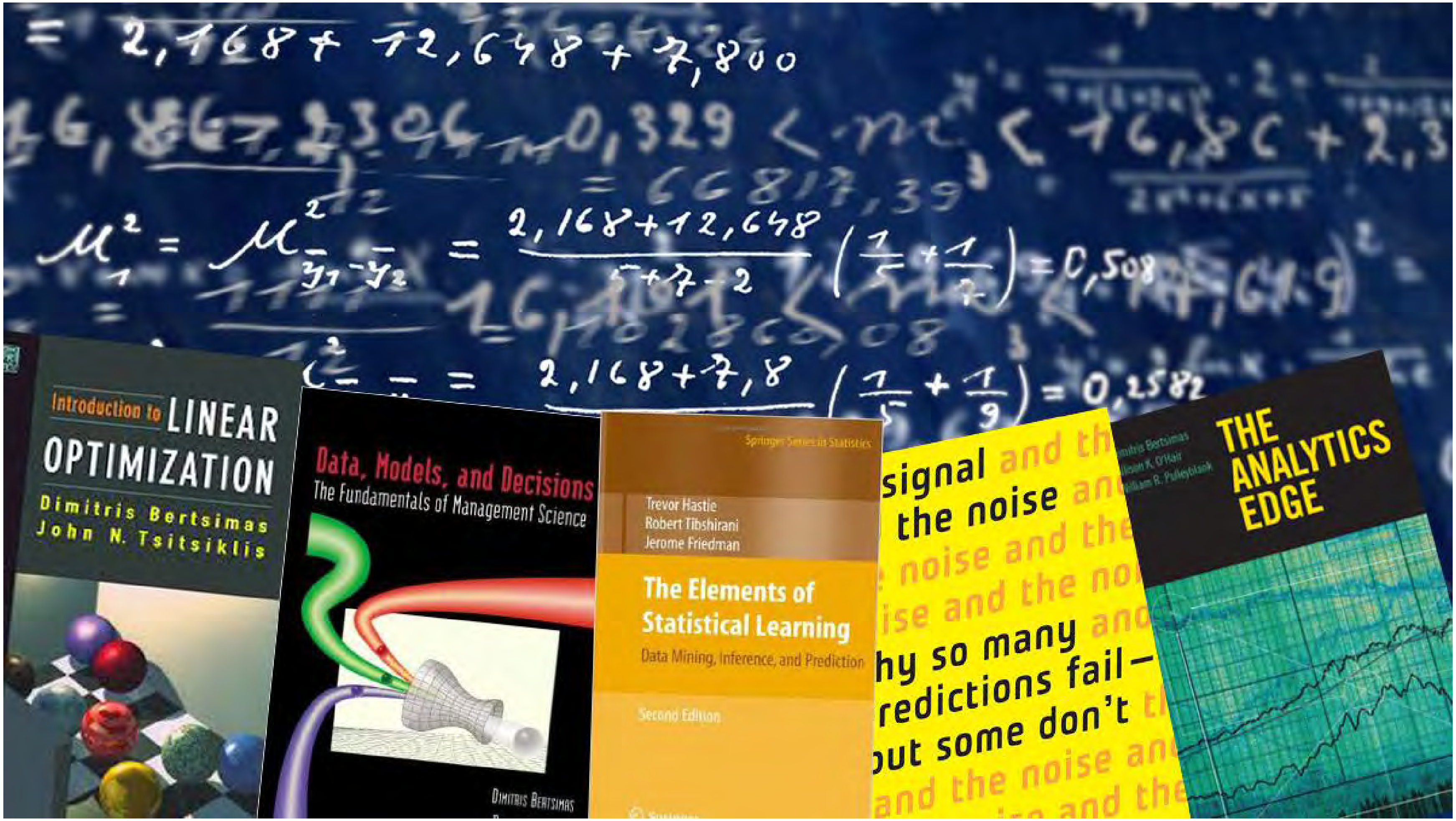


PRIZE PAYOUTS	
1st	$4,000.00
2nd	$3,000.00
3rd	$2,000.00
4th	$1,500.00
5th	$1,000.00
6th	$750.00
7th - 8th	$600.00
9th - 10th	$500.00
11th - 12th	$400.00
13th - 15th	$300.00
16th - 20th	$200.00
21st - 25th	$150.00
26th - 35th	$100.00
36th - 45th	$75.00
46th - 55th	$50.00
56th - 65th	$40.00
66th - 85th	$30.00
86th - 115th	$25.00
116th - 165th	$20.00
166th - 265th	$15.00
266th - 465th	$12.00
466th - 1065th	$10.00
1066th - 2190th	$8.00
2191st - 4390th	$6.00

100% of the money in the top 20% lineups

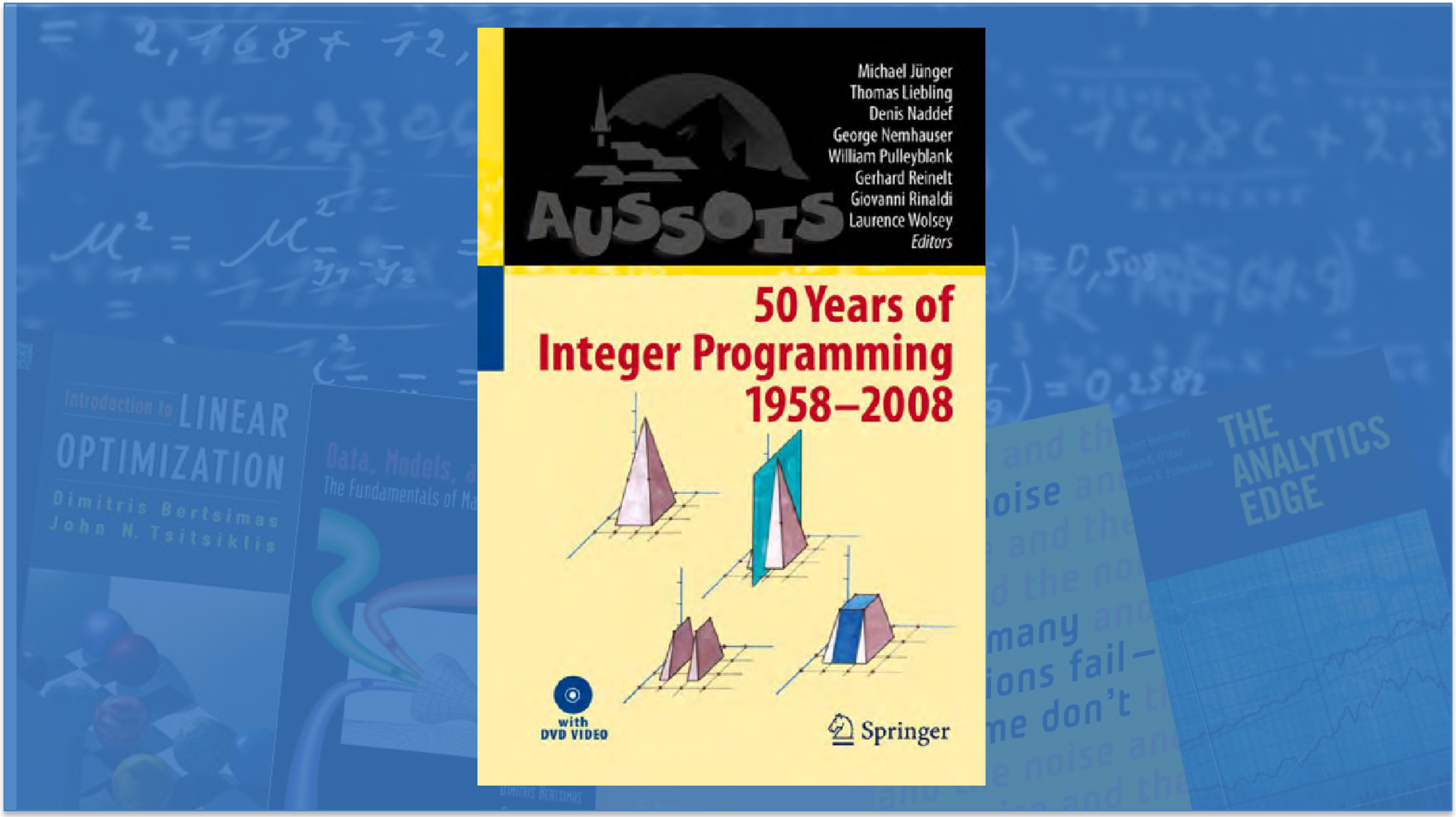
26% of the money in the top 10 lineups (0.04%)

Previous Knowledge: Analytics


Introduction to LINEAR OPTIMIZATION
Dimitris Bertsimas
John N. Tsitsiklis
Data, Models, and Decisions
The Fundamentals of Management Science
Dimitris Bertsimas
Springer Series in Statistics
Trevor Hastie
Robert Tibshirani
Jerome Friedman
The Elements of Statistical Learning
Data Mining, Inference, and Prediction
Second Edition
signal and the noise
why so many predictions fail—but some don't
Dimitris Bertsimas
Allison K. O'Hair
William R. Pulleyblank
THE ANALYTICS EDGE

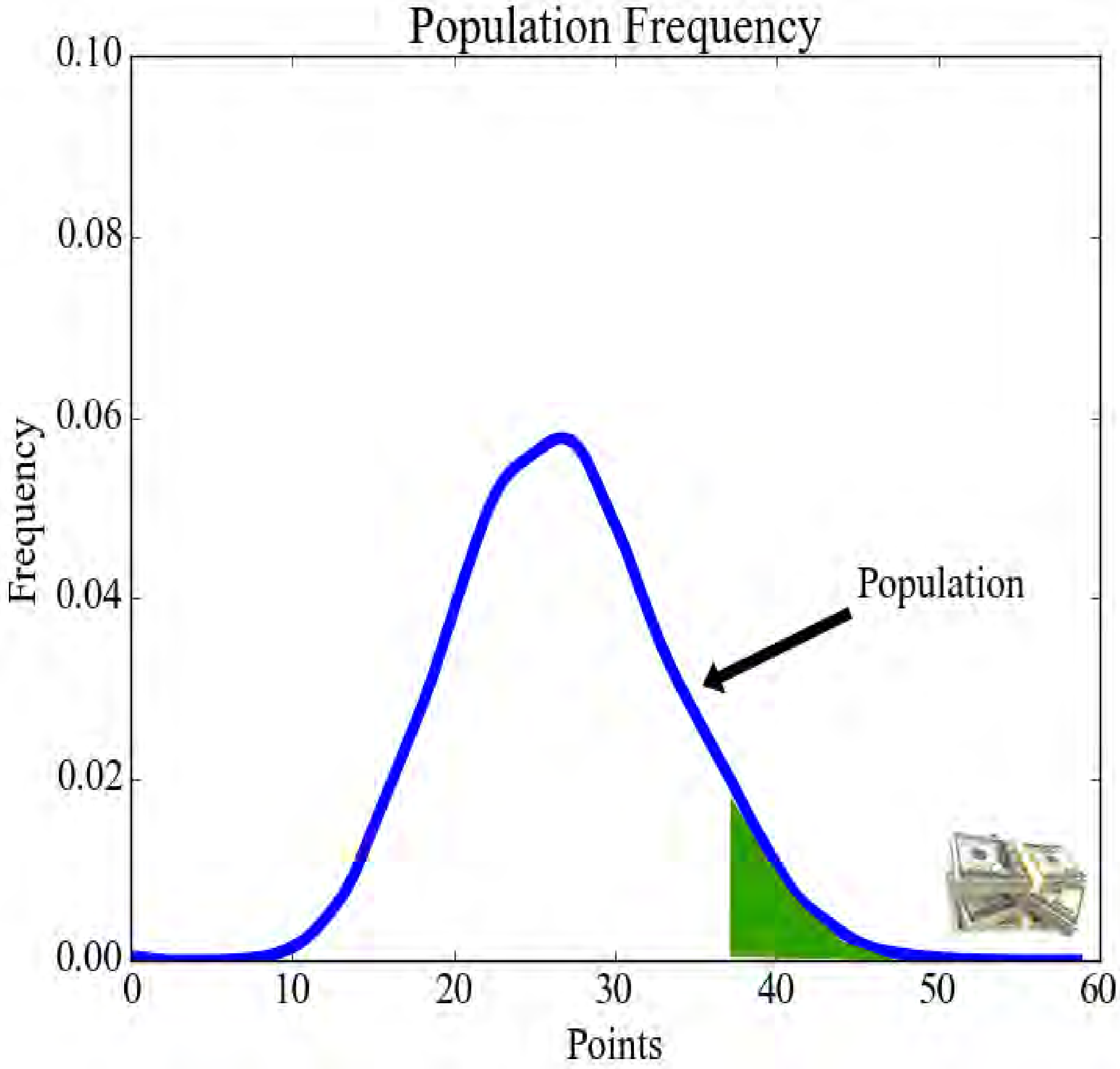
Previous Knowledge: Analytics

#KnightsOfMIP

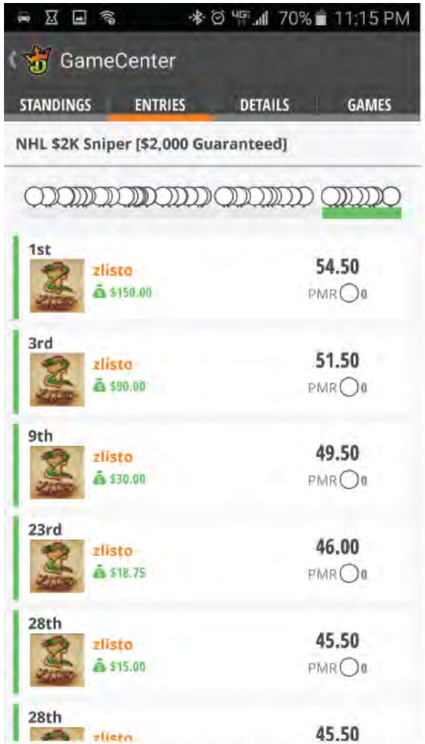


Building a Lineup

Using this knowledge…



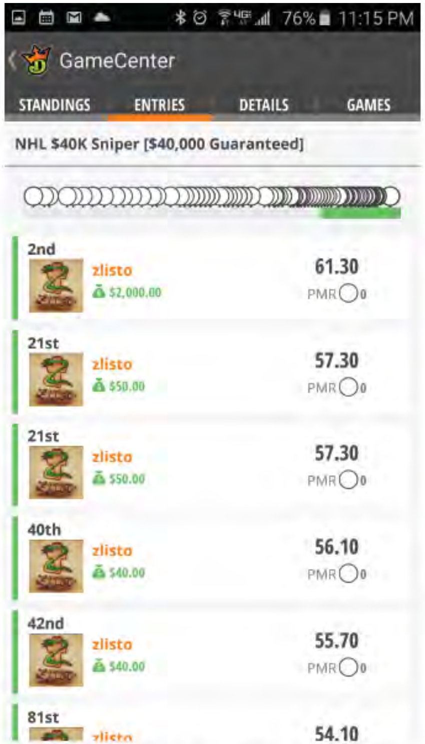
Were we able to do it?



NHL $2K Sniper [$2,000 Guaranteed]

1st	zlisto	$150.00	54.50	PMR 0
3rd	zlisto	$90.00	51.50	PMR 0
9th	zlisto	$30.00	49.50	PMR 0
23rd	zlisto	$18.75	46.00	PMR 0
28th	zlisto	$15.00	45.50	PMR 0
28th	zlisto		45.50	

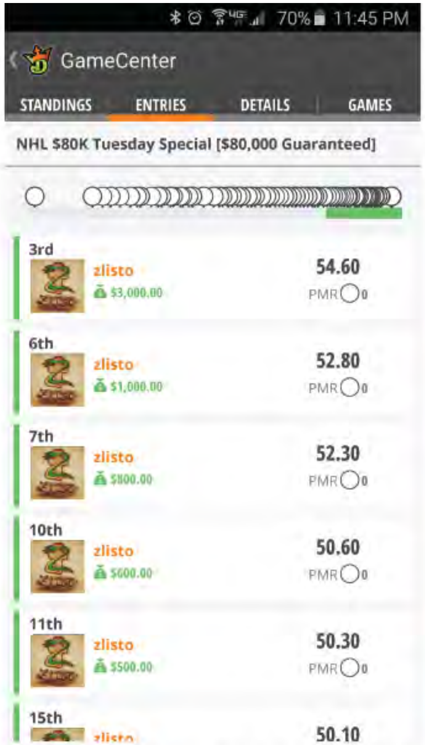
November 15, 2015



NHL $40K Sniper [$40,000 Guaranteed]

2nd	zlisto	$2,000.00	61.30	PMR 0
21st	zlisto	$50.00	57.30	PMR 0
21st	zlisto	$50.00	57.30	PMR 0
40th	zlisto	$40.00	56.10	PMR 0
42nd	zlisto	$40.00	55.70	PMR 0
81st	zlisto		54.10	

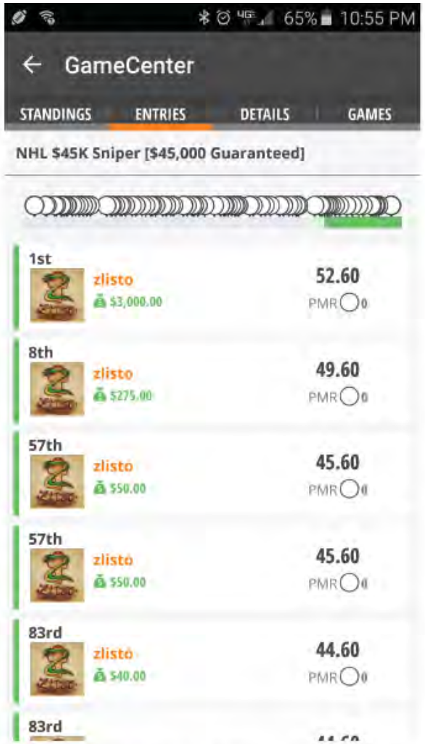
November 16, 2015



NHL $80K Tuesday Special [$80,000 Guaranteed]

3rd	zlisto	$3,000.00	54.60	PMR 0
6th	zlisto	$1,000.00	52.80	PMR 0
7th	zlisto	$800.00	52.30	PMR 0
10th	zlisto	$600.00	50.60	PMR 0
11th	zlisto	$500.00	50.30	PMR 0
15th	zlisto		50.10	

November 17, 2015



NHL $45K Sniper [$45,000 Guaranteed]

1st	zlisto	$3,000.00	52.60	PMR 0
8th	zlisto	$275.00	49.60	PMR 0
57th	zlisto	$50.00	45.60	PMR 0
57th	zlisto	$50.00	45.60	PMR 0
83rd	zlisto	$40.00	44.60	PMR 0
83rd				

November 23, 2015

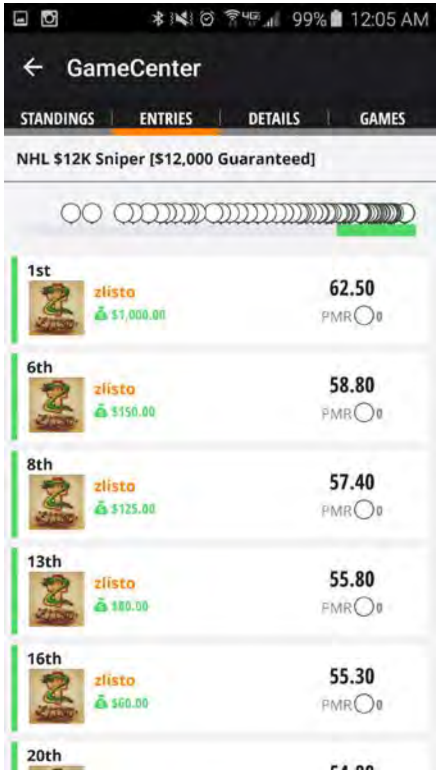
200 lineups

Policy Change

Policy Change

200 lineups -> 100 lineups

Were we able to continue it?



NHL $12K Sniper [$12,000 Guaranteed]

1st	zlisto	$1,000.00	62.50	PMR ○ 0
6th	zlisto	$150.00	58.80	PMR ○ 0
8th	zlisto	$125.00	57.40	PMR ○ 0
13th	zlisto	$80.00	55.80	PMR ○ 0
16th	zlisto	$60.00	55.30	PMR ○ 0
20th				

December 12, 2015

100 lineups

Legal Disclaimer: All profits are in the process of being donated to charity.

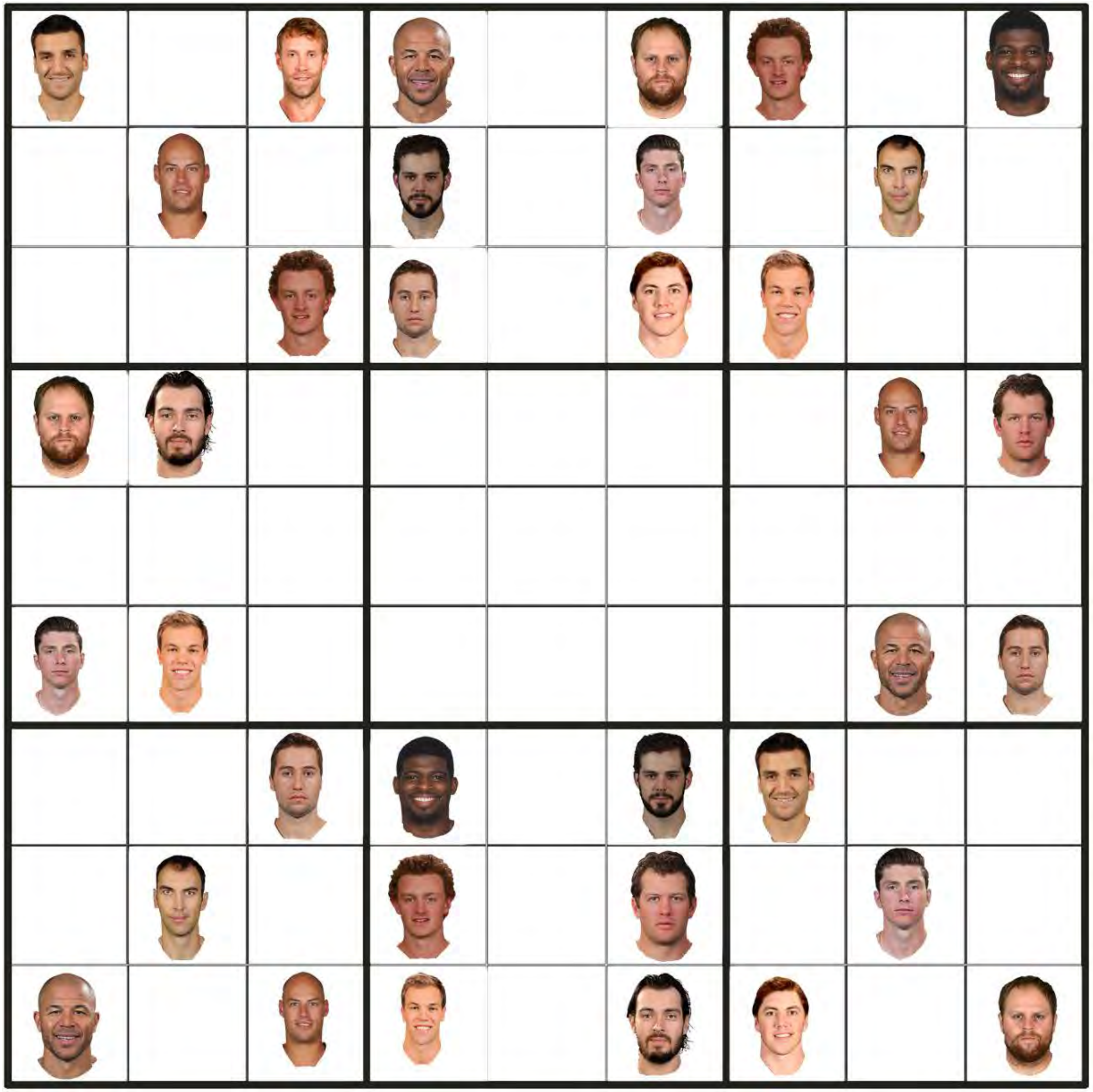
Integer Programming Formulation

- We will make a bunch of lineups consisting of 9 players each
- Use an integer programming approach to find these lineups

Decision variables

$$x_{pl} = \begin{cases} 1, & \text{if player } p \text{ in lineup } l \\ 0, & \text{otherwise} \end{cases}$$

First Attempt...



Basic Feasibility

- 9 different players
- Salary less than $50,000

Basic constraints

$$\sum_{p=1}^{N} c_p x_{pl} \leq \$50,000, \quad \text{(budget constraint)}$$

$$\sum_{p=1}^{N} x_{pl} = 9, \quad \text{(lineup size constraint)}$$

$$x_{pl} \in \{0,1\}, \quad 1 \leq p \leq N.$$

Position Feasibility

- Between 2 and 3 centers
- Between 3 and 4 wingers
- Between 2 and 3 defensemen
- 1 goalie

Position constraints

$$2 \leq \sum_{p \in C} x_{pl} \leq 3, , \quad \text{(center constraint)}$$

$$3 \leq \sum_{p \in W} x_{pl} \leq 4, \quad \text{(winger constraint)}$$

$$2 \leq \sum_{p \in D} x_{pl} \leq 3, \quad \text{(defensemen constraint)}$$

$$\sum_{u \in G} x_{pl} = 1 \quad \text{(goalie constraint)}$$

Team Feasibility

- At least 3 different NHL teams

Team constraints

$$
\begin{aligned}
& t_i \leq \sum_{p \in T_i} x_{pl}, \qquad \forall\, i \in \{1,\ \ldots\ , N_T\} \\
& \sum_{i=1}^{N_T} t_i \geq 3, \\
& t_i \in \{0,\ 1\}, \qquad \forall\, i \in \{1,\ \ldots\ , N_T\}.
\end{aligned}
$$

First Attempt...

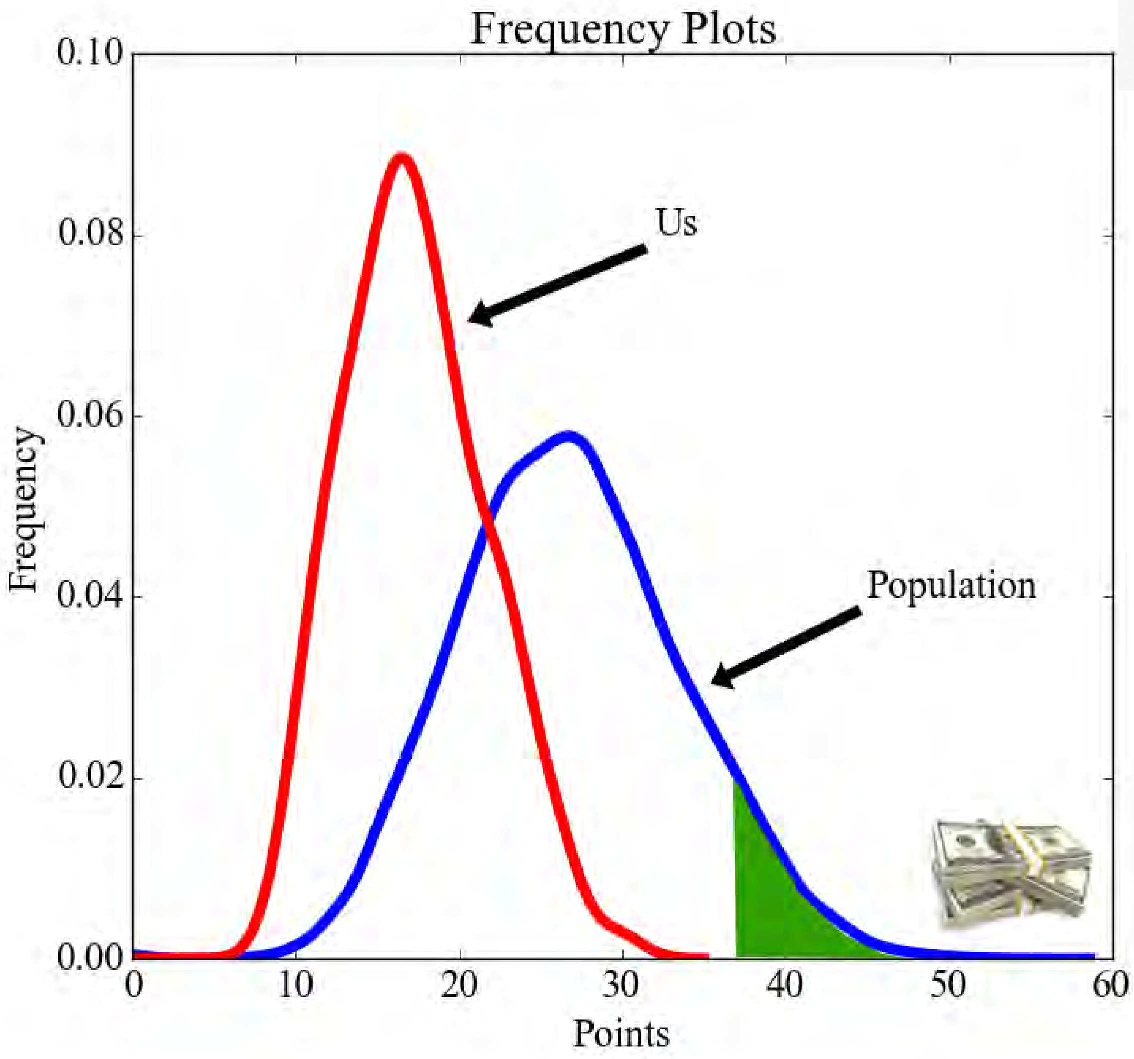
$6400	$7200	$4200	$4100	$5300	$4400	$4800	$5800	$7800
W	UTIL	D	D	C	C	W	W	G

Feasible...

But Not Good

> 3 Different Teams

First Attempt…



Second Attempt…

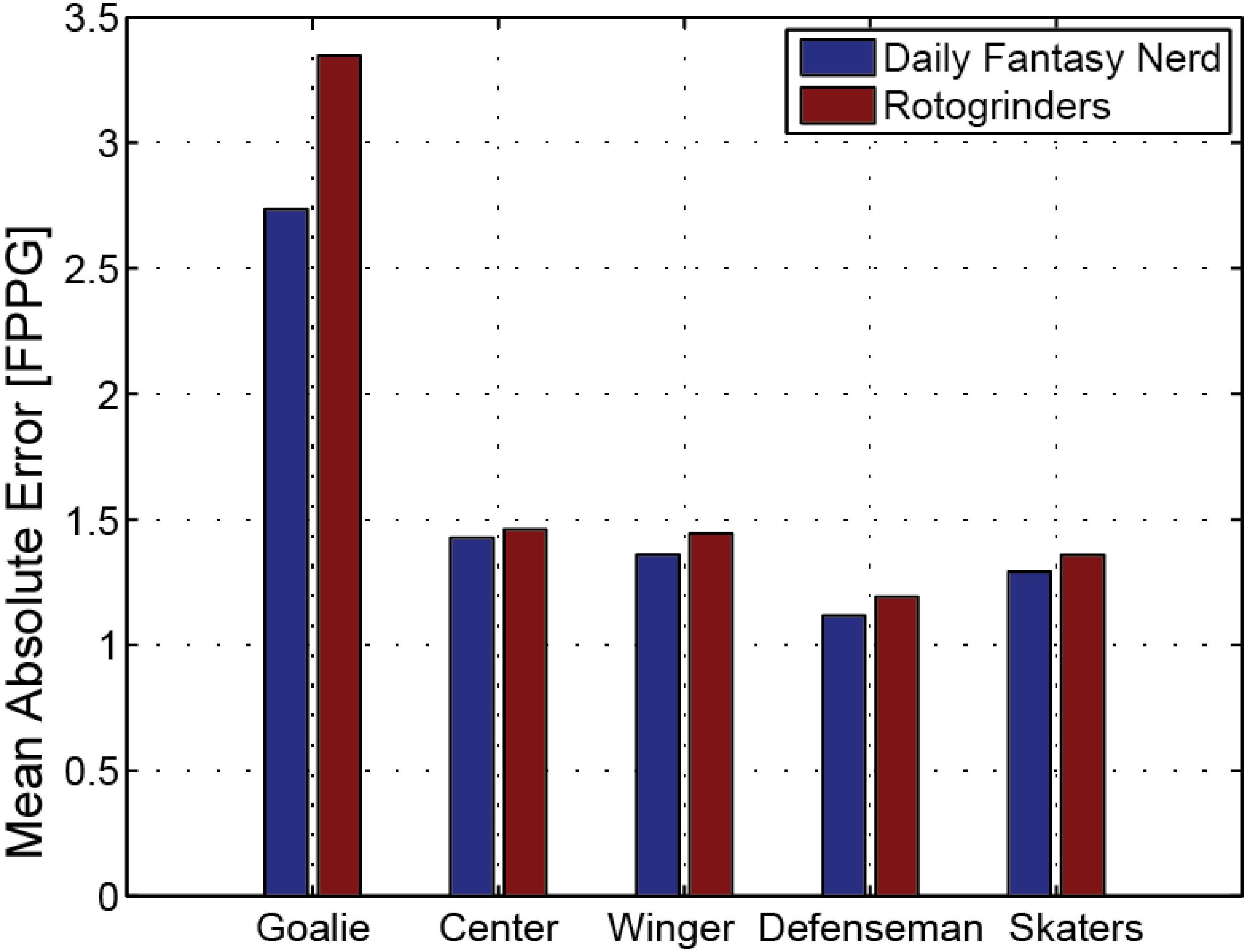
- Must increase our mean points…
- Solution:

USE EXPERT PREDICTIONS

RotoGrinders
The Daily Fantasy Games Authority

dfnerd

Prediction Errors



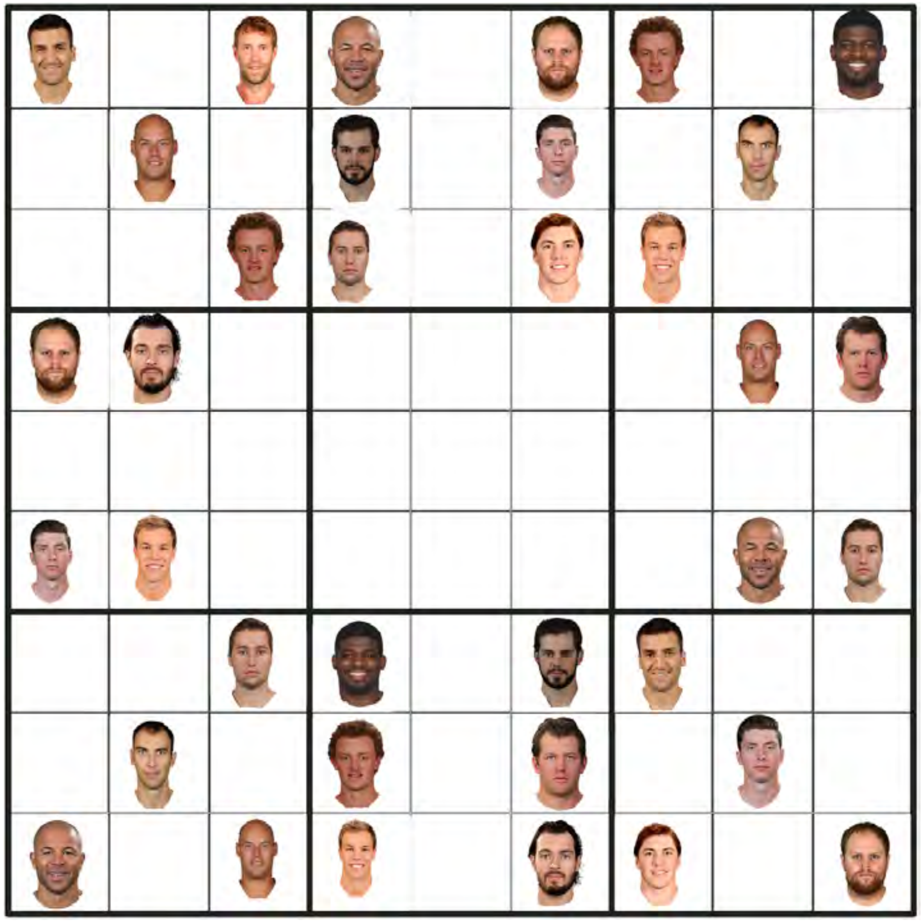
Maximize Points

- Forecasted points for player p: f_p
- You get to choose what the forecasts are

Points Objective Function

$$\sum_{p=1}^{N} f_p x_{pl}$$

Second Attempt…



+ Maximize points

$$\max_{x} \sum_{p=1}^{N} f_p x_{pl}$$

Old Lineup

$6400	$7200	$4200	$4100	$5300	$4400	$4800	$5800	$7800
W	UTIL	D	D	C	C	W	W	G

Feasible…

But Not Good

12 points on average

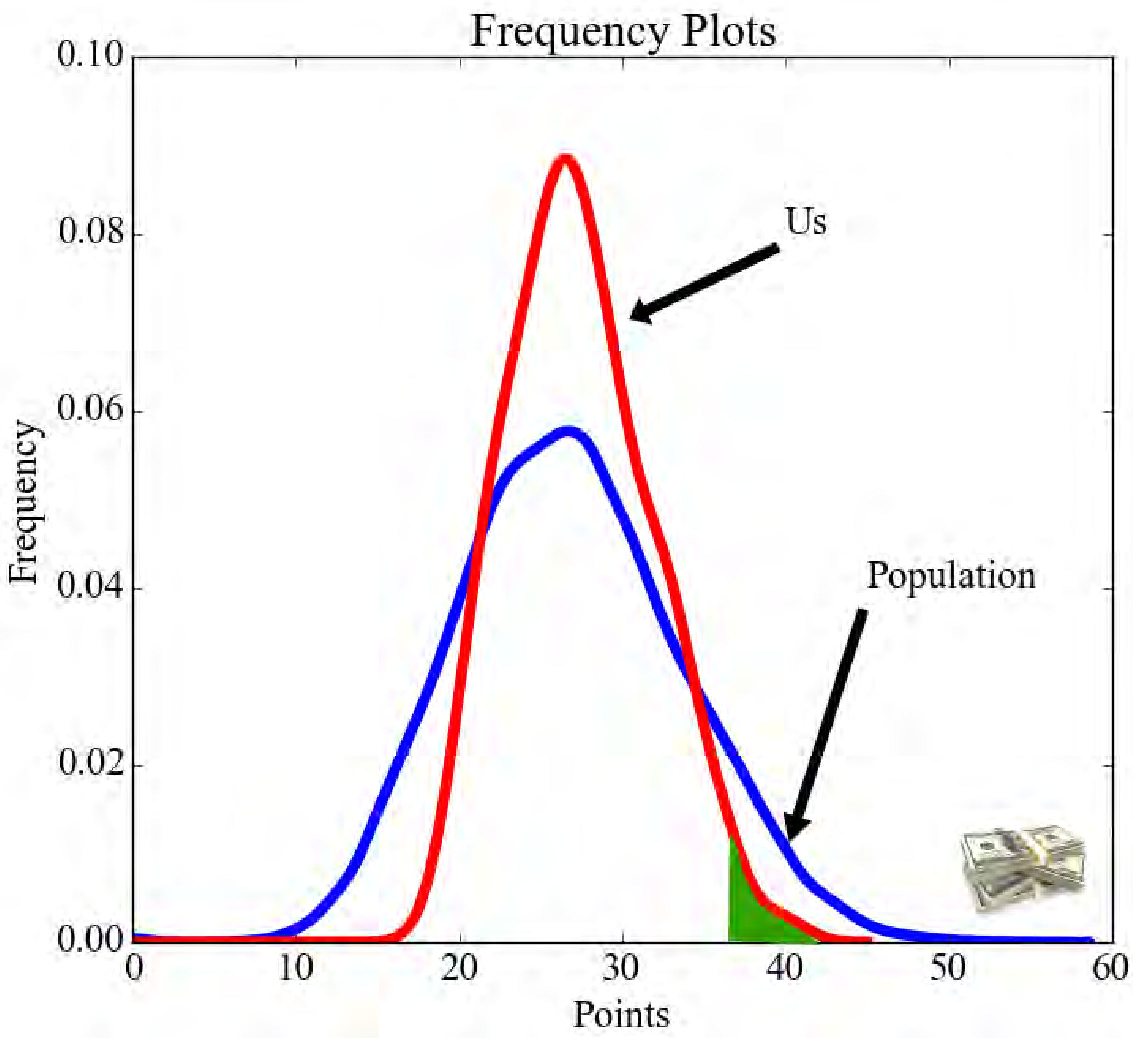
New Lineup

Projections:	5.4	2.5	3.4	3.0	3.2	4.2	3.5	3.4	5.7
	$9500	$2700	$4600	$3800	$4600	$6400	$5200	$5100	$8000
	W	UTIL	D	D	C	C	W	W	G

Better...

23 points on average

Second Attempt…

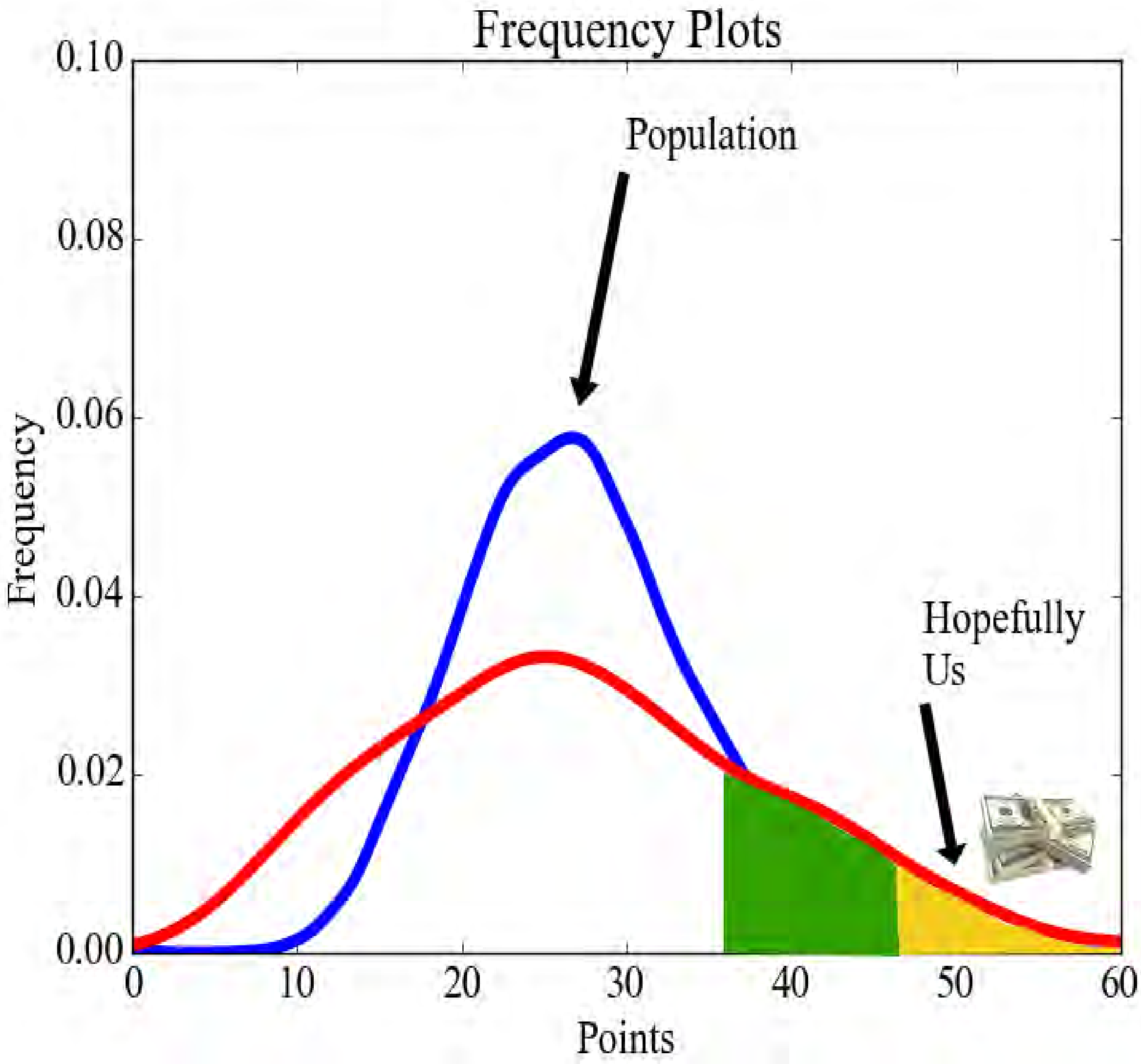


How can we do better?

- Three D’s of Finance:

So what do we do?

By doing this…

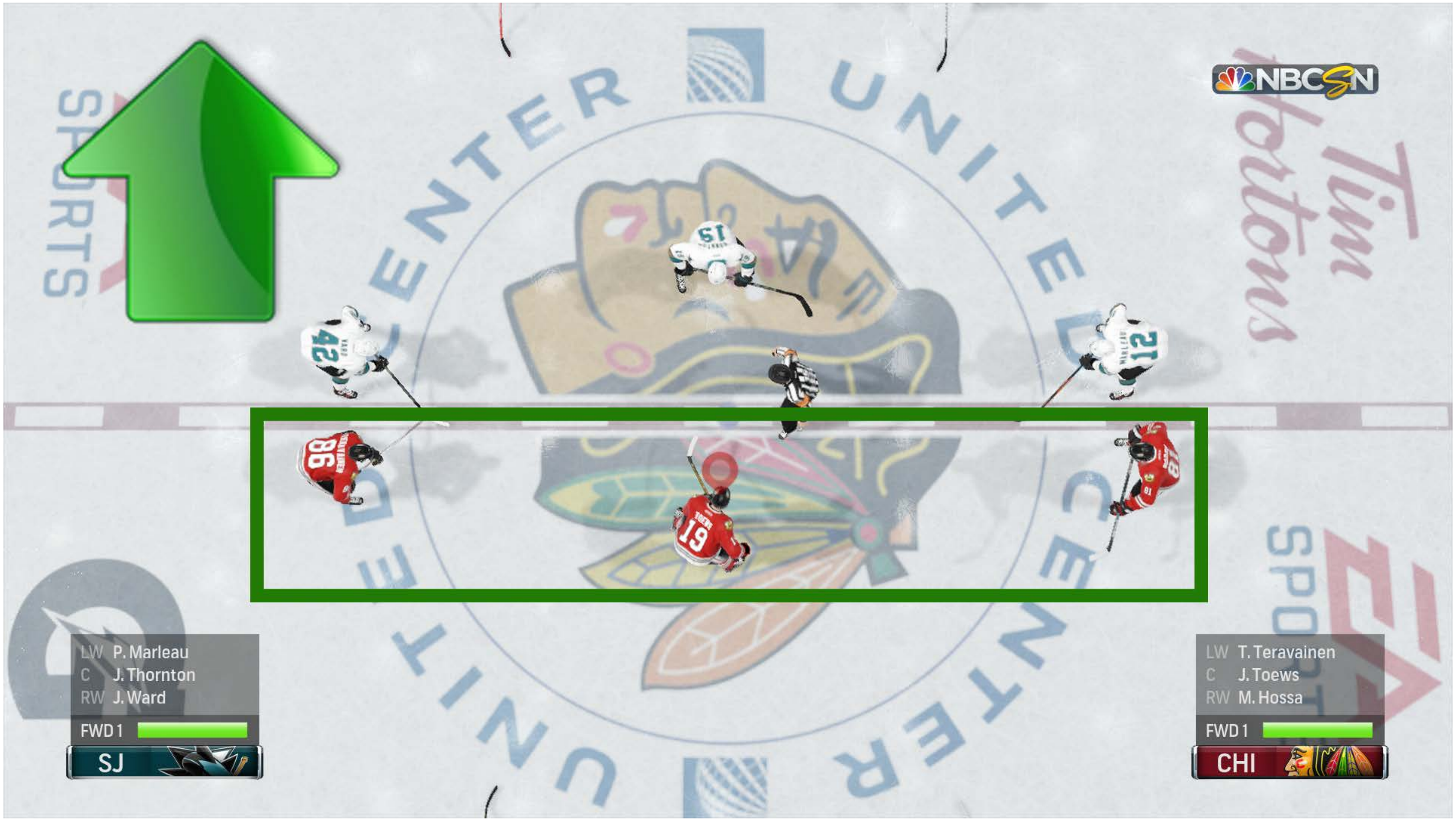


Stacking Lineups

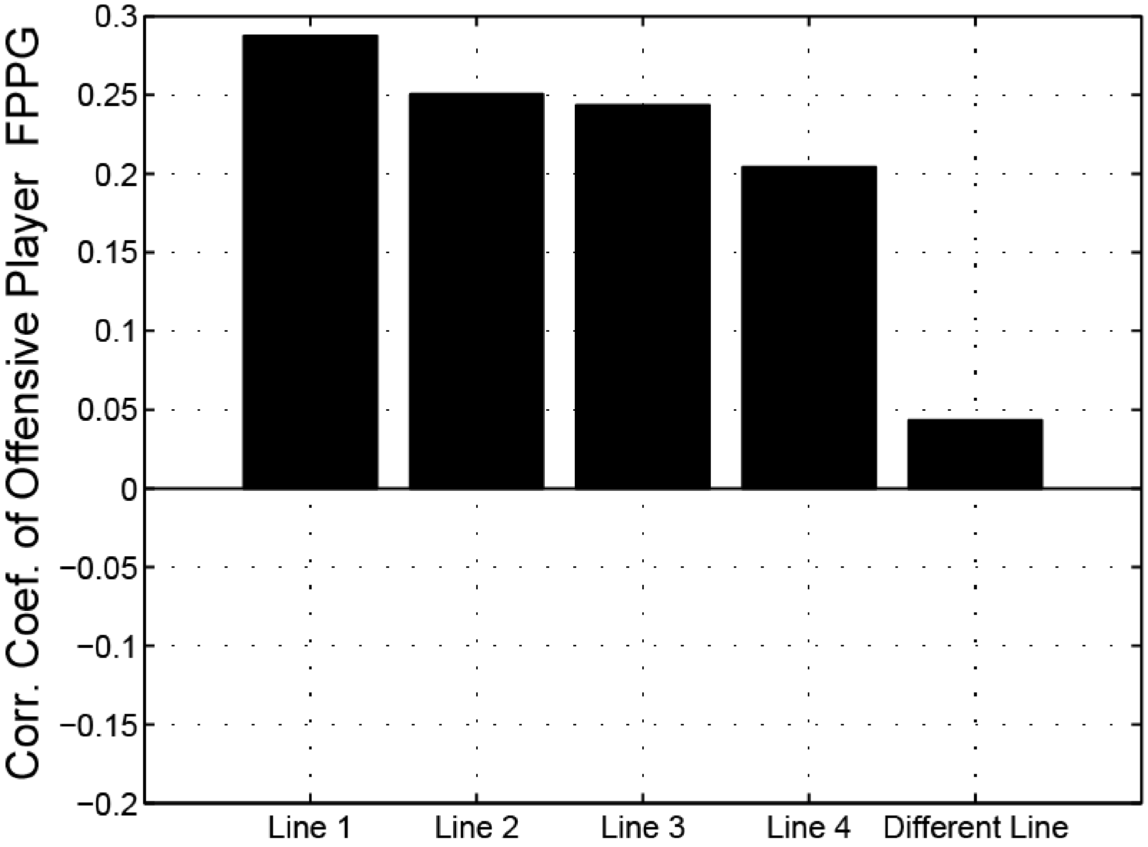
- Stacking means putting players on a single lineup that have a positive correlation
- Either the players pop off together -> tons of points
- Or the players crap out -> few points
- We stack using “structural correlations”

Structural Correlations - Teams

Structural Correlations - Lines



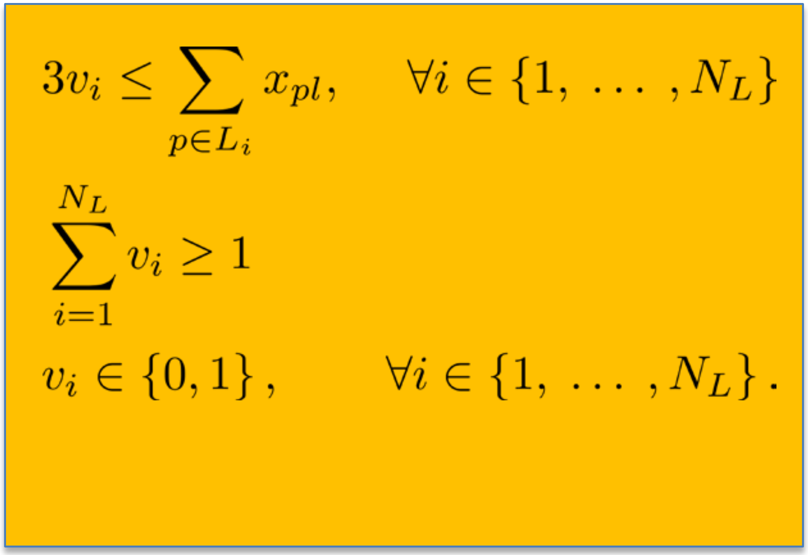
Structural Correlations - Lines



Structural Correlations - Lines

- At least 1 complete line (3 players per line)
- At least 2 partial lines (at least 2 players per line)

1 complete line constraint

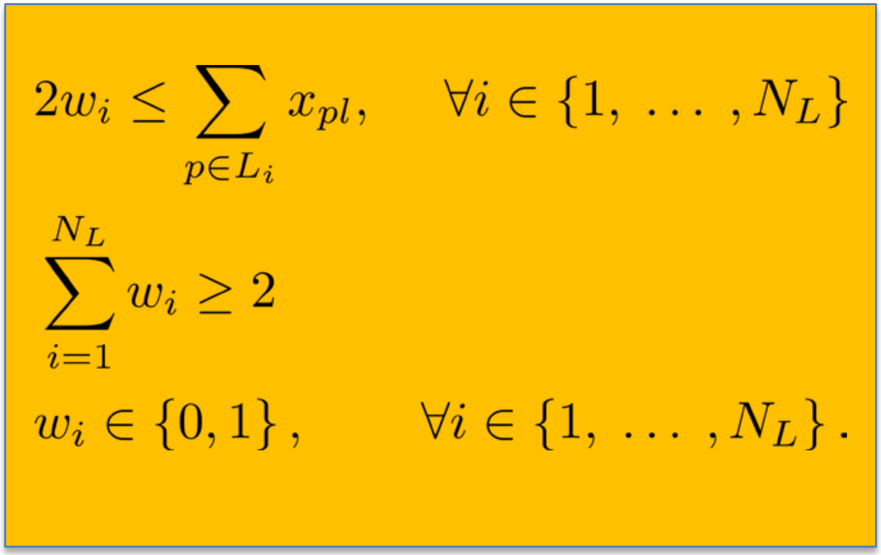


$$3v_i \leq \sum_{p \in L_i} x_{pl}, \qquad \forall i \in \{1, \ldots, N_L\}$$

$$\sum_{i=1}^{N_L} v_i \geq 1$$

$$v_i \in \{0,1\}, \qquad \forall i \in \{1, \ldots, N_L\}.$$

2 partial lines constraint



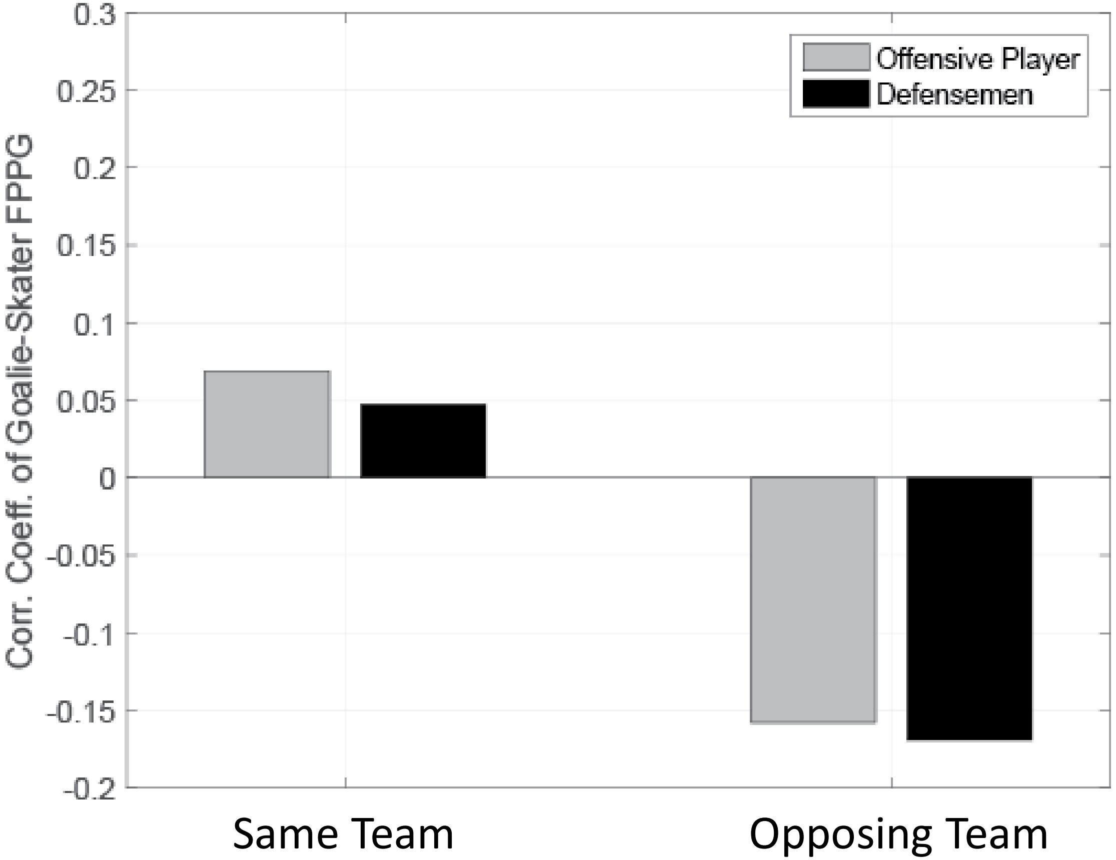
$$2w_i \leq \sum_{p \in L_i} x_{pl}, \qquad \forall i \in \{1, \ldots, N_L\}$$

$$\sum_{i=1}^{N_L} w_i \geq 2$$

$$w_i \in \{0,1\}, \qquad \forall i \in \{1, \ldots, N_L\}.$$

Structural Correlations – Goalie Against Opposing Players

Structural Correlations – Goalie Against Opposing Players

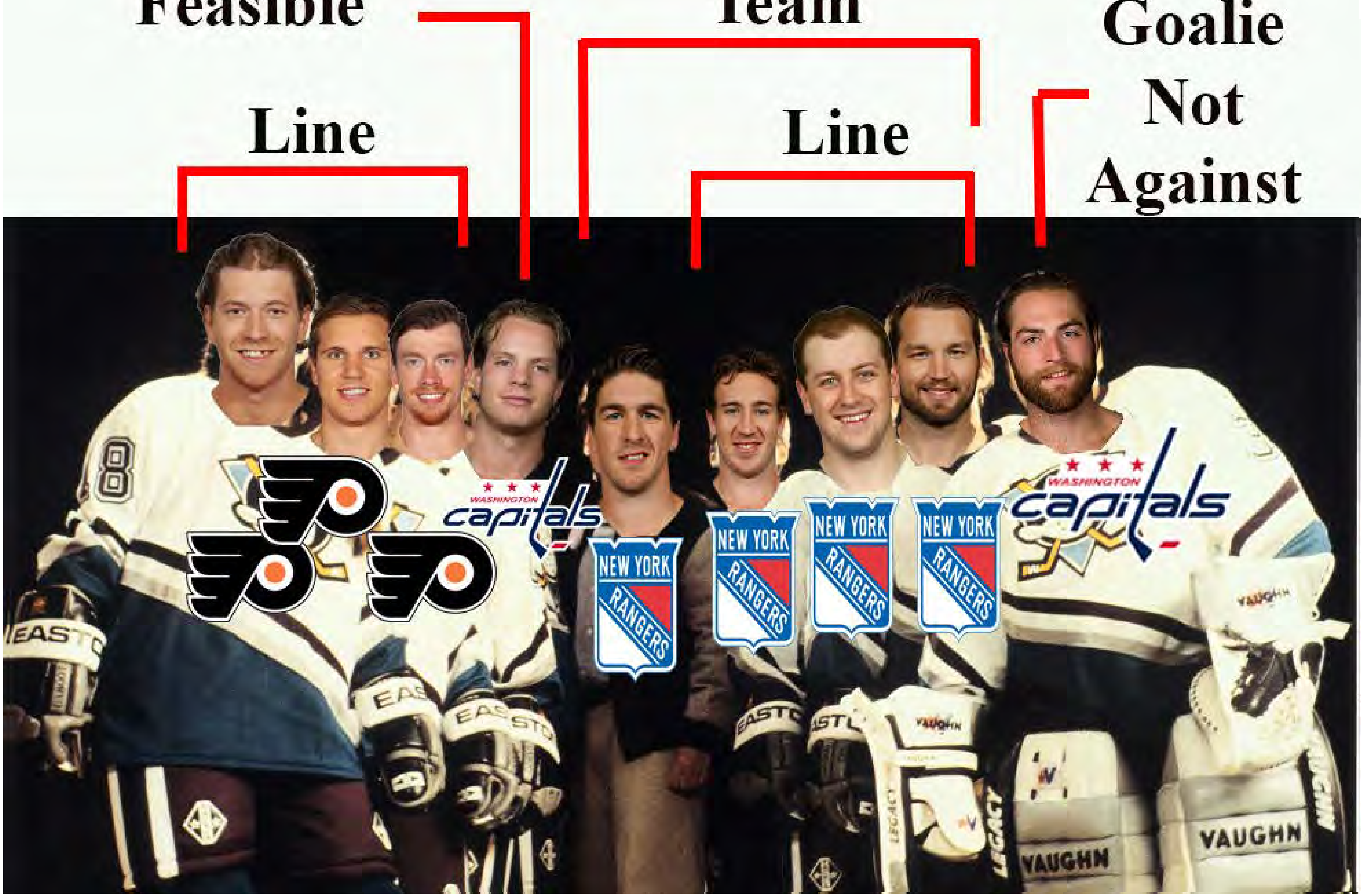


Structural Correlations – Goalie Against Skaters

- No skater against goalie

No skater against goalie constraint

$$6x_{pl} + \sum_{q \in Opponents_p} x_{ql} \leq 6, \quad \forall p \in G$$

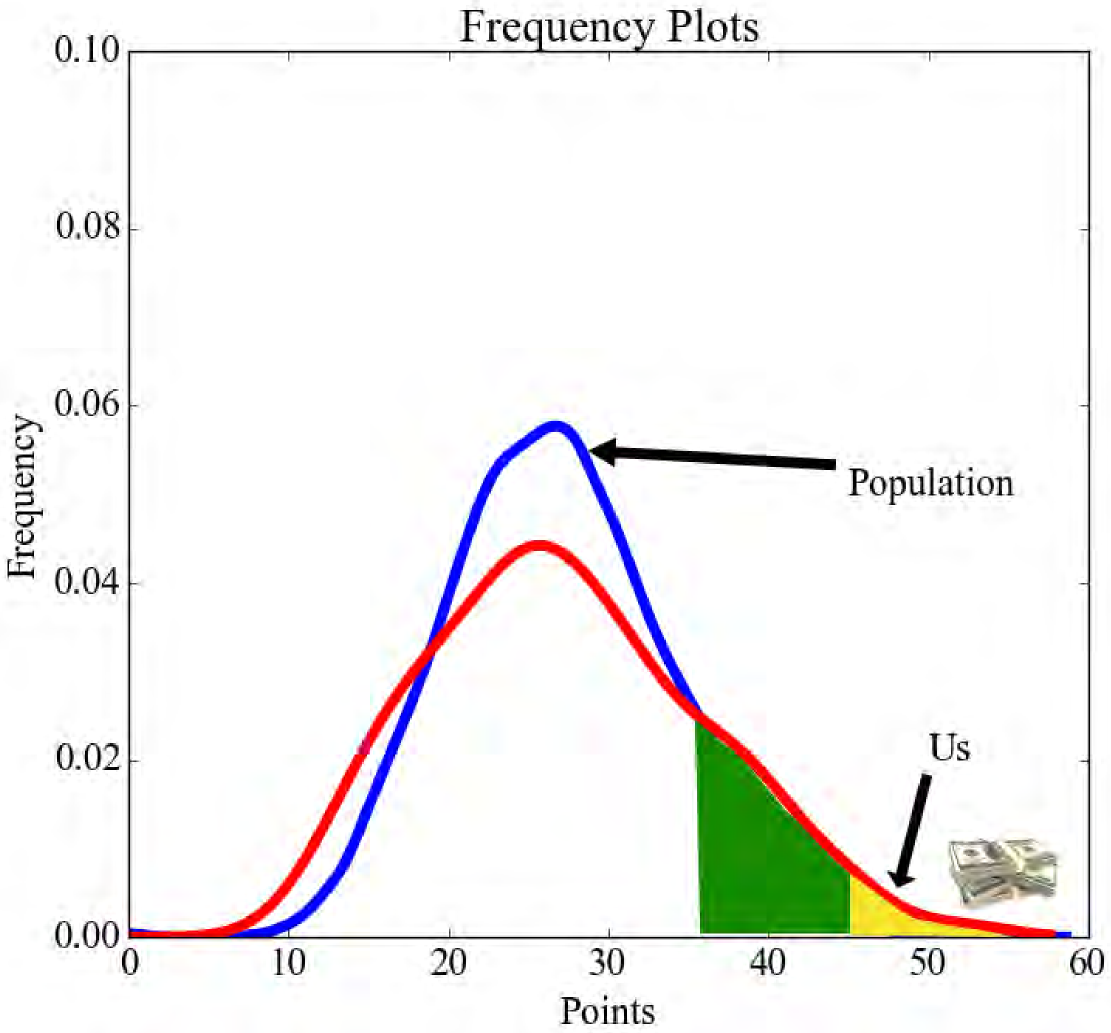


Feasible Line

Team Line

Goalie Not Against

Second Attempt…



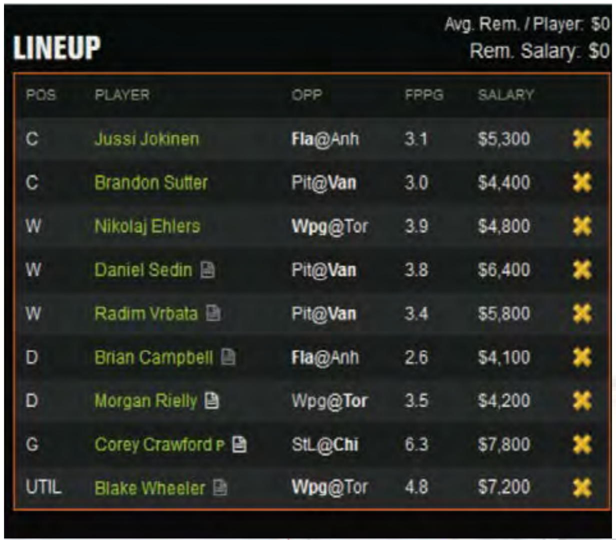
Lineup Diversity

- Make sure lineup l has no more than γ players in common with lineups 1 to l-1

Diversity constraint

$$\sum_{p=1}^{N} x^*_{pk} x_{pl} \leq \gamma, k = 1, \ldots, l - 1$$

To Review…



LINEUP

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Rem. Salary: $0

POS	PLAYER	OPP	FPPG	SALARY	
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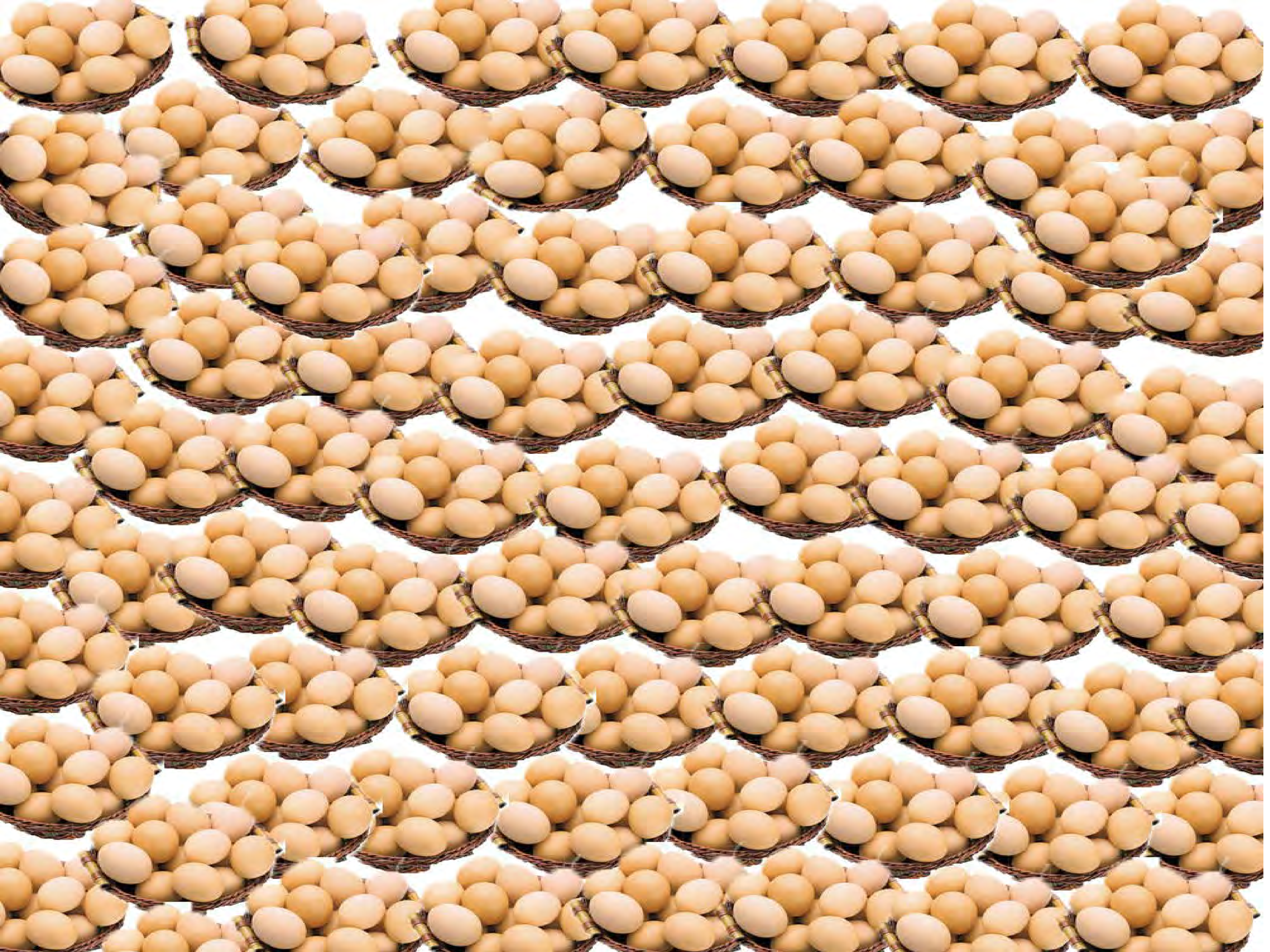
W	Nikolaj Ehlers	Wpg@Tor	3.9	$4,800	✖
W	Daniel Sedin	Pit@Van	3.8	$6,400	✖
W	Radim Vrbata	Pit@Van	3.4	$5,800	✖
D	Brian Campbell	Fla@Anh	2.6	$4,100	✖
D	Morgan Rielly	Wpg@Tor	3.5	$4,200	✖
G	Corey Crawford P	StL@Chi	6.3	$7,800	✖
UTIL	Blake Wheeler	Wpg@Tor	4.8	$7,200	✖

RotoGrinders.com

dfnerd

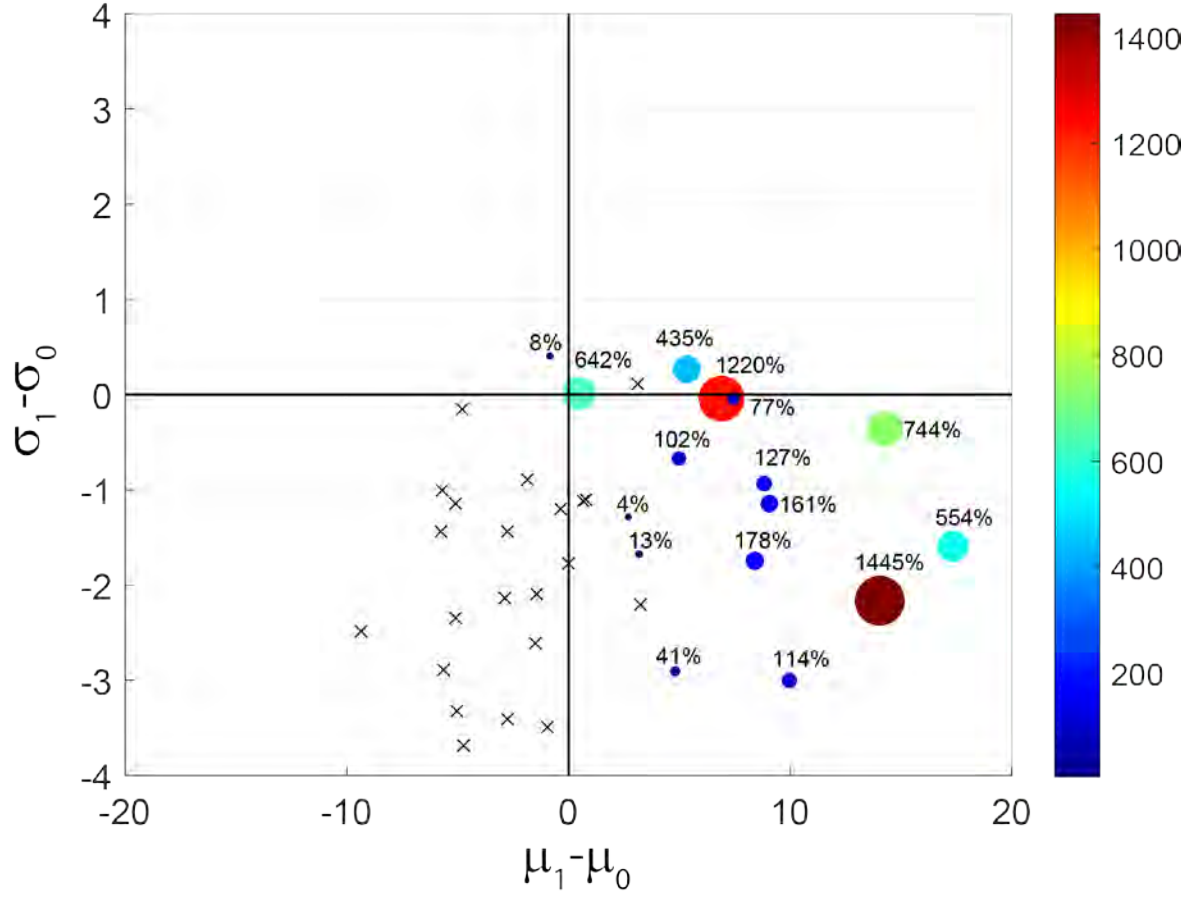


PERFORMANCE ON REAL CONTESTS

Performance on Real Contests

- Each point is a contest, with profit margin shown
- Used all stacking, a maximum overlap of 7, and 200 lineups

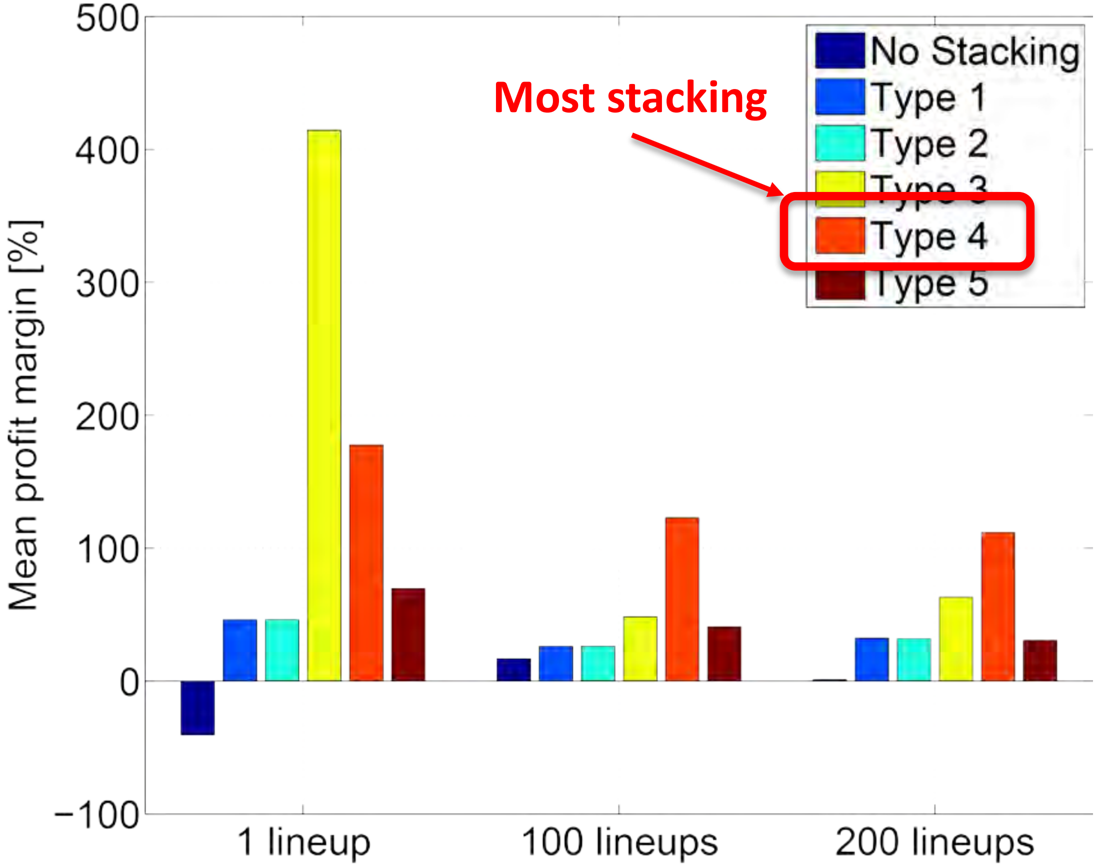
Edge in standard deviation (fantasy points)



Edge in mean (fantasy points)

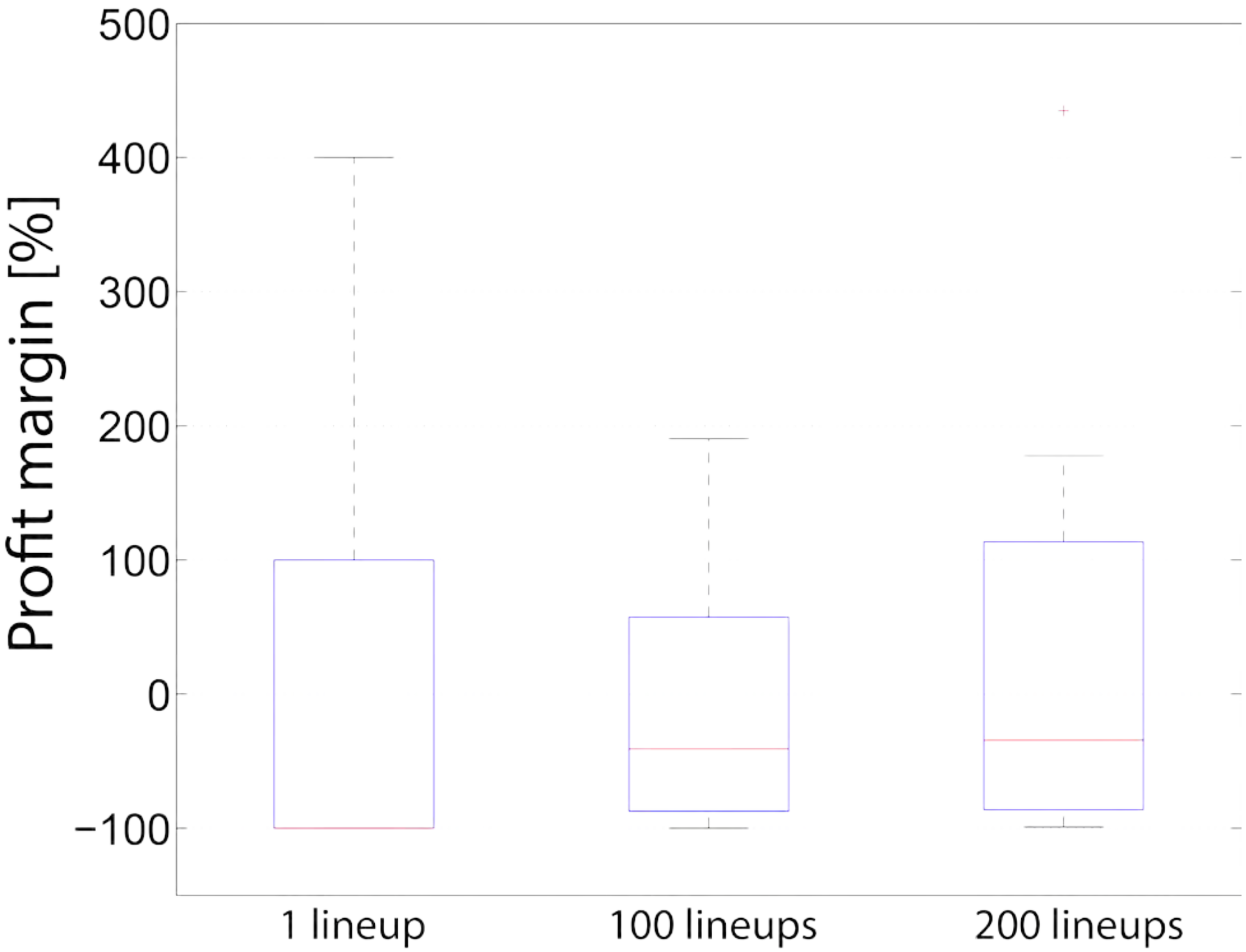
Impact of Stacking

- Used a maximum overlap of 7, and 200 lineups



More stacking is better

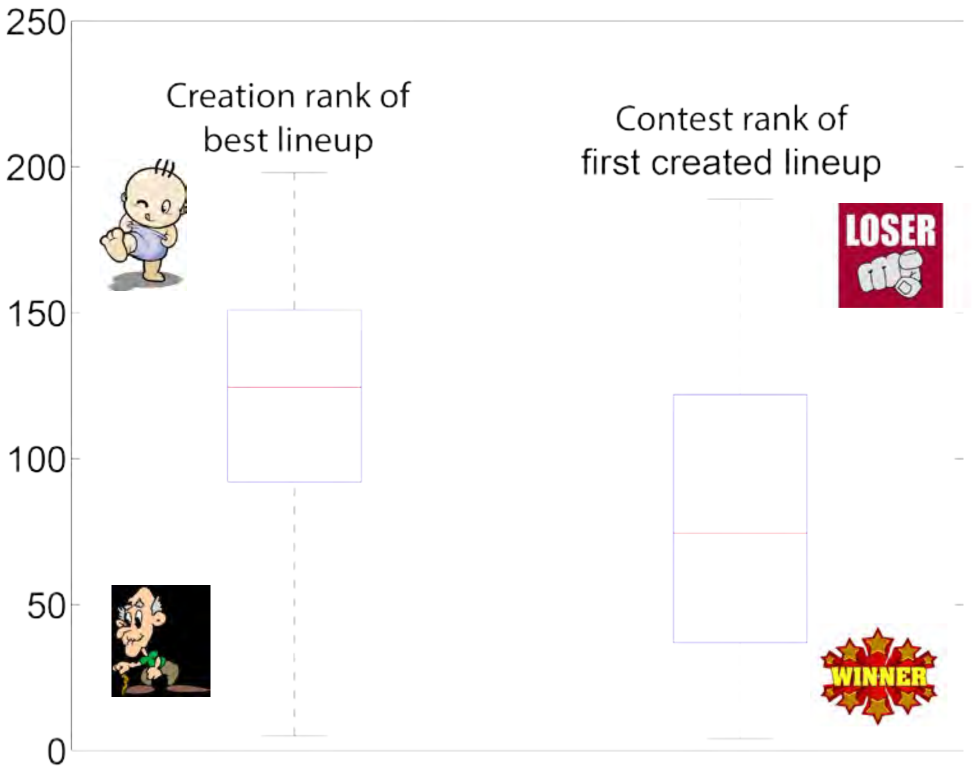
Impact of Number of Lineups



More lineups is better

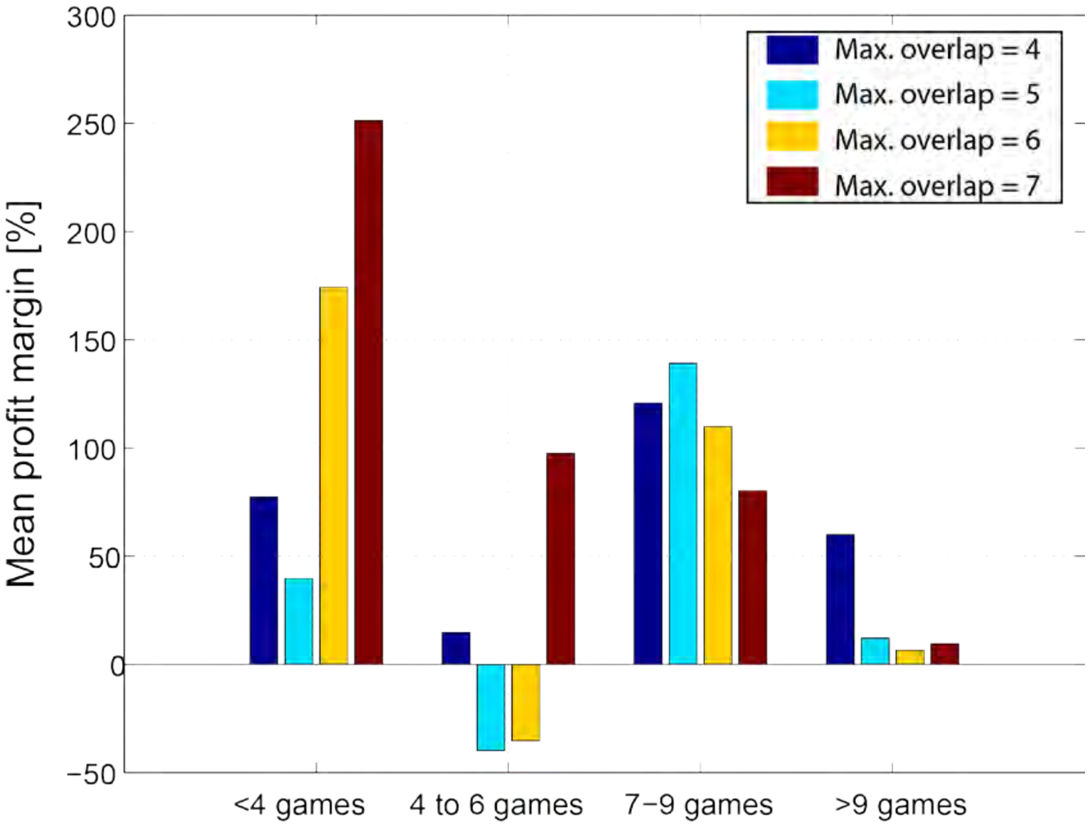
Impact of Lineup Birth Order

- We create lineups sequentially
- Are the best lineups the "oldest" lineups?



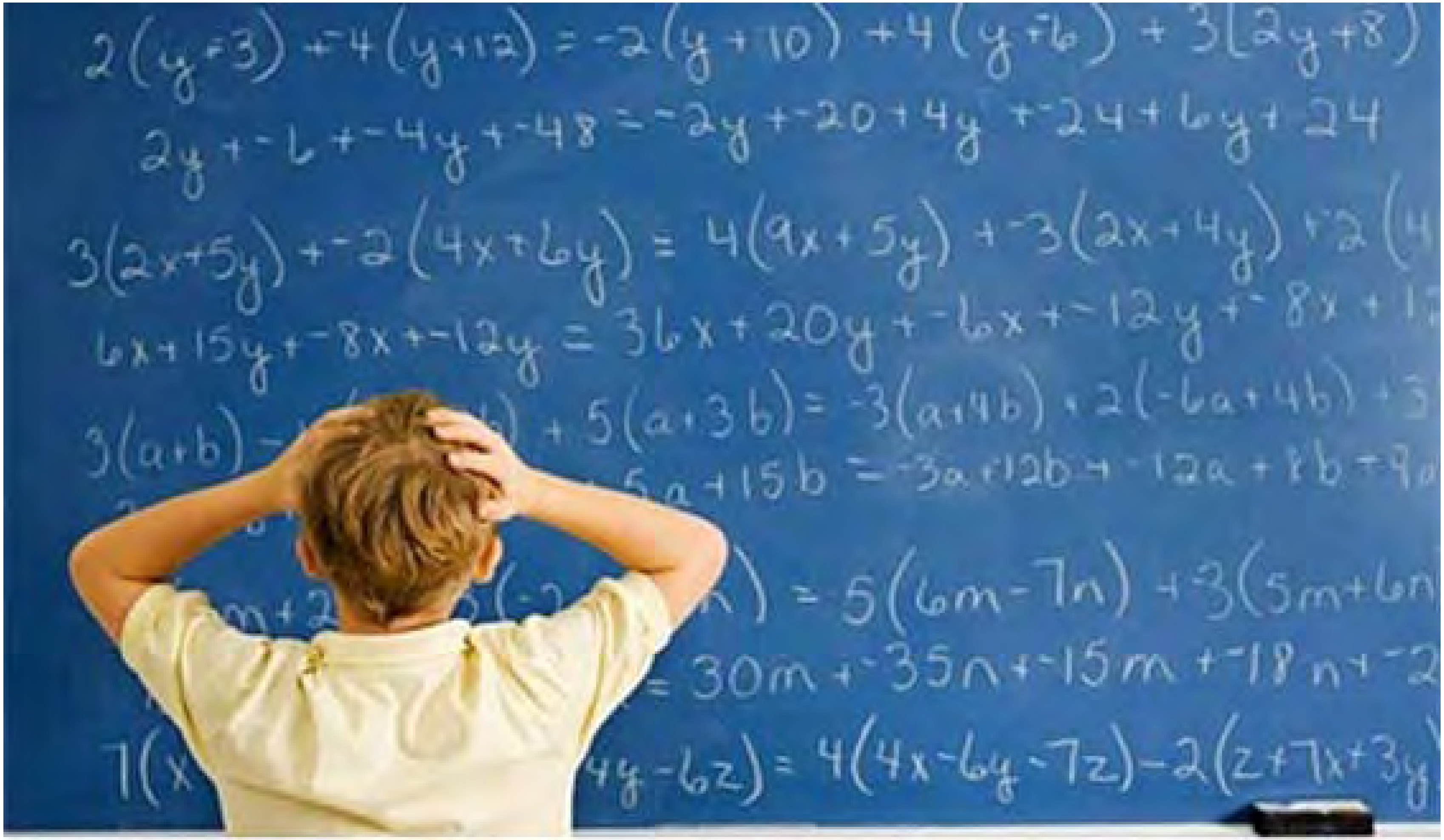
First lineup isn't usually the best lineup

Impact of Diversity



More games -> Use more diversity

How can you do it?


2(y+3) + -4(y+12) = -2(y+10) +4(y+6) + 3(2y+8)
2y + -6 + -4y + -48 = -2y + -20 + 4y + -24 + 6y + 24
3(2x+5y) + -2(4x+6y) = 4(9x+5y) + -3(2x+4y) + 2(4
6x+15y + -8x + -12y = 36x + 20y + -6x + -12y + -8x + 1
3(a+b) + 5(a+3b) = -3(a+4b) + 2(-6a+4b) + 3
6a+15b = -3a+12b + -12a + 8b + 9a
m+2 = 5(6m-7n) + 3(5m+6n
= 30m + -35n + -15m + -18n + -2
7(x 4y-6z) = 4(4x-6y-7z) - 2(z+7x+3y

Lineup Construction Procedure

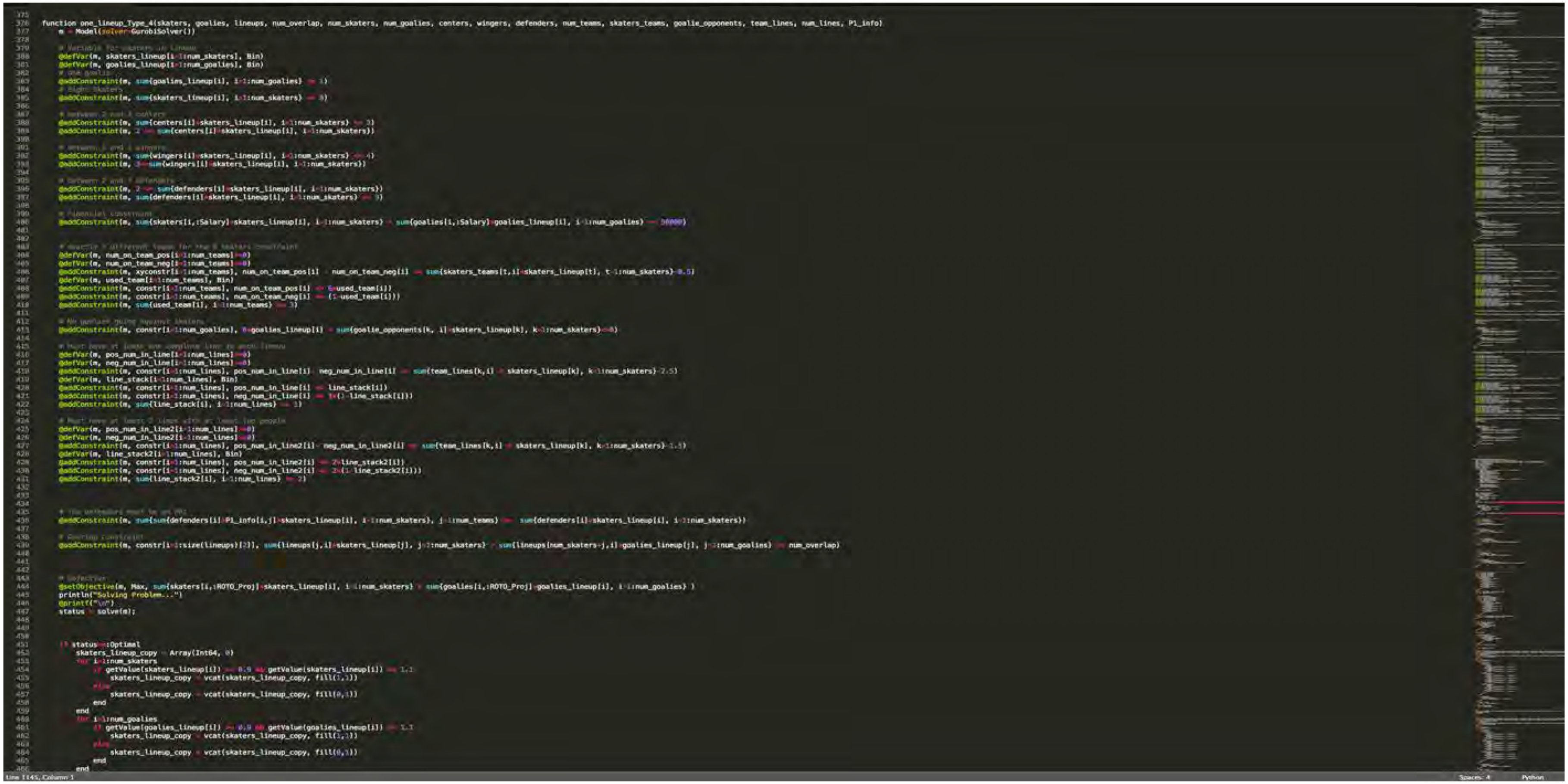
- Get projection data
  - Make sure you wait until the starting goalies are announced
- Solve integer program for each lineup one at a time
  - But add in the new diversity constraints for each new lineup

< 30 Minutes

How can you do it?

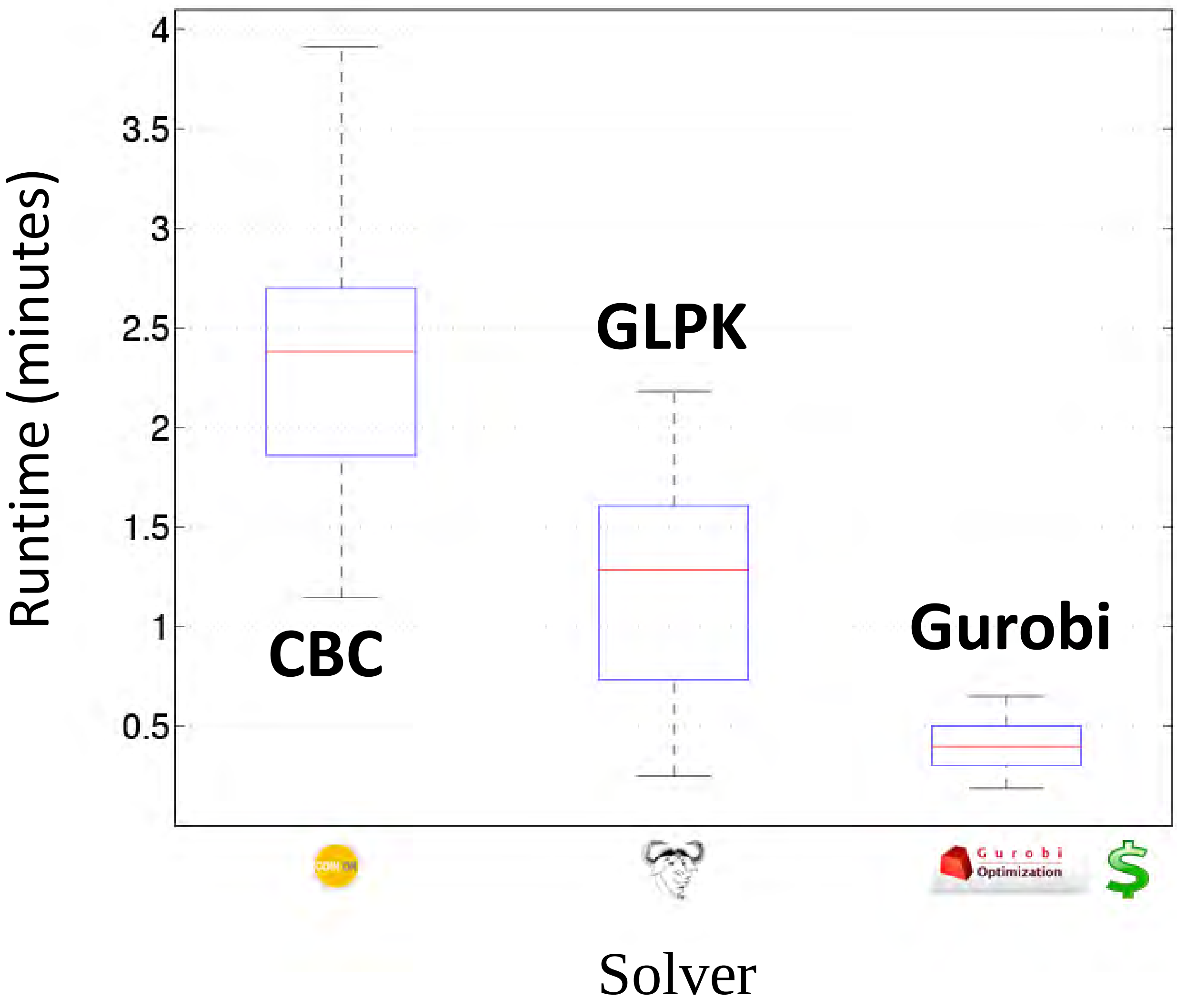
Download Code from Github:

https://github.com/dscotthunter/Fantasy-Hockey-IP-Code



```
function one_lineup_Type_4(skaters, goalies, lineups, num_overlap, num_skaters, num_goalies, centers, wingers, defenders, num_teams, skaters_teams, goalie_opponents, team_lines, num_lines, P1_info)
    m = Model(solver=GurobiSolver())

    # Variable for skaters in lineup
    @defVar(m, skaters_lineup[i=1:num_skaters], Bin)
    @defVar(m, goalies_lineup[i=1:num_goalies], Bin)
    # One goalie
    @addConstraint(m, sum{goalies_lineup[i], i=1:num_goalies} == 1)
    # Eight Skaters
    @addConstraint(m, sum{skaters_lineup[i], i=1:num_skaters} == 8)

    # between 2 and 3 centers
    @addConstraint(m, sum{centers[i]*skaters_lineup[i], i=1:num_skaters} <= 3)
    @addConstraint(m, 2 <= sum{centers[i]*skaters_lineup[i], i=1:num_skaters})

    # between 3 and 4 wingers
    @addConstraint(m, sum{wingers[i]*skaters_lineup[i], i=1:num_skaters} <= 4)
    @addConstraint(m, 3<=sum{wingers[i]*skaters_lineup[i], i=1:num_skaters})

    # between 2 and 3 defenders
    @addConstraint(m, 2 <= sum{defenders[i]*skaters_lineup[i], i=1:num_skaters})
    @addConstraint(m, sum{defenders[i]*skaters_lineup[i], i=1:num_skaters} <= 3)

    # Financial Constraint
    @addConstraint(m, sum{skaters[i,:Salary]*skaters_lineup[i], i=1:num_skaters} + sum{goalies[i,:Salary]*goalies_lineup[i], i=1:num_goalies} <= 50000)


    # exactly 3 different teams for the 8 skaters constraint
    @defVar(m, num_on_team_pos[i=1:num_teams]>=0)
    @defVar(m, num_on_team_neg[i=1:num_teams]>=0)
    @addConstraint(m, xyconstr[i=1:num_teams], num_on_team_pos[i] - num_on_team_neg[i] == sum{skaters_teams[t,i]*skaters_lineup[t], t=1:num_skaters}-0.5)
    @defVar(m, used_team[i=1:num_teams], Bin)
    @addConstraint(m, constr[i=1:num_teams], num_on_team_pos[i] <= 6*used_team[i])
    @addConstraint(m, constr[i=1:num_teams], num_on_team_neg[i] <= (1-used_team[i]))
    @addConstraint(m, sum{used_team[i], i=1:num_teams} == 3)

    # No goalies going against skaters
    @addConstraint(m, constr[i=1:num_goalies], 6*goalies_lineup[i] + sum{goalie_opponents[k, i]*skaters_lineup[k], k=1:num_skaters}<=6)

    # Must have at least one complete line in each lineup
    @defVar(m, pos_num_in_line[i=1:num_lines]>=0)
    @defVar(m, neg_num_in_line[i=1:num_lines]>=0)
    @addConstraint(m, constr[i=1:num_lines], pos_num_in_line[i]-neg_num_in_line[i] == sum{team_lines[k,i] * skaters_lineup[k], k=1:num_skaters}-2.5)
    @defVar(m, line_stack[i=1:num_lines], Bin)
    @addConstraint(m, constr[i=1:num_lines], pos_num_in_line[i] <= line_stack[i])
    @addConstraint(m, constr[i=1:num_lines], neg_num_in_line[i] <= 3*(1-line_stack[i]))
    @addConstraint(m, sum{line_stack[i], i=1:num_lines} >= 1)

    # Must have at least 2 lines with at least two people
    @defVar(m, pos_num_in_line2[i=1:num_lines]>=0)
    @defVar(m, neg_num_in_line2[i=1:num_lines]>=0)
    @addConstraint(m, constr[i=1:num_lines], pos_num_in_line2[i]-neg_num_in_line2[i] == sum{team_lines[k,i] * skaters_lineup[k], k=1:num_skaters}-1.5)
    @defVar(m, line_stack2[i=1:num_lines], Bin)
    @addConstraint(m, constr[i=1:num_lines], pos_num_in_line2[i] <= 2*line_stack2[i])
    @addConstraint(m, constr[i=1:num_lines], neg_num_in_line2[i] <= 2*(1-line_stack2[i]))
    @addConstraint(m, sum{line_stack2[i], i=1:num_lines} >= 2)



    # The defenders must be on PP1
    @addConstraint(m, sum{sum{defenders[i]*P1_info[i,j]*skaters_lineup[i], i=1:num_skaters}, j=1:num_teams} ==  sum{defenders[i]*skaters_lineup[i], i=1:num_skaters})

    # Overlap Constraint
    @addConstraint(m, constr[i=1:size(lineups)[2]], sum{lineups[j,i]*skaters_lineup[j], j=1:num_skaters} + sum{lineups[num_skaters+j,i]*goalies_lineup[j], j=1:num_goalies} <= num_overlap)



    # Objective
    @setObjective(m, Max, sum{skaters[i,:ROTO_Proj]*skaters_lineup[i], i=1:num_skaters} + sum{goalies[i,:ROTO_Proj]*goalies_lineup[i], i=1:num_goalies} )
    println("Solving Problem...")
    @printf("\n")
    status = solve(m);



    if status==:Optimal
        skaters_lineup_copy = Array(Int64, 0)
        for i=1:num_skaters
            if getValue(skaters_lineup[i]) >= 0.9 && getValue(skaters_lineup[i]) <= 1.1
                skaters_lineup_copy = vcat(skaters_lineup_copy, fill(1,1))
            else
                skaters_lineup_copy = vcat(skaters_lineup_copy, fill(0,1))
            end
        end
        for i=1:num_goalies
            if getValue(goalies_lineup[i]) >= 0.9 && getValue(goalies_lineup[i]) <= 1.1
                skaters_lineup_copy = vcat(skaters_lineup_copy, fill(1,1))
            else
                skaters_lineup_copy = vcat(skaters_lineup_copy, fill(0,1))
            end
        end
```

Performance Time



In the paper…

- Consider several strategies
- Different Integer Programming formulations
- Varying prediction models
- Number of lineups
- http://arxiv.org/pdf/1604.01455v1.pdf

YOU CAN DO IT